TIGHT BINDING BOOK

ELEMENTARY EXPERIMENTAL STATICS FOR SCHOOLS

WRITTEN BY

A. P. McMULLEN, M.A.

LATE HEAD OF THE SCIENCE DEPARTMENT,
ROYAL NAVAL COLLEGE, DARTMOUTH

REVISED FOR THE PRESS, WITH SOME ADDITIONAL MATTER AND A PREFACE

BY

E. W. E. KEMPSON, B.A.

LATE ASSISTANT MASTER AT RUGBY SCHOOL

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PREFACE

THIS book treats from an experimental point of view questions of bodies which are in equilibrium, and attempts a few simple generalisations from experiments; it does not enter upon the necessary and sufficient conditions for a body to be in equilibrium, and, except for one or two simple extensions, does not go beyond concurrent (and parallel) forces in one plane.

The book divides naturally into three parts. The first part, Chapters I to IV, discusses machines and the principle of work derived from experiments with machines, and includes friction. The second part, Chapters V to VII, discusses the principle of moments, centre of gravity, and stability. The third part, Chapters VIII and IX, discusses the triangle of forces and the resolution of forces. About 500 examples are included.

The order of the book—Work—Moments—Triangle of Forces—is a result of experience, for the writers have found that boys are specially interested in machines for transmission of work, and have relatively little difficulty in grasping the idea of mechanical work; less difficulty than they have with the idea of the moment of a force, and much less than they have with the question of the composition and resolution of forces by the 'triangle.'

Familiarity with the use of sines, cosines and tangents has been taken for granted.

It is very far from the intention of the writers to encourage the learning of formulas at this stage. Symbolic expression

has been used wherever it has been found convenient for discussing general rather than particular cases, and expressions like $P = a + b \,.\, W$ do occur; but the reader is not expected to commit to memory any of these expressions, and indeed he should be warned against it.

The book was written, all but the articles 110, 111 and Chapter IX, by Mr McMullen and the revision of the MS. and preparation for the press up to that point has been approved by him. While the book remained still unfinished Mr McMullen had a severe accident which made it impossible for him to do any more work before the time at which the book must go to press. The duty of writing the last few articles and the final correction of the proofs has therefore, much to his regret, fallen solely upon the writer of the preface.

Mr J. Watt of the Royal Naval College, Dartmouth, gave valuable help in the early stages, and it is regretted that he was unable to fulfil his original intention of collaborating with Mr McMullen throughout.

E. W. E. K.

KEW.
June 1924.

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CHAPTER I

FORCES IN ONE STRAIGHT LINE

1. Force. When we pull on the handle of an oar, press on the pedal of a bicycle, or try to raise a box, we are conscious of exerting *Force* on the object in question. Our idea of force, then, is derived from the sensations we experience when we make use of our muscles. When any other agent produces an effect of the same kind as we ourselves produce by muscular exertion, we realise that it also exerts force. Thus, a prop exerts force on a clothes-line; the wind exerts force on the sail of a boat; a book exerts force on a table. Observe that in general our muscular efforts are directed towards moving a body or towards altering in some way the motion it already possesses. Thus, a bowler exerts force on a cricket ball to give it motion; a batsman to alter its motion; a fielder to stop its motion. We may therefore define **force** as *any action which tends to move a body or tends to alter the motion it already possesses.*

2. Magnitude of a force. Although our muscular sensations convince us that when pulling a garden roller up a slope we are exerting a greater force than when pulling it along the level, we are well aware that these sensations provide us with a very imperfect means of comparing forces. For example, we should find it difficult to compare the force with which we *pulled* a roller in one instance with that with which we *pushed* it in another. This difficulty we experience even in the case of those forces which we are most accustomed to estimate, namely, weights. Can we, for instance, trust ourselves to guess the weight (and hence the postage) of a heavy envelope or parcel by the feel of it?

None the less, when we estimate that the fish we have caught weighs one pound, or that the trigger of our gun has a seven-pound pull, however incorrect our estimate, we know quite well what we mean to imply, for we are accustomed to regard the downward pull of a one pound weight or of a seven pound weight as a definite force and assume without argument that one is seven times the other. So in order that we may measure forces we require a set of weights marked in pounds.

Note that we have used the word 'weight' to denote both a standard piece of metal and the force which it exerts on our hand when we hold it. No harm will be done in continuing this practice since there is usually no doubt as to the sense in which the word is employed.

In denoting a force which is equal to the weight of, say, 5 pounds, we shall write it 5 lbs. wt.

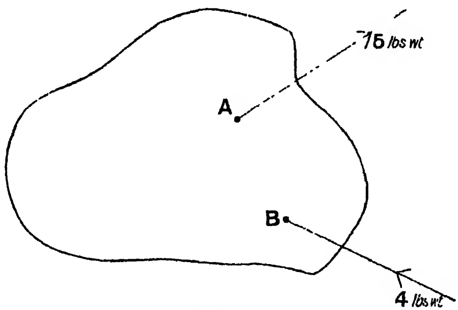


Fig. 1.

If we wish to shew in a figure a force acting on a body at a given point we shall indicate its direction by a line drawn from this point. An arrow is placed on this line to indicate what is called the 'sense' of the force, that is, to shew which way along the line the force is acting. Thus in Fig. 1 we indicate that the body is acted upon by a pull of 5 lbs. wt. at *A* and a push of 4 lbs. wt. at *B*, in the directions shewn by the lines.

3. The use of a spiral spring to measure forces. To measure a force we commonly use a spring-balance, the essential part of which is a spiral spring of steel wire. Before describing the balance we will examine the behaviour of a spiral spring under the actions of forces which cause it to stretch. To do this, the spring is suspended by one end from a fixed support and a weight-carrier is attached to the lower end. Beside it is clamped vertically a graduated scale so that the lower edge of the carrier is opposite to a convenient division of the scale (Fig. 2). We now place a weight of 1 lb. on the carrier and measure the extension produced. On adding another pound, thereby exerting on the spring a force of 2 lbs. wt., we find the extension produced by this force is twice that caused by the force of 1 lb. wt. For instance, if the spring stretches 1 inch for a force of 1 lb. wt., a force of 2 lbs. wt. is found to stretch it 2 inches and, on continuing the loading, we find a force of 3 lbs. wt. stretches it 3 inches, and so on. If we now remove the weights we find, unless the load has been excessive, the spring returns to its original length. It should now be clear that we can use this spring to measure forces and moreover that these forces need not necessarily be weights. For example, suppose we pull on the free end and observe that the extension produced is $3\frac{1}{2}$ inches. Knowing that a force of 1 lb. wt. produces an extension of 1 inch, we rightly infer that the force we are exerting is one of $3\frac{1}{2}$ lbs. wt.

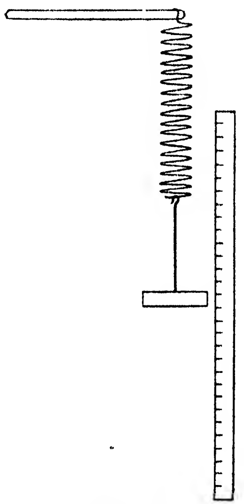


Fig. 2.

4. Spring-balances. One form of this instrument, which we shall use for measuring pulling forces, is shewn in Fig. 3. Its

construction is as follows. The ends of a spiral spring are attached internally to the closed ends of two brass tubes, one of which slides inside the other. The inner tube carries a small stud, which works in a longitudinal slot cut in the outer tube, and serves as an index to shew how much the spring has been extended. When the balance is to be graduated, it is hung by the ring from a fixed support. In this position the spring is slightly extended by the action of its own weight and that of the inner tube, and the index descends a little. As we do not wish to take into account these forces when we use the balance to measure others, we put a scratch on the edge of the slot, opposite to the point where the index comes to rest, and mark this 0. If the spring has been designed to enable the balance to read up to 4 lbs. wt., a load of 4 lbs. is now suspended from the hook and another scratch is made to mark the position of the index. The distance between 0 and this mark measures the extension of the spring for 4 lbs. wt. By dividing this distance into 4 equal parts we obtain the points at which the index will rest when the stretching force is 1, 2, and 3 lbs. wt. These distances are further subdivided into 16 equal parts so that a force may be measured to the nearest oz. wt.

Fig. 3.

Notice that it is only when the balance is in a vertical position that the full weight of the inner tube acts on the spring. Consequently the balance reads quite correctly only when it is used to measure a downward vertical pull. If it is employed to measure a force in any other direction the reading is slightly too small, but so slightly that as a rule we can disregard the difference.

When we want to measure pushing forces we use what are sometimes known as compression balances, two useful types of which are shewn in Figs. 4 and 5. That represented in Fig. 4 is constructed and graduated so that when one end is fixed, the

balance measures either a pushing or a pulling force applied to the other end in the direction of its length. If you examine the inside of these balances you will find they are only modifications of the extension balance described above: the push applied to the end, or pan, is communicated to the free end of a spring (or springs) in such a way as to stretch it, the extension produced thus being proportional to the force applied.

Fig. 4.

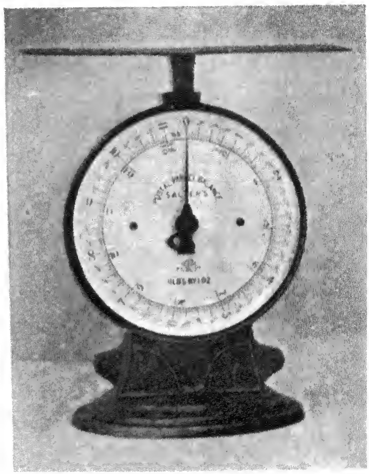


Fig. 5.

5. How to apply a known force to a body in any direction. If, instead of hanging a weight of, say, 2 lbs. directly on the hook of a spring-balance, we suspend this weight from the hook by a light piece of string, the balance reads 2 lbs. wt. just the same. Now let us pass this string over a pulley as shewn in Fig. 6. If we use for our purpose a pulley as free from friction as possible, the balance still reads very nearly 2 lbs.

wt. and continues to do so in whatever position we hold it. Moreover, if we raise or lower the weight by moving our hand and observe the balance during the motion, we find the reading is still 2 lbs. wt., always provided that the motion is a perfectly

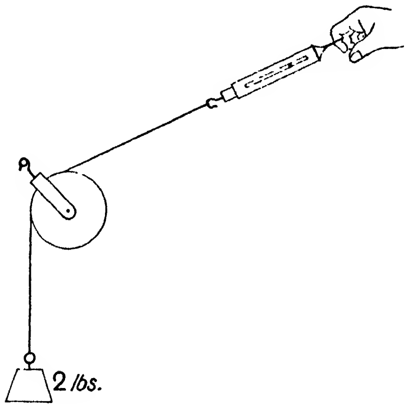


Fig. 6.

steady one. By 'steady' we mean 'of constant speed.' If the motion is jerky, or even if the speed increases smoothly or decreases smoothly, the reading of the spring-balance will differ from 2 lbs. wt.

We shall frequently take advantage of the above device to apply a known pulling force to a body in any required direction. Thus, if we wish to exert a force of 4 lbs. wt. in a given direction, we have only to pass a string coinciding with that direction over a pulley, and load the free end with 4 lbs. An illustration of this use of a pulley immediately follows.

6. Equilibrium of a body under the action of two forces. Two strings are fastened to a light ring and then passed over pulleys clamped some distance apart (Fig. 7). To the free ends of the strings equal weights are attached. That

the ring should remain at rest appears natural, for we know that it is acted upon by equal and opposite forces (Fig. 7 *a*). These forces are said to *balance* and the ring on which they act is said to be *in equilibrium*.

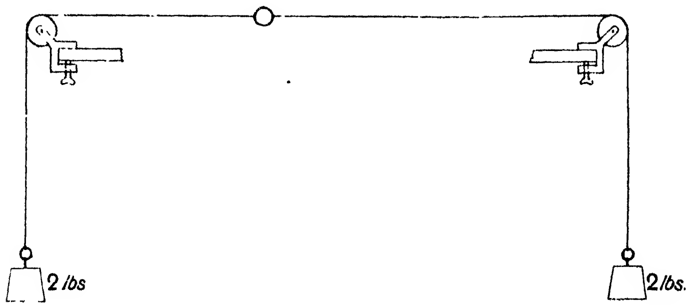


Fig. 7.

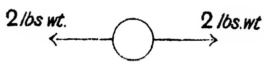


Fig. 7 *a*.

If either of these forces is increased by making one of the weights greater than the other, the ring no longer remains at rest but moves in the direction of the greater force. Note also that the ring will not remain at rest unless the forces lie in the same straight line; if we pull the ring aside and then let it go, we find that it always returns to its original position where the strings, and therefore the forces, lie in the same straight line. Hence we conclude that when any body remains at rest under the actions of two forces only, these two forces are equal and act in opposite senses along the same straight line.

We will now bring the ring up to one pulley and start it moving in the direction of the other. If the pulleys are free from friction, the ring keeps moving at the same speed; that is, the motion given to it is undisturbed by the forces exerted by the strings. Since these forces are the same during steady

motion as when the ring is at rest (Art. 5), they continue to balance one another and the ring is still said to be in equilibrium. This expression must therefore be taken to imply either 'at rest' or 'moving at constant speed.'

7. Supported body. Fig. 8 represents a light ring to which a weight of P lbs. is attached, the ring being supported on a fixed horizontal peg. Let us first ask ourselves why the ring is at rest. We see that it is acted upon by a downward force of P lbs. wt. To keep it in equilibrium we know that the peg must exert upon it an equal force upwards (Fig. 9). To make this quite clear we have only to attach to the ring a vertical string passing over a pulley and fasten to the other end of the string a weight of P lbs. If we now remove the peg the ring remains at rest (Fig. 10), shewing that the upward force of P lbs. wt. now exerted upon it by the upper string, is the same as that which the peg exerted in the first place.

Let us now consider the weight itself in Fig. 8. We see from Fig. 10 that this weight is acted upon by an upward force of P lbs. wt. It follows therefore that there must be acting upon it an equal force downwards (Fig. 11). We realise that this force which is acting on the weight downwards is due in some way to the Earth and we call it the Force of Gravitation or, simply, Gravity. The magnitude of the force with which the Earth attracts a body is called the *weight* of the body. Notice that, up to this point, we have (as in ordinary life) talked of the weight of a body as the force which the body exerts on anything which supports it; in future we must think of the weight of a body as a force acting upon the body itself.

It can be shewn that any body has a point of balance, and that if the body is supported, *e.g.* upon a pivot, at this point, it will remain in equilibrium in any position, and, so far as the body affects its support, it behaves as though its weight were a single force acting through the point of balance. We shall discuss this question more fully in Chapter VI; in the meantime we shall

speak of the weight of a body as a single force acting through the centre of balance of the body. Anticipating Chapter VI. we

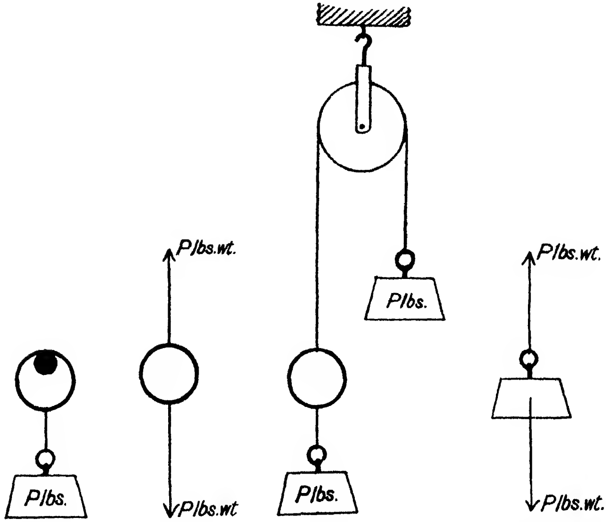


Fig. 8. Fig. 9. Fig. 10. Fig. 11.

may call the point of balance the 'centre of gravity.' For instance, we shall speak of the weight of a uniform straight rod as a single force acting through the rod's centre.

8. Equilibrium of a body under the action of several forces in one straight line. We will now fit up the arrangement shewn in Fig. 12. Three strings, carrying loads of 5, 3, and 2 lbs. respectively, are passed over three separate pulleys and fastened to a small ring. The pulleys are supported in a straight line so that the strings carrying the loads of 2 and 3 lbs. lie parallel to another. The ring is now acted upon by three forces in one line, namely, a pull of 5 lbs. wt. to the left and two pulls of 2 and 3 lbs. wt. to the right.

As we should expect, these forces balance and the ring is in

equilibrium. So also, when any number of forces act on a body in one straight line, the body is in equilibrium when the sum of

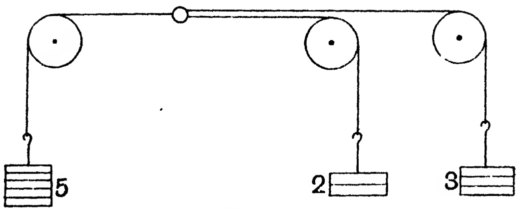


Fig. 12.

the forces of one sense is equal to the sum of the forces of opposite sense.

9. Resultant. Observe that, if we replace the weights of 2 and 3 lbs. in the preceding case by one weight of 5 lbs., the ring is still in equilibrium. Hence a force of 5 lbs. wt. acting to the right has the same effect as the two forces of 2 and 3 lbs. wt. acting together. A force which, acting alone, has the same effect as two or more forces acting together, is called the *resultant* of these forces. We see that the resultant of forces acting in the same straight line, and of the same sense, is equal to their sum.

Further, since we may regard the force of 2 lbs. wt. acting to the right as balancing the opposite forces of 5 and 3 lbs. wt., it is clear that these last two forces are equivalent to one of 2 lbs. wt. acting to the left. That is, the resultant of forces in the same line and of opposite sense is equal to their difference.

We shall find experimentally the resultant of a number of forces by first finding a single force which balances them, and then assuming that the resultant is equal in magnitude, but of opposite sense, to this 'balancing force.'

10. Experimental exercises. The following pieces of apparatus are designed to illustrate the foregoing principles. The reader should fit up for himself several of these, or similar, arrangements, and should think out the answers to the following

questions with the apparatus before him. He should then verify his answers as far as possible. Since the spring-balances only weigh about 2 or 3 ozs. each, their weights may be neglected in comparison with the other forces acting. If the readings of the balances are found to differ slightly from the values you predict, endeavour in each case to find the reason for this. In the figures the numbers against the weights indicate pounds; a shaded block represents a fixed support.

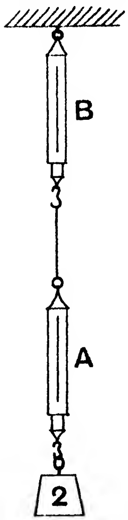


Fig. 13.

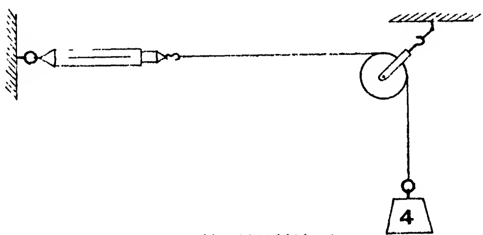


Fig. 14.

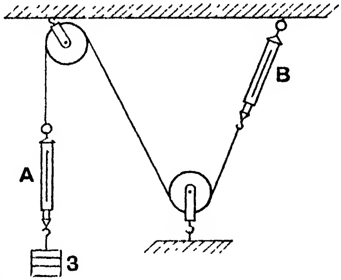


Fig. 15.

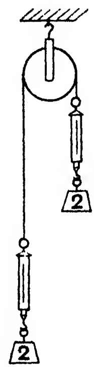


Fig. 16.

Ex. 1. (Fig. 13.) (*a*) What are the readings of the balances *A* and *B*? (*b*) Specify the forces acting on the weight of 2 lbs.

Ex. 2. (Fig. 14.) (*a*) What is the reading of the balance? (*b*) Will the balance read the same if attached the other way round? (*c*) What forces keep the balance in equilibrium?

Ex. 3. (Fig. 15.). (*a*) What are the readings of the balances *A* and *B*? (*b*) What forces are acting upon these balances and by what are they exerted?

Ex. 4. (Fig. 16.) (*a*) What do the balances read? (*b*) If the right-hand weight is given a start downwards, what do the balances read during the motion?

Ex. 5. (Fig. 17.) (*a*) What are the readings of the balances *A* and *B*?
(*b*) Specify the forces acting on the weight of 1 lb.

Ex. 6. (Fig. 18.) (*a*) What forces are acting on the weight of 2 lbs.?
(*b*) What is the reading of the balance?

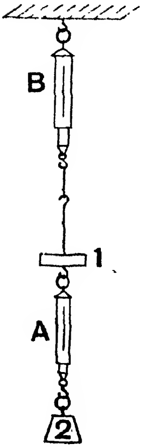


Fig. 17.

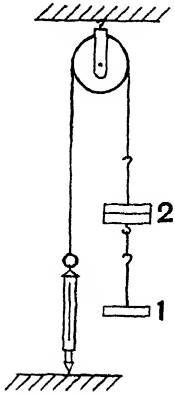


Fig. 18.

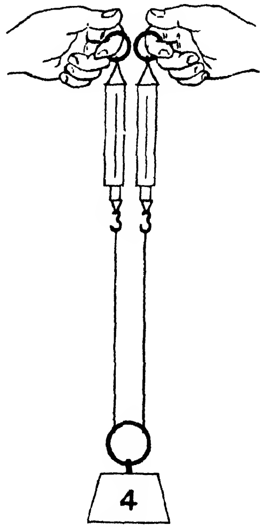


Fig. 19.

Ex. 7. (Fig. 19.) The weight is fastened by two separate strings to the balances whose rings are supported by your hands.

(*a*) What is the sum of the readings of the balances?

(*b*) What is the resultant of the forces which the strings exert on the weight?

(*c*) If you separate your hands so that the strings are inclined to each other, is the sum of the readings of the balances the same as before?

Ex. 8. (Fig. 20.) (*a*) What force does each string exert on the balance, and what is their resultant?

(*b*) What is the reading of the balance?

(*c*) What force balances the pulls of the strings and by what is it exerted?

Ex. 9. (Fig. 21.) (*a*) What are the readings of the balances *A* and *B*?
(*b*) What forces are acting on the weight of 2 lbs.?

Ex. 10. (Fig. 22.) (*a*) What is the reading of *B*?

(*b*) What forces are acting on the weight of 7 lbs.?

(*c*) What is the reading of *A*?

Ex. 11. (Fig. 23.) The balance *A* is pulled upwards by the hand.

(*a*) If *A* reads *P* lbs. wt. and *B* reads *Q* lbs. wt., specify all the forces acting on the weight of 7 lbs.

(*b*) What is the value of $P+Q$?

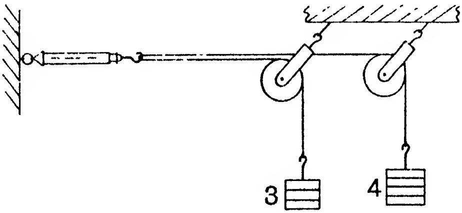


Fig. 20.

Ex. 12. (Fig. 24.) Pieces of the same thread are used for the connections. Increase the load *W* till the thread breaks.

(*a*) Which piece of thread will break and why?

(*b*) If the greatest pull the thread can stand in 5 lbs. wt., what is the value of *W*?

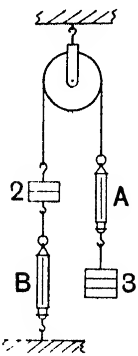


Fig. 21.

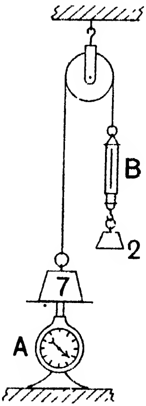


Fig. 22.

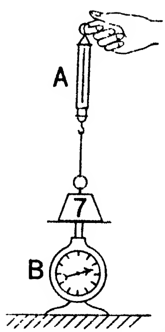


Fig. 23.

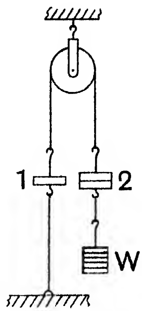


Fig. 24.

Ex. 13. (Fig. 25.) (*a*) What forces are acting on the weight of 3 lbs.? (*b*) What is the reading of the balance?

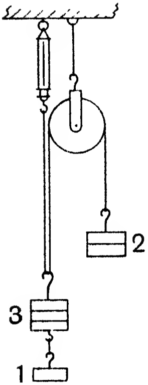


Fig. 25.

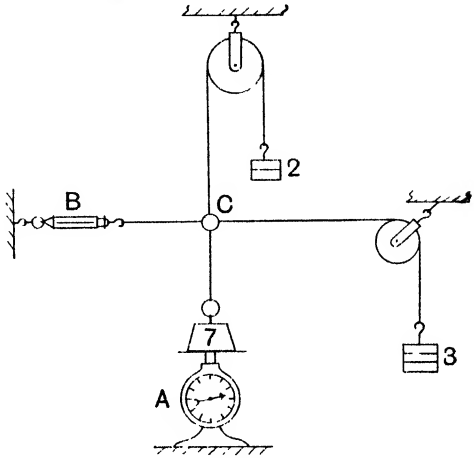


Fig. 26.

Ex. 14. (Fig. 26.) The horizontal strings are first attached to the ring *C* and then the other strings are arranged vertically without disturbing them.

(*a*) What are the forces acting on the weight of 7 lbs.?

(*b*) What are the readings of the balances?

(*c*) Draw a diagram shewing the forces acting on the ring *C*.

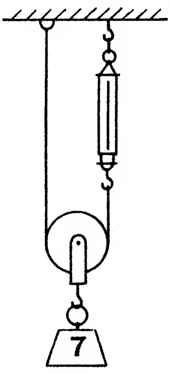


Fig. 27.

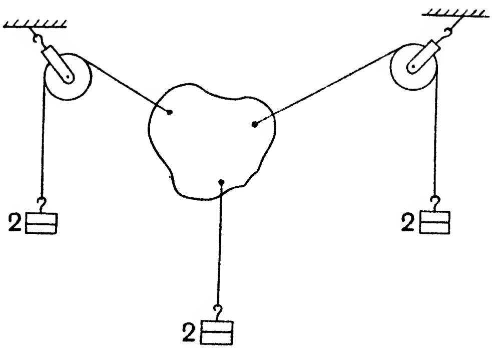


Fig. 28.

Ex. 15. (Fig. 27.) The string passes round a movable pulley to which the weight is fastened, the string on each side being vertical.

What is the reading of the balance?

Ex. 16. (Fig. 28.) A *light* piece of cardboard is suspended as shewn.

(*a*) What forces are acting on the cardboard?

(*b*) Sketch the position in which the cardboard will come to rest when the bottom weight is removed.

11. Action and Reaction. Stress. From the foregoing examples we see that, in considering the equilibrium of a body, we must be careful to recognise and confine our attention to all the forces acting *on* it. These forces, however, are exerted *by* other bodies and if we turn our attention to these bodies, we realise that they also are acted upon by the body in question. Let us examine this mutual action between bodies.

If you press with your right hand on your left, you realise that your left hand presses back on your right. If you pull on a fixed string you feel the string pulling back on your hand. Thus we observe that, whenever a force is exerted, we get not a single force but a pair of forces which act in opposite directions on different bodies. Moreover, these two forces are always of equal magnitude. To demonstrate this, we hook together two spring-

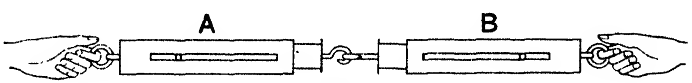


Fig. 29.

balances as shewn in Fig. 29 and pull the rings of the balances apart. We see that the readings of the two balances are always equal, shewing that the pull of A on B (indicated by the reading of B) is always equal to the pull of B on A (indicated by the reading of A). Thus, when we pull on the hook of a spring-balance, it pulls back with an equal and opposite force, and therefore the reading of the balance may be taken to register either the force acting upon it or the force which it exerts.

In the same way by taking two compression balances, placing them end to end and pressing them together, we may shew that

the push which the first exerts on the second is equal and opposite to that which the second exerts on the first.

These are illustrations of a more general result which may be stated thus: whenever one body exerts force upon another, the second body exerts an equal and opposite force on the first. This latter force, which is called into play by the *action* of the first body on the second, may be called a *reaction*. Hence this principle is usually expressed shortly by saying that 'To every Action there is an equal and opposite Reaction.'

Since it is convenient to have a name for this pair of forces which constitute a mutual action between two bodies (or two parts of a body) we call it a *Stress*.

As a further example of what is meant by these terms, take the case of a boy standing on the ground. His feet press on the ground with a total downward force which is equal to his weight. The ground presses back on his feet with an equal upward reaction. These two forces together constitute a stress. We do not always think of the reaction of the ground, but it becomes painfully obvious when we walk with bare feet on a pebbly beach.

12. Friction. Just as important as the vertical stress between our feet and the ground is the horizontal stress set up when we start to walk. In this case the reaction brought into play acts along the surface of contact and is caused by what is called *Friction*. Let us examine the nature of this friction.

When we hold up a book or a plate of glass with its plane vertical, by pressing it between the finger and thumb, why does it not slip downwards? The horizontal forces exerted upon it by the finger and thumb balance one another but have no direct effect in preventing the object from falling. We therefore infer that upward forces acting on the object have been called into play at the places where our finger and thumb are pressed against it. Again, consider a book resting on a table. The only forces acting upon it are its weight downwards and the

equal upward reaction of the table. If we now apply a gentle horizontal push or pull to it we find it does not move. By our action we have evidently called into play an equal and opposite reaction which prevents the book from moving. Further, if we increase the applied force until the book starts to move, we find that we must continue to exert a certain force to keep it in steady motion. Hence, in this case also there must be acting on the under-surface of the book an equal horizontal reaction opposing its motion.

This force which prevents or tends to prevent the sliding of one surface over another is called the *Force of Friction* or *Frictional Resistance.* It owes its existence to the inequalities of the surfaces in contact; the rougher these are, the greater is the force of friction which can be called into play between them. We cannot slide far on a stone floor, however smooth it appears to be. With the same effort we can slide much farther on a sheet of ice, and farther still if we put on skates and thus substitute polished steel runners for the comparatively rough surface of the soles of our boots. Now, no surface can be made absolutely smooth; even the surface of polished silver, when viewed under a microscope, is seen to be covered with innumerable scratches. Hence it follows that when any two surfaces (however highly polished) are pressed together, minute projections on one surface interlock to some extent with minute hollows on the other, and consequently force is required to make one slide over the other. The following simple experiment will help you to realise the nature of friction.

Experiment. Connect a spring-balance by a string to one end of a thin flat piece of wood. Place this on a smooth table

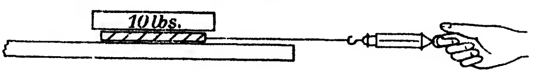


Fig. 30.

or horizontal plane of some kind and load it with a weight of 10 lbs. (Fig. 30). Pull the ring of the balance horizontally. If

the pull is applied gradually the wooden slide at first remains at rest. As the pull increases, the friction increases and always opposes to it an equal and opposite force. Hence the reading of the balance gives the force of friction at any moment. But we find that, as we increase the pull, the friction cannot increase beyond a certain limit, for when the pull exceeds a certain value the slide starts to move.

The maximum value of the force of friction, which is indicated by the reading of the balance when the slide is just on the point of moving, is called the *Limiting Friction* or *Starting Friction.* If we now read the balance while we pull the slide along with steady motion we obtain the value of what is called the *Sliding Friction.* The forces acting on the wooden slide when it is in equilibrium, *i.e.* when at rest or moving steadily, are shewn in Fig. 31, the magnitude of the friction in each case being equal

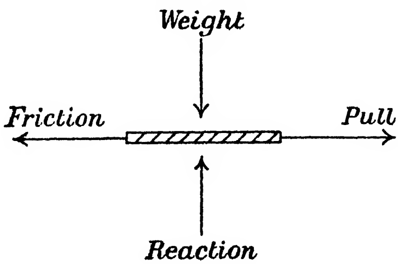


Fig. 31.

to the pull we exert. We find that the pull necessary to start the motion is always somewhat greater than that necessary to maintain it, that is, the Starting Friction is greater than the Sliding Friction. The magnitude of the Limiting Force of Friction, which one surface can exert on another, is found to obey certain simple laws, but we will defer the experimental investigation of these laws till Chapter IV.

We may remark here, however, that common experience tells us the chief factors which govern the Limiting Friction between

surfaces. Thus, we know it largely depends on the materials in contact; for instance, there is less tendency to slip on a dry surface if we have rubber soles to our shoes. Again, friction increases with the pressure or normal stress between the surfaces. This fact we make use of instinctively; for instance, to diminish the tendency of a cricket bat to slip in our hands we grasp it more tightly; to increase the friction between the brake and the rim of the wheel of a bicycle we press harder on the operating lever.

13. Effects of Friction. We have only to examine any machine in action, for instance, a screw-jack or any kind of engine, to observe that surfaces between different parts of the machine are sliding over one another. Sliding friction is therefore brought into play and opposes the motion which it is necessary to maintain. Clearly the effect of this is that a force has to be applied to work the machine greater than would be necessary if there were no friction. For instance, in Art. 5 we shewed that we could raise a weight by a string passing over a pulley, by pulling on the free end with a force very nearly equal to the weight, providing the pulley was specially constructed so as to be as free from friction as possible.

If we repeat this experiment with an ordinary commercial article such as a common ship's pulley block, we shall find that to raise a weight of 56 lbs. we shall have to pull with a force of about 63 lbs. wt. We can regard this force as the sum of two forces, namely, one of 56 lbs. wt., which is instrumental in raising the load, and one of 7 lbs. wt., which is instrumental in making the sheave of the pulley block turn against the opposing frictional resistance at its axle.

Having indicated the ill-effect of friction in impeding motion, we should at the same time point out that friction has often a very useful effect in being indirectly the cause of motion. For example, but for friction between our feet and the ground we could not walk. Unless there were friction between the driving-

wheels of a locomotive and the rails, no train could be moved. It is owing to friction that motion can be transmitted by means of belts and pulleys as, for instance, in motor bicycles, or by friction-clutches, as in motor cars. It is to increase friction that a locomotive is provided with an apparatus for putting sand on the rails when these are slippery. For similar reasons a billiard player chalks his cue, a gymnast puts resin on his hands, and a bowler uses sawdust on a wet day.

14. Methods of reducing the effect of friction. Friction can be greatly reduced by interposing between the surfaces a lubricant, such as oil or grease. In such cases the surfaces are not in actual contact but separated by a thin film of the lubricant over which they slide. For lubricated surfaces the friction depends on the quality of the lubricant and

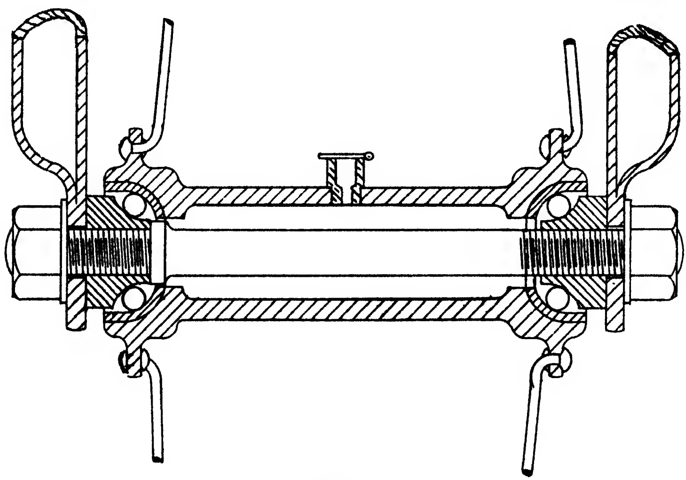


Fig. 32.

on the extent to which it succeeds in keeping the surfaces clear of one another. We can still further reduce the resistance offered to the movement of one surface over another by interposing

between them a number of small hard spheres, which roll as the movement takes place. Thus, if we rest a book on a number of large-sized shot placed on a table, we find that a very small horizontal force will produce and maintain motion. The shot do not slide but roll over the surfaces.

There is a little resistance to rolling (so-called Rolling Friction) which is found to be due to the fact that the surfaces at the points of contact become to some extent indented. Better results are therefore obtained by using harder materials, as we can well illustrate by placing steel balls between two plates of glass and comparing the resistance to motion with that of the book on the shot. Steel balls are used to reduce the axle friction of bicycles and motor cars. Fig. 32 shews the ordinary adjustable ball-bearings of a bicycle wheel, the free running of which is familiar to every one. The hub of the wheel is separated from the axle about which it revolves by a circle of hard steel balls at each end. An end section in Fig. 33 shews the arrangement of the balls between the hardened steel surfaces of the cone and hub.

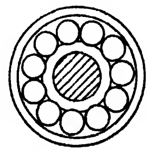


Fig. 33.

15. Transmission of force. Tension and Compression. If one end of a cord is fastened to a staple in the wall and we pull on the other end, the force we exert is transmitted unchanged to the staple irrespective of the length of the cord (Fig. 34). Even supposing we were not aware of this fact at the outset, the experimental demonstrations in Arts. 5 and 10 should have made it familiar. We will now examine this more closely.

If the pull we exert on the end of the cord is P lbs. wt., the staple must exert an opposite force of P lbs. wt. on the other end to keep the cord in equilibrium (Fig. 34*a*). These two forces tend to stretch the cord; the material in resisting extension exerts equal and opposite reactions at its ends on the hand and on the staple (Fig. 34*b*). Thus the staple is being pulled by

the cord with a force equal to that exerted by our hand and this is clearly independent of the length of the cord.

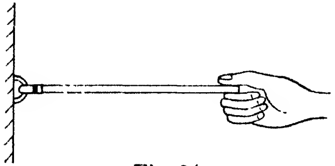


Fig. 34.

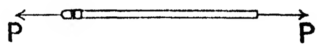


Fig. 34*a*.

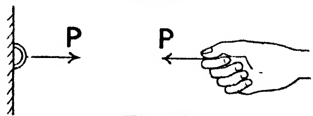


Fig. 34*b*.

The cord is said to be *in tension*, and the two equal and opposite forces acting on its ends are said to constitute a *Tension*, since they tend to stretch the material in the direction of the forces. The tension in the cord in this case is said to be P lbs. wt., that is, the tension is measured by the magnitude of one of the forces which constitute it.

If we substitute a light stiff rod for the cord above and press one end of it against the staple in the wall by applying a pushing force to the other end, the push we exert is likewise transmitted by the rod to the staple. The rod in this case is in equilibrium under the action of two pushing forces at its ends which tend to shorten it and it is therefore said to be *in compression*. The rod in resisting this shortening effect reacts on the hand and on the staple with forces equal and opposite to those exerted upon it, with the result that the force which we apply to the end of the rod is transmitted unchanged to the staple, whatever the length of the rod may be.

16. Internal condition of a body under stress. Up to this point we have confined our attention to the stresses set up at the ends of a cord or rod. We will now consider what happens to the material itself when in tension or in compression. If we hold the ends of a stout piece of rubber cord with our hands and then pull them apart, we notice that the cord becomes longer and also thinner throughout its length (Fig. 35). This

shews that when the cord is in tension, every part of the material is stretched in the direction of the applied forces. Forces therefore must be pulling on the ends of every piece of the cord along its length. Also, since the whole and every part of the cord are in equilibrium, it follows that all these forces must be equal, that is, the tension must be the same throughout the

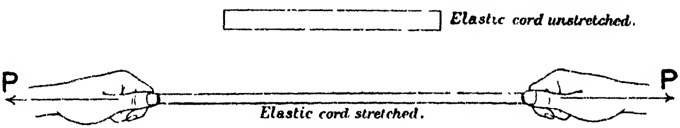


Fig. 35.

length of the cord. If we pull on the ends of a rod, a piece of string, or a wire, instead of a rubber cord, the material behaves in the same manner, but this is not apparent because the alterations in the dimensions produced by the tension are too small to be observed by eye. That the material really is stretched, however, we can prove by the use of very accurate measuring instruments. The action and reaction between any two parts of the material tending to pull one part away from the other constitute an *internal stress.*

The alteration in the shape of the material caused by the stress is called the *strain.* This strain in the case of tension consists, as we have seen, of an increase in the length of the material in the direction of the stress and a corresponding decrease in its other dimensions. In the case of compression the strain produced is of exactly the opposite kind. For instance, when we press on the ends of a block of rubber, we observe that the material is shortened in the direction of the forces and its section at right angles to this direction is increased. Thus we see that each part of the block is shortened and hence it is clear that forces must be pushing on the ends of every piece of the material along its length. For equilibrium we know that all

these forces must be equal, that is, the compression must be the same throughout the length of the block.

The internal stress in this case consists of the action and reaction between any two parts of the material tending to press one part into the other.

Although we have only considered, above, forces which, acting on a body, increase or decrease its length in their own direction, we may notice here that forces may produce other effects on the material. For instance, the forces acting on a body may tend to make one part of the material slide over another, when the material is said to be subjected to *shear stress.* A stress of this kind is set up in a piece of thick cardboard when we cut it with a pair of scissors. This internal stress is of a similar nature to the external frictional stress set up between two surfaces (Art. 12).

17. Rigid bodies. Transmissibility of Force. We see then that all bodies when acted upon by forces are subjected to internal stresses which tend to break them and which produce in them some change of shape. It is important always to bear these facts in mind. At the same time we know that this change of shape is often so small as to be inappreciable. Now, in Statics, our aim is to discover, and to use, the relations existing between forces which keep a body in equilibrium. Hence, in this book, we shall consider the tendencies of forces to move a body as a whole, rather than the effects which these forces produce on the material of the body. For this reason we shall deal chiefly with bodies whose material is capable of withstanding the actions of applied forces without undergoing any appreciable distortion. Such bodies are said to be *rigid.* In considering the equilibrium of a rigid body it is important to observe that we may regard any given force applied to it as acting at any point in the line of action of this force. That is, the tendency of a force to move a body as a whole is not altered by altering its point of applica-

tion, provided this point remains in the line of action of the force and is rigidly connected to the body. For instance, suppose we hang a sheet of cardboard from a spring-balance and suspend a weight by a string from a hook passing through the cardboard at A (Fig. 36).

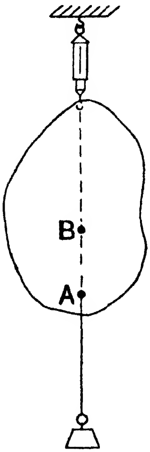


Fig. 36.

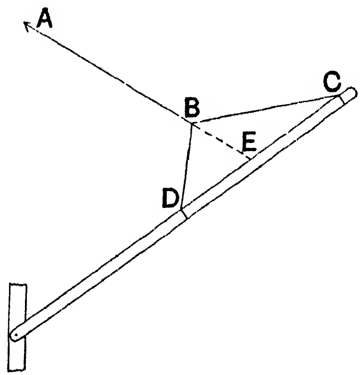


Fig. 37.

If we now transfer this hook from A and pass it through the cardboard at any point which lies in the direction of the string, such as B, we do not thereby alter the position of the cardboard or the reading of the spring-balance. As regards the equilibrium of the cardboard, we may therefore consider the force which the weight exerts upon it as acting at any point in the line of action of the force.

As another example, consider the arrangement in Fig. 37. A rigid spar pivoted at its lower end is held in position by applying a force to the end of the rope AB which is connected to two ropes BC and BD attached to the spar at C and D. In considering the equilibrium of the spar we may consider this

force as acting upon it at E, for if, instead of the arrangement shewn, we were to tie a rope at E and pull it with the same force in the direction BA the spar would still be found to be in equilibrium. The ropes BA, BD and BC are in tension and, under these circumstances, behave as parts of a rigid body. The different arrangements of the ropes have the same effect upon the equilibrium of the spar as a whole though different effects upon the material of the spar.

18. Rods hinged or pinned at each end. Ties and Struts. Let us take a light stiff rod having a hole drilled through it at each end. Holding a pencil in each hand, let us pass them through these holes (Fig. 38).

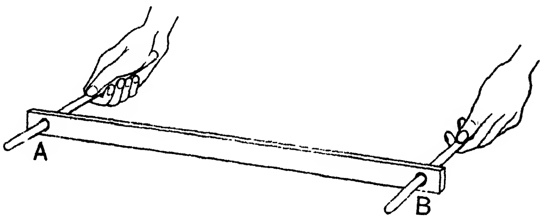


Fig. 38.

In what ways can we now exert forces on the ends of the rod so as to keep it in equilibrium?

The forces acting on the rod will be (1) the action of the pencil at A, (2) the action of the pencil at B, (3) the weight of the rod itself. If the weight of the rod is small in comparison with the other forces, we can neglect it. This being so, the forces exerted by the two pencils must balance one another, and this is only possible if they are equal and act in the same straight line, and this line must be AB. Hence the rod must be acted upon either by a direct pull or a direct thrust at each end, that is, it must be in tension or compression only. Note carefully, however, that this is no longer true if any other force is

applied to the rod between A and B in a direction inclined to it. Now rods, pinned or hinged at each end, like the one above, are used in the construction of many mechanisms and of many structures, such as those used for supporting roofs and bridges. For instance, consider the connecting-rod of a steam-engine, the jib of a crane (Fig. 133 on p. 257), the bar DB in Fig. 84 on p. 147. In all such cases as these, if we can neglect the weights of the members themselves, these members are acted upon only by forces exerted by the pins at their ends and hence must be simply in tension or compression. Those members which are in a state of tension are called *ties*; those in a state of compression are called *struts*.

An important difference between a tie and a strut should be noted. In a tie the pulls on the two ends tend to keep the rod straight, while in a strut, if the rod becomes but slightly bent, the thrusts on its ends tend to bend it more. Consequently, ties are usually made of solid rods of a simple section, round or rectangular, while struts are made of a section specially designed to resist buckling. For instance, the members of a bicycle frame are made of a hollow section since a tube offers greater resistance to bending than a solid rod of the same weight.

EXAMPLES I.

1. A spiral spring, when hung from a fixed support and loaded at the lower end with a weight of 6 lbs., is found to stretch 4 inches. What extension would be produced in this spring by a load of 4 lbs.?

2. If a pull of 8 lbs. wt. is required to increase the length of a certain spring by 3 inches, what pull will be required to increase its original length by $5\frac{1}{2}$ inches?

3. Given a piece of stout indiarubber cord, explain how you would construct, and graduate, an instrument to measure forces.

4. A spiral spring was suspended from a fixed support and various weights were attached to its lower end. The heights of the upper and

lower ends of the spring above the table were measured for each load. The results in the following table were obtained :

Stretching force in lbs. wt.	Height of upper end of spring in inches	Height of lower end of spring in inches	Length of spring in inches
0·6	14·5	9·3	
0·8	14·5	8·6	
1·0	14·5	7·9	
1·2	14·5	7·2	
1·4	14·5	6·5	
1·6	14·5	5·85	
1·8	14·5	5·1	

Obtain the numbers for the last column.

Plot a graph on squared paper to shew how the length of the spring varies with the stretching force, measuring the latter along the horizontal axis.

Find from your graph the probable length of the spring (*a*) when unstretched, (*b*) when the stretching force is 0·2 lb. wt., and (*c*) when the stretching force is 1·5 lbs. wt.

5. When a weight of 3 lbs. is suspended from the lower end of a spiral spring, the length of the spring is 6·9 inches. When the weight is increased to 5 lbs., the length of the spring is 7·5 inches.

What is the unstretched length of the spring? What will its length be when the suspended weight is 4 lbs.?

6. A spring-balance is hung from a fixed support and carries a load at its lower end. If the balance reads 6 lbs. wt., what forces are acting on the ends of the spring?

7. A 12-stone man standing on the ground pulls downwards on a vertical rope which passes over a pulley and carries a load at its other end. What is the greatest load the man can raise in this manner, if the friction of the pulley is negligible?

8. When is a body said to be in equilibrium? What is the condition for equilibrium in the case of a body acted upon by forces in one straight line?

9. A sack of coals weighing 1 cwt. is being raised by a rope at a steady speed. What force is the rope exerting on the sack?

10. A steam tug, in towing a vessel at a certain steady speed, exerts on the tow rope a force of 5 cwt. What force does the rope exert on the tug? What resistance does the water offer to the vessel's motion?

11. The tension in a certain string is 5 lbs. wt. Explain what this means. What forces are being exerted by this string? If a spring-balance is inserted in this string at any point without altering the tension, what will the balance read?

12. If the strings in Fig. 7 on p. 7, instead of being fastened to the ring, are fastened to the ends of a light spring-balance, what will the balance read? If the balance is now moved steadily to the right or to the left, what will it read during the motion?

13. What is meant by the resultant of two forces?

One man pulls a cart with a force of 60 lbs. wt. while another pushes behind with a force of 45 lbs. wt. What is the resultant of these forces?

If the motion of the cart is uniform, what is the resistance to its motion?

14. Explain what is meant by Friction. Under what circumstances is friction called into play between two surfaces?

15. Describe the action and reaction which are brought into play between two bodies when one is made to slide over the other. Explain the cause of this frictional stress.

16. A man pulls with a horizontal force of 8 lbs. wt. on the end of a plank resting on a horizontal bench, but the plank does not move. What is the force of friction acting on the plank?

When another man, at the same time, pushes horizontally on the other end of the plank with a force of 7 lbs. wt., the plank just starts to move. What is the magnitude of the Limiting Friction?

When once started, one man can keep the plank moving steadily by exerting a force of 11 lbs. wt. What is the magnitude of the Sliding Friction?

17. (Fig. *a*.) A wooden slide A, which weighs $\frac{1}{2}$ lb., and carries a load

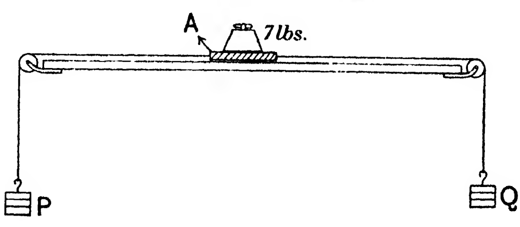


Fig. (*a*).

of 7 lbs., rests on a horizontal table. The ends of the slide are connected by strings over pulleys (whose friction is negligible) to the weights P and Q.

(i) It is found that when $P=2$ lbs. and $Q=3\cdot5$ lbs., no motion is produced. What is the force of friction acting on the slide and in what direction does this force act?

Draw a sketch of the wooden slide alone, shewing all the forces acting upon it.

(ii) When $P=3\cdot5$ lbs. wt. and $Q=6\cdot3$ lbs. wt., the slide, on being started, moves steadily along the table. What is the force of the Sliding Friction?

(iii) If $Q=7\cdot5$ lbs. wt., what must be the value of P so that on being started the slide will move steadily to the (a) left, (b) right?

(iv) If there were no friction, what would be the resultant force acting on the slide in each case?

18. A book weighing $\frac{1}{2}$ lb. is at rest when pressed against a vertical wall by a horizontal force of 1·4 lbs. wt.

Draw a figure shewing the horizontal and vertical forces acting on the book.

If the book is on the point of sliding down, what is the magnitude of the Limiting Friction between the surfaces of the book and the wall?

19. Describe any part of machinery you have seen in which the force of friction serves a useful purpose. Describe some other part in which the existence of friction is a drawback.

20. Why will a coin, when given the same shove on a table, travel farther on its edge than on its face?

When travelling on its edge, why does it go farther when there is no cloth on the table than when there is?

21. If a pulley wheel works stiffly, we oil it. Explain how this reduces the friction.

22. To raise a weight of 85 lbs. by means of a rope passing over a pulley, a man finds he has to exert a pull of 96 lbs. wt. How much of this force is required to overcome the friction of the pulley?

23. Describe how you would design a laboratory form of pulley to have as little friction as possible.

24. A pile of 5 note-books rests on a table. If you apply a horizontal force to the top book, why does it alone slide? If you apply a horizontal force to the third book from the top, why do the top three books move together?

25. A row of light spring-balances are hooked together and one end of the row is fastened to a fixed support. The other end is now pulled with a force of 8 lbs. wt. What does each balance register? What forces are acting on the ends of each of the springs? What forces are exerted by each spring?

26. A chain is fastened at one end and is pulled at the other end with a force of 80 lbs. wt. Sketch one link of the chain and shew what forces are acting upon it. The weight of the link need not be considered. What forces are being exerted by this link on the links with which it is coupled?

27. A pile of 5 books, each weighing 2 lbs., is placed on a table. What is the stress between the lowest book and the table? What is the stress between the third and fourth books from the top?

28. A telegraph pole is partly supported by a wire stay fastened to a stake in the ground. What is meant by saying that the tension is the same throughout the length of the stay? If this tension is 60 lbs. wt., what forces are acting *on* the stay, and *by* what bodies are these forces exerted?

What forces does the stay itself exert, and on what bodies are these forces exerted?

29. Fig. (*b*) represents a pair of Dutch draw tongs, hung from a bar *B*. The end of a wire *ab*, which has a tension of 300 lbs. wt., is clamped between the jaws *jj* of the tongs. What is the resultant force of friction of the jaws on the wire?

What force does the tension in the wire produce on the bar *B*?

If the weight of the tongs is 12 lbs., what is the reaction of the bar *B*?

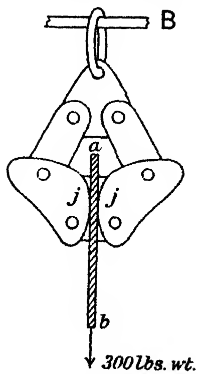


Fig. (*b*).

CHAPTER II

MACHINES. PRINCIPLE OF WORK

19. Machines. However strong a man may be, there is a limit to the muscular force which he can exert. No man, for instance, can directly lift a body weighing a ton. It is true that if the body is capable of being divided up into a number of parts, say, 40 parts each of 56 lbs., he can then raise the body, for he has only to exert a force of 56 lbs. wt. Notice, however, that to raise the body any given height he has to exert this force through a distance 40 times the height which the whole body is raised. But this method is often impossible or inconvenient in practice. He can, however, bodily lift a load of a ton by using a suitable machine. For example, if he wishes to raise one end of a motor car until the wheels are clear of the ground, he uses a screw-jack, for by applying a small force to this machine and by continuing to exert this force through a considerable distance, he gives the machine the capacity of exerting a very large force through the necessary few inches. A more common type of machine which we use to multiply our effort is a simple lever. For instance, when we employ an iron bar to prise open the lid of a box, a large resistance is overcome by one end of the bar when we apply a comparatively small effort to the other end. Notice again that we exert this effort through a much greater distance than that through which the resistance or load is overcome.

Besides levers and screw-jacks, there are many other kinds of machines, such as winches and various arrangements of pulleys, which we shall presently describe. In each of these, also, a comparatively small force, called the *effort*, acting through a certain distance, is used to overcome a large resistance, called the *load*, through a much smaller distance. There are also certain

machines in which the reverse effect is produced. We shall see that a bicycle and a hydraulic crane are examples of this class. In these a large force acting through a comparatively small distance is used to overcome a smaller force through a much greater distance.

A machine, then, is a mechanical contrivance by means of which a small force acting through a large distance may be made to overcome a larger force through a smaller distance; or vice versa.

What is the principle underlying this valuable property of machines whereby force can be multiplied?

Before searching for this principle we may remark that we know from experience that friction has an important effect on the behaviour of a machine. If a machine works more stiffly than usual we attribute this to excessive friction, and oil it. We can, however, construct certain simple machines in which the effect of friction may be rendered negligible and it is with such machines that we shall first experiment.

20. Experiment on a Simple Lever. The following experiment with a simple straight lever will enable us to compare the forces exerted on and by this machine, and the distances through which these forces act.

A straight rod AB is pivoted at C, the pivot (or *fulcrum* as it is called) being as frictionless as possible (Fig. 39). The lever may be balanced so that it will remain at rest at any inclination, by

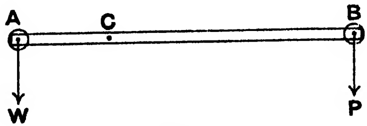


Fig. 39.

placing a counterpoise weight on the shorter arm AC. To find what downward effort (P) applied at B will overcome a given resistance (W) at A, a load of W lbs. is placed on a pin passing

through the rod at A, and then weights (P) are added to a similar pin at B until, on being given a start, the end B moves down with steady motion.

By measuring the vertical heights of the pins at A and B from the table when the rod is horizontal, and again when the rod has been displaced through any angle, we find the vertical distances through which these weights have moved, namely the distances DA and BE (Fig. 40).

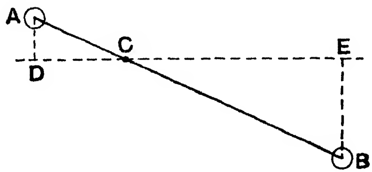


Fig. 40.

If EB is the vertical distance through which the effort P has acted, DA is the corresponding vertical height by which the load W has been raised. We see from the similar triangles in the figure (or alternatively we may prove by direct measurement) that the distance through which the effort acts and the height by which the load is raised are in the same proportion one to the other as the corresponding arms of the lever,

$$i.e.\ EB/AD = EC/CD.$$

Let us use a lever 1 metre long so arranged that we can fix the fulcrum at any point we please; let us make the distance of C from A successively 50 cms., $33\frac{1}{3}$ cms., 25 cms. and 20 cms., and therefore the corresponding distances of C from B 50 cms., $66\frac{2}{3}$ cms., 75 cms. and 80 cms. In the first case the arms of the lever are equal and the weight is raised a height equal to the distance through which the effort acts; in the second case the effort arm of the lever is twice the length of the weight-lifting arm and the weight is raised a height equal to half the distance through which the effort acts; similarly in the third case the weight is raised a height equal to one-third, and in the fourth case

equal to one-fourth of the distance through which the effort acts.

We may now find by experiment for each of these positions of the fulcrum what effort is required to raise a given load of say 12 lbs. (Note that in the first position of the lever, *i.e.* when it has equal arms, it is not necessary to counterpoise it before loading; in each of the three remaining positions it is necessary to counterpoise the lever afresh before loading.)

The following are the results of this experiment tabulated:

Position	Vertical height the weight is raised	Vertical distance the effort acts	Load	Effort
1	1 inch	1 inch	12 lbs.	12 lbs.
2	1 "	2 inches	12 "	6 "
3	1 "	3 "	12 "	4 "
4	1 "	4 "	12 "	3 "

We have used the lever to do the same thing in each case, *i.e.* to raise a load of 12 lbs. 1 inch. We have changed the distance through which the effort acts, making it first 1 inch, then twice as much, then three times as much, and then four times as much; in doing so we have found it possible to reduce the effort in the same proportion as we have increased the distance through which it acted. Putting the result in other words, we have found that however we have changed the effort, the product of the effort and the distance through which it acted has remained the same.

The lever we used moved almost without friction, and we notice that not only is the product of the effort and the distance through which it acted the same in each case, but also this product is equal to the product of the weight of the load and the height the load is raised.

21. Work. If we give the name WORK to the product of a force and the distance through which in its own direction a force

acts, we can express what we have learnt of the action of the lever still more shortly:

A certain quantity of WORK is done in raising the load of 12 lbs. a height of 1 inch.

The WORK done by the effort is the same, whether a smaller effort acts through a longer distance or a larger effort through a shorter distance.

The WORK done on the load is equal to the WORK done by the effort.

Using the word 'work' in this sense, we have given a special meaning to a common word, and therefore it is important strictly to define what we have done.

22. Definition of Work in Mechanics. Whenever we exert ourselves in any way, either physically or mentally, we are accustomed, in ordinary language, to say that we do work. In Mechanics, however, 'work' is a term which is used in a more limited sense. When we exert force on a body we may, or may not, do mechanical work. As a rule, our efforts are directed towards making some body move in opposition to some resistance. If we succeed in moving the body against this resistance we are said to do mechanical work. But if the body does not move in the direction of the force applied, we do no mechanical work upon it, however great this force may be. Suppose, for example, we exert ourselves to lift a heavy weight from the floor and fail to do so. The exertion tires us and thus, in a sense, we do work in making the effort; but as long as the weight does not move in response to the force we do no mechanical work in the sense in which hereafter we intend to use the word 'work' in this book. If we succeed in raising the weight against the opposing force with which the earth attracts it, then, while doing so, we are exerting force through a certain distance and are thereby doing mechanical work. A pillar supporting a roof or a bridge is exerting enormous forces but is doing no work because these forces are not being exerted through any distance. A man when cutting a lawn

with a mowing machine does work, for he exerts a force on the machine sufficient to overcome the resistance offered and thereby exerts this force through a certain distance. *Work, then, is said to be done when a force acts through a certain distance along its line of action.*

23. Measurement of Work. When a labourer is engaged on piece-work, he is paid for the quantity of work he does. Upon what does quantity of work depend? In the first place note that in estimating work we take no account of time. For instance, if a man has a lawn to cut he has a certain 'job of work' to do. Clearly he will do the same amount of work whether he takes one hour or several hours to finish his job. In considering this man at work, it would seem that work is a quantity depending directly on two things only, namely, the force he exerts and the distance through which he exerts it. Thus, if the grass is long, he has to exert a large force on the machine and he will tell you that it is 'hard work.' Again, when he has cut half the lawn he has done only half his job, and has only exerted the required force through half the distance necessary to cut the whole.

If, therefore, we agree that a man does the same amount of work when he exerts a force of 6 lbs. wt. through 2 inches as when he exerts a force of 12 lbs. wt. through 1 inch, and to this conclusion our experiment with the lever led us, we shall measure work correctly by the product *force* × *distance*.

More precisely we say that, **The Work done by a force is measured by the product of the number of units in the force and the number of units in the distance through which the force is exerted along its line of action.**

The usual unit, in terms of which work is measured, is called a *foot-pound* (written ft. lb.). It is the work done when a weight of 1 lb. is lifted to a height of 1 foot, that is, when a force of 1 lb. wt. is exerted through a distance of 1 foot along its line of action.

For example, to lift 14 lbs. to a height of 5 feet we have to do 5×14 ft. lbs. $= 70$ ft. lbs., that is, seventy times as much work as when we lift a weight of 1 lb. to a height of 1 foot.

In measuring large amounts of work the foot-ton is often the unit employed, this being the work done when a force of 1 ton wt. acts through a distance of 1 foot.

Ex. 1. How much work is done in raising 20 gallons of water from a well 90 feet deep?

Since 1 gall. of water weighs 10 lbs., the required force is 200 lbs. wt. and this is exerted through 90 feet, hence

Work done $= 200 \times 90$ ft. lbs. $=$ **18000** ft. lbs.

Ex. 2. If a locomotive exerts a steady pull of $1\frac{1}{2}$ tons wt., how much work does it do on the train in travelling 1 mile?

Here, a force of $1\frac{1}{2}$ tons wt. is exerted through 5280 feet, hence

Work done $= 1\frac{1}{2} \times 5280$ ft. tons $=$ **7920** ft. tons.

Ex. 3. A man applies a steady force of 14 lbs. wt. at right angles to a handle attached to a crank 9″ long (Fig. 41). If the direction of the force is always at right angles to the crank, calculate the work done in turning the handle once. Here, although the direction of the force is continually changing, its point of application is always moving in its direction; hence the distance through which the force is exerted is equal to the circumference of a circle of 9″ radius,

i.e. distance through which force is exerted $= 2\pi \times 9$ inches

$$= \frac{18\pi}{12} \text{ feet.}$$

$$\text{Work done} = \frac{14 \times 18\pi}{12} \text{ ft. lbs.} = \mathbf{66} \text{ ft. lbs.}$$

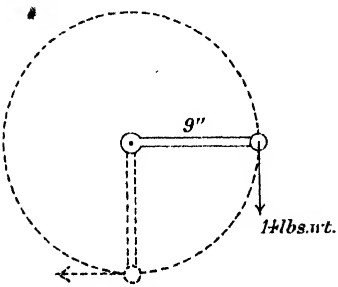


Fig. 41.

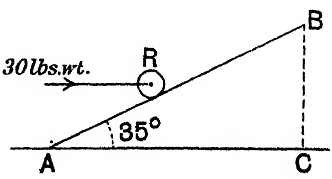


Fig. 42.

Ex. 4. A roller *R* is pushed up a slope from *A* to *B* by applying a horizontal force of 30 lbs. wt. (Fig. 42). *AB* is 10 feet, and the angle *BAC* is 35°. Calculate the work done by the force.

Since the force always acts horizontally, the distance through which it is exerted *in its own direction* is the horizontal distance *AC*.

Now $\frac{AC}{AB} = \cos 35°$, or $AC = AB \cos 35 = 10 \times \cdot 819 = 8\cdot 19$ feet.

Hence the work done by the force $= 30 \times 8\cdot 19 = \mathbf{245\cdot 7}$ ft. lbs.

24. The Principle of Work. In our experiment with a lever (Art. 20) we found that

Load × distance through which the load is raised
= Effort × distance through which the effort acts.

We expressed this by saying that *the work done on the load is equal to the work done by the effort* during a movement of the machine, when there is no friction. This is called the *Principle of Work*. This principle underlies the action of all machines. It suggests that, could we eliminate friction from any machine, we should get as much work out of the machine as we put into it.

Since, however, no practical machine is free from friction, we will now enquire how we must modify the expression of the principle to apply it to all working machines.

25. Further experimental illustrations of the Principle of Work. Inclined Plane. A solid metal roller is fitted with a stirrup pivoted loosely on the ends of its axle

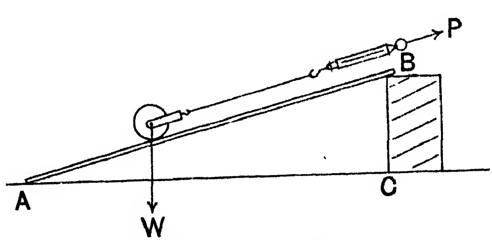


Fig. 43. Cylinder rolling.

can be pulled in the same manner as a garden roller

(Fig. 43). By pulling on a spring balance which is attached by a string to the stirrup, the roller is moved with steady motion up an inclined plane which may well consist of a plate of glass supported at one end by a block of wood. Since the hard surface of the cylinder rolls over the hard surface of the plane and since the sliding friction at the bearings is very small, the total resistance due to friction is inappreciable. This is shown by the fact that, whether the roller is at rest or moving up or down with steady motion, the balance reads very nearly the same.

In a certain experiment the following results were obtained: Weight of roller = 2 lbs. Reading of spring-balance $\frac{1}{2}$ lb wt.

Length $AB = 2$ feet. Height $BC = \frac{1}{2}$ foot.

From these results we see that, in pulling the roller from A to B,

Work done by effort $= \frac{1}{2} \times 2 = 1$ ft. lb.

Now, the load of 2 lbs. is thereby raised vertically through the distance CB which is $\frac{1}{2}$ foot, hence

Work done on load $= 2 \times \frac{1}{2} = 1$ ft. lb.

That is, the work done by the effort = work done on the load, when the friction is negligible. In the same way we find that, however we alter the inclination of the plane,

$P \times AB$ always equals $W \times CB$.

This experiment is now repeated in a slightly different form; instead of rolling, the cylinder is made to slide lengthwise up the plane by attaching the string to one end (Fig. 44). The reading of

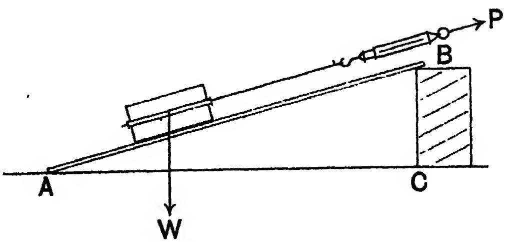


Fig. 44. Cylinder sliding.

the balance is now greater, owing to friction opposing the motion. With the plane inclined as in the above example, it was found, for instance, that the effort required to slide the cylinder up with steady motion was $\frac{3}{4}$ lb. wt. instead of $\frac{1}{2}$ lb. wt. Hence the friction accounts for $\frac{1}{4}$ lb. wt., and the work done against friction when the cylinder is pulled from A to B, a distance of 2 feet, is

$$\tfrac{1}{4} \times 2 \quad \text{or} \quad \tfrac{1}{2} \text{ ft. lb.}$$

The useful work done on the load is the same as before, namely 1 ft. lb., and the total work done by the effort is $\frac{3}{4} \times 2 = 1\frac{1}{2}$ ft. lb. Hence in this case we extend the Principle of Work by saying,

Work done by the effort = Work done on the load
+ Work done against friction.

Wheel and Drum. This is a type of machine frequently met with. In the form in which we are going to use it for experimental purposes it consists of a drum and a wheel fixed to a spindle. This

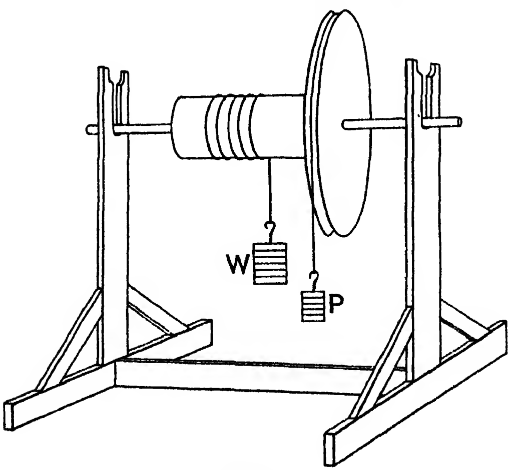
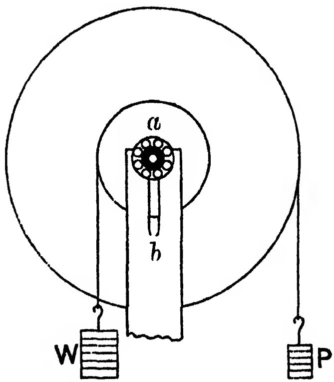


Fig. 45.

spindle rests horizontally in two wooden bearings (Fig. 45). The load W is attached to a cord fastened to the drum and the effort P

is exerted on a string fastened to and wrapped round the wheel. This machine is often known as a 'Wheel and Axle,' since it is possible to dispense with the drum by winding the cord attached to the load directly on the axle of the wheel. In experimenting with this machine we begin by eliminating friction as much as possible by substituting ball-bearings for the ordinary bearings. This can be quickly done by lifting out the spindle, and inserting its ends into two well-fitting ball journals which are placed in semi-circular notches in the supports just above the ordinary bearings. This arrangement can be understood from Fig. 46, which shews the ball-bearing at one end.



(*a*) Ball-bearing Fig. 46. (*b*) Ordinary bearing

To find what effort P is required to raise any particular load, such as 28 lbs., we add weights to the end of the string attached to the wheel until, on being given a start, P moves down steadily. To find the relation between the distances moved by P and W, we first measure the heights of the lower edges of these weights from the table with a long wooden scale. Then, standing the scale vertically beside W, P is pulled down until W is raised a convenient distance. In this way it is found with this particular machine that when W is raised $\frac{1}{2}$ foot, P descends 2 feet;

when W is raised 8 inches, P descends 32 inches. That is, P always moves through four times the distance that W is raised.

In an experiment done with this machine it was found that a load of 28 lbs. was raised by an effort of 7·06 lbs. wt. Suppose the load to be raised a distance of $\frac{1}{2}$ foot. Then we have

$$\text{Work done on the load} = 28 \times \tfrac{1}{2} = 14 \text{ ft. lbs.}$$

During this movement P has descended 2 feet, and

$$\text{Work done by the effort} = 7{\cdot}06 \times 2 = 14{\cdot}12 \text{ ft. lbs.}$$

These results are so nearly equal that we are led to believe that they would agree exactly if we could entirely eliminate friction. This is another illustration of the Principle of Work, namely, that when friction is negligible,

$$\text{Work done by the effort} = \text{Work done on the load.}$$

Next the spindle is replaced in its ordinary bearings, and the experiment repeated. It is now found that to raise a load of 28 lbs. an effort of 8·3 lbs. wt. is required. In this case the work done to raise the load $\frac{1}{2}$ foot is the same as before, namely

$$28 \times \tfrac{1}{2} = 14 \text{ ft. lbs.,}$$

but the work done by the effort is

$$8{\cdot}3 \times 2 = 16{\cdot}6 \text{ ft. lbs.}$$

It is reasonable to assume, as we shewed was the case in pulling a weight up a slope, that the difference between the work done by the effort and the work done on the load, which in this case is $16{\cdot}6 - 14 = 2{\cdot}6$ ft. lbs., is the work required to overcome the frictional resistances of the machine, in accordance with the Extended Principle of Work, namely,

$$\begin{aligned}\text{Work done by the effort} = {} & \text{Work done on the load} \\ & + \text{Work done against friction.}\end{aligned}$$

26. Explanation of terms applied to Machines. Velocity-Ratio. In the case of the Wheel and Drum in the preceding article, we found that the effort is always exerted

through four times the distance the load is raised. We express this by saying that the *velocity-ratio* of the machine is 4. That is, the velocity-ratio is given by the relation

$$\text{Velocity-ratio} = \frac{\text{Displacement of effort}}{\text{Corresponding displacement of load}}.$$

This ratio does not depend in any way on the load or on the friction of the machine. It is solely governed by the design, and can always be calculated from the dimensions of the machine.

For instance, with the Wheel and Drum in Fig. 45, we see that when the spindle turns once, the displacement of the effort is equal to the circumference of the wheel, and the corresponding displacement of the load is equal to the circumference of the drum if we neglect the thickness of the cord in each case; hence the velocity-ratio of this machine is given by

$$\frac{\text{Circumference of the Wheel}}{\text{Circumference of the Drum}}.$$

It is advisable, however, always to check the calculated value of the velocity-ratio by direct measurement. The origin of the term velocity-ratio or speed-ratio becomes clear when we remark that, in a machine whose velocity-ratio is 20, the effort moves 20 times as far as the load in the same time, and hence always moves 20 times as fast.

Mechanical advantage. The mechanical advantage of a machine is the ratio of the load to the corresponding effort required, that is

$$\text{Mechanical advantage} = \frac{\text{Load}}{\text{Effort}}.$$

This ratio is determined by experiment, that is, by finding the effort required to overcome a given load. For instance, if with a certain machine we find that a load of 1 cwt. can be raised by applying an effort of 12 lbs. wt., then the mechanical advantage in this particular case is $\frac{112}{12} = 9\frac{1}{3}$. We shall find that the mechanical advantage of any machine is not constant

but depends on the load. If, however, there is no friction in a machine, we can shew that the mechanical advantage is constant. For in this case we have

$$\text{Work done on the load} = \text{Work done by the effort};$$

that is,

$$\text{Load} \times \text{displacement of load} = \text{Effort} \times \text{displacement of effort},$$

or
$$\frac{\text{Load}}{\text{Effort}} = \frac{\text{Displacement of effort}}{\text{Displacement of load}} = \text{Velocity-ratio}.$$

So we see that to obtain a machine with a large mechanical advantage we must design it to have a large velocity-ratio.

Efficiency. A knowledge of the velocity-ratio of a machine enables us to calculate what fraction of the work done on the machine is usefully employed in raising the load, when we have found the necessary effort by experiment. This fraction is called the *Efficiency* of the machine, thus

$$\text{Efficiency} = \frac{\text{Useful work done on the load}}{\text{Work done by the effort}}$$

during any given time or during any given movement of the machine.

For example, suppose that for a Wheel and Axle whose velocity-ratio is 20, it is found that an effort of 12 lbs. wt. is required to overcome a load of 1 cwt.

Considering the work done on and by the machine during any convenient movement, for instance, when the load is overcome through 1 foot, we have

$$\text{Useful work done on the load} = 112 \times 1 = 112 \text{ ft. lbs.}$$

Since the velocity-ratio is 20 we know that during the movement the effort is exerted through 20 feet, hence

$$\text{Work done by the effort} = 12 \times 20 = 240 \text{ ft. lbs.}$$

That is,
$$\text{Efficiency} = \tfrac{112}{240} = 0{\cdot}467$$

This means that for every 1 ft. lb. of work done on the machine

(in raising the particular load) we only get back 0·467 ft. lb. in a useful form, or, for every 100 ft. lbs. we get back

$$100 \times 0{\cdot}467 = 46{\cdot}7 \text{ ft. lbs.}$$

This is expressed by saying that the efficiency is 46·7 per cent. In this particular case the remaining 53·3 per cent. of the work done on the machine is expended in overcoming the internal frictional resistances of the machine.

When we come to test machines we shall find that the efficiency, like the mechanical advantage, is not constant for any machine but increases as the load increases.

Friction-effect. We know that friction occurs wherever one part of a machine slides over another with which it is in contact. At this stage, however, we shall not concern ourselves with the actual forces of friction at the various rubbing surfaces within the machine; we shall only consider the general effects which friction produces without enquiring exactly where it occurs. As we are already aware, the presence of friction requires more work to be expended than would be necessary if friction were absent; and to perform this extra work an additional effort is necessary. This 'wasted' effort we will call the *Friction-effect.* This force is clearly the difference between the actual effort required in any given case, and the effort which would be required if there were no friction. Now we have already shewn, from the Principle of Work, that, if there is no friction, $\dfrac{\text{Load}}{\text{Effort}} = \text{Velocity-ratio}$. That is, the effort without friction, or what we will call the 'useful' effort, is equal to $\dfrac{\text{Load}}{\text{Velocity-ratio}}$. Hence, by subtracting this from the actual effort we obtain that part of this effort which is instrumental in overcoming the unknown resistances within the machine, that is, we obtain the Friction-effect.

Let us illustrate this with reference to the foregoing example of a Wheel and Axle having a velocity-ratio of 20, and with which

it is found possible to raise 1 cwt. by an effort of 12 lbs. wt. If there were no friction in this machine, the effort required would be $\frac{112}{20} = 5{\cdot}6$ lbs. wt. This means that $12 - 5{\cdot}6$ or $6{\cdot}4$ lbs. wt. is that part of the effort which is effective in making the machine work against the opposing resistances within it. That is, we say the Friction-effect is $6{\cdot}4$ lbs. wt.

In the next chapter we shall shew that the friction-effect increases with the load and that a simple law connects these two forces.

27. Applications—Simple arrangement of Pulleys

(Fig. 47). A cord, fastened at *A*, passes round a movable pulley from which the load is suspended and then over a fixed pulley. The load is raised by pulling on the free end of the cord at *P*.

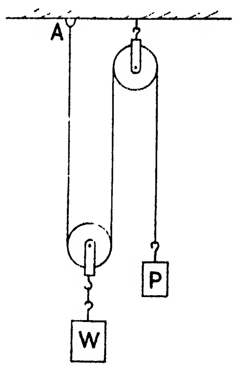


Fig. 47.

To calculate the velocity-ratio we will find how far *P* moves when the load is raised through 1 foot. Since the movable pulley rises 1 foot, each part of the cord supporting it is shortened by this amount. Hence the cord forming the whole loop is shortened by 2 feet. That is, *P* descends 2 feet or twice as far as the load is raised. The velocity-ratio is therefore 2.

In a certain experiment it was found that to raise a load of 28 lbs. an effort of $18{\cdot}5$ lbs. wt. was required.

To calculate the efficiency we will find the work done on the load and the work done by the effort when the load is raised 1 foot. Here the work done on the load $= 28 \times 1 = 28$ ft. lbs. Since the effort is exerted through 2 feet, the work done by the effort $= 18{\cdot}5 \times 2 = 37$ ft. lbs.

Hence $\quad$ Efficiency $= \frac{28}{37} \times 100\ {}^\circ/_\circ = 75{\cdot}67\ {}^\circ/_\circ$.

To calculate the friction-effect we first find what effort would

be required if there were no resistance to be overcome in the machine itself; this would be $\frac{28}{2}$ or 14 lbs. wt. Subtracting this from 18·5 lbs. wt. we find that 4·5 lbs. wt. is the force necessary to overcome the internal resistances.

Although we call this force of 4·5 lbs. wt. the friction-effect, it should be remarked that the internal resistances to be overcome in this case are not caused solely by friction, for part of this force is used in raising the movable pulley. For instance, if this pulley weighs 0·6 lb., the force required to raise it is 0·3 lb. wt. Strictly then, the friction of the machine in this case is 4·5 − 0·3 or 4·2 lbs. wt. Therefore, in defining the friction-effect as that part of the actual effort which is not usefully employed, we should note that, with those lifting machines such as the above where parts of the machine itself are raised, this friction-effect includes the small constant force necessary to raise these parts.

28. Worm and Worm-Wheel. This machine is shewn in Fig. 48 in a form in which we can directly experiment with it. A worm on a horizontal spindle AB is in gear with a toothed wheel (called the Worm-Wheel). Fixed to the horizontal spindle of the worm-wheel is a drum D, round which a cord is wound. From the end of this cord the load is suspended. The effort is applied to another cord wound on the wheel B which is fixed to the spindle of the worm.

In this particular machine, the diameter of the wheel B is 9 inches and the diameter of the drum D is 4 inches. The worm-wheel has 30 teeth.

To calculate the velocity-ratio it will be found convenient to find the displacements of W and P when the drum D turns once. In this case

$$\begin{aligned}\text{the displacement of the load } W &= \text{circumference of drum } D\\ &= 4\pi \text{ inches.}\end{aligned}$$

Now, since the worm is single-threaded, it will pass one tooth along for each revolution it makes and hence it must revolve

30 times to pass 30 teeth along, *i.e.* to make the worm-wheel revolve once. That is, during the above displacement of the

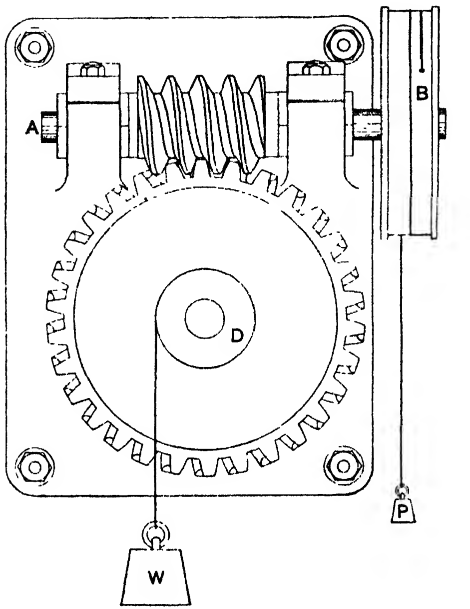


Fig. 48.

load, the effort is exerted through a distance equal to 30 times the circumference of the wheel B,

$$\text{or displacement of effort} = 30 \times 9\pi \text{ inches.}$$

Hence $$\text{the velocity-ratio} = \frac{30 \times 9\pi}{4\pi} = 67\tfrac{1}{2}.$$

In a certain experiment with this machine it was found that a load of 50 lbs. could be raised by an effort of 2·6 lbs. wt. What are the Efficiency and the Friction-effect of the machine for this load? [*Ans.* 28·5 %; 1·86 lbs. wt.]

The low efficiency of this machine is due to the considerable sliding friction between the threads of the screw and the teeth of the wheel, in addition to the friction at all the bearings. However, a convenient result of this excessive friction is that the machine does not 'overhaul,' that is to say the load does not run down when the effort is removed. We shall find that a machine will not overhaul when its efficiency is less than 50 %.

29. The Bicycle. So far we have only dealt with machines designed to multiply the effort exerted upon them; that is, in each case the mechanical advantage, and hence the velocity-ratio, have been greater than unity. We will now briefly consider two examples of machines designed to produce the reverse effect.

To calculate the velocity-ratio of a bicycle. A bicycle is driven by thrusting on the pedals with each foot alternately. Although not strictly true, we will assume that this thrust is always exerted at right angles to the crank. If the cranks are 7 inches in length, then for each complete revolution which they make, the effort is exerted through a distance of $7 \times 2\pi$ inches. The motion of

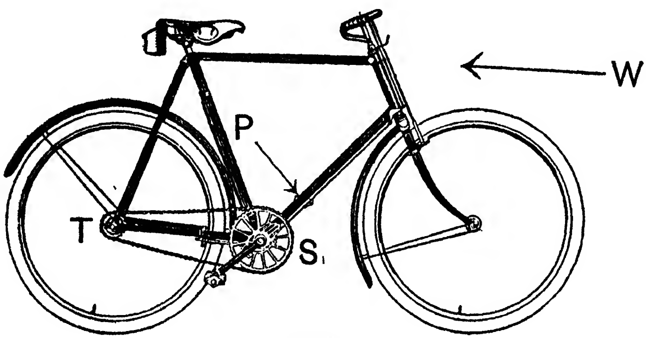


Fig. 49.

the pedals is communicated to the back wheel by an endless chain, whose links engage with the teeth of the sprocket wheels *S* and *T* (Fig. 49). *S* is mounted on the axle to which the

cranks are fixed, and T is fixed to the hub of the back wheel. S is always of a greater diameter than T. Suppose S to have 36 teeth and T to have 14 teeth and the diameter of the wheels of the bicycle to be 28 inches. When the chain moves through any distance it must engage with exactly the same number of teeth on each sprocket wheel. Consequently T must rotate more quickly than S. Now when 36 links of chain pass over S it makes one complete revolution: when 36 links pass over T it makes $\frac{36}{14}$ revolutions. Therefore for each revolution of the pedals the back wheel makes $\frac{36}{14}$ revolutions, and the machine travels $\frac{36}{14} \times 28\pi$ inches.

Hence the velocity-ratio $= \dfrac{7 \times 2\pi}{\frac{36}{14} \times 28\pi} = \frac{7}{36}$.

Notice that the velocity-ratio is less than 1. We are making a large force, acting through a certain distance, overcome a small force through a larger distance. The resistance which has to be overcome is chiefly that of the air, for the friction of the machine is very small owing to the adoption of ball-bearings for all the rotating parts. The old type high bicycle, common twenty-five years ago, and now to be seen in the South Kensington Museum, had a very large wheel in front directly driven from the pedals. When we say that a bicycle is geared up to 80, we mean that it will go as far for one revolution of the pedals as would a direct driven bicycle whose wheel had a diameter of 80 inches.

What is the gear of the bicycle discussed above? Call it D. This means that for one revolution of the pedals the bicycle would travel πD inches.

Therefore $$\pi D = \tfrac{36}{14} \times 28\pi,$$
or $$D = 72.$$

30. Hydraulic crane. Fig. 50 shews the hydraulic cylinder which operates the wire rope by which the load is raised. There are two pulley wheels on the end of the cylinder at B and also two similar wheels on the end of the plunger at A. The wire rope is fastened to the cylinder and passes round the wheels

at A and B in turn and then passes over pulleys on the crane to the load. The effort is exerted by the pressure P of the water on the plunger. If the stroke of the plunger is 6 feet, then during a stroke, each of the 4 parts of the wire rope passing along the

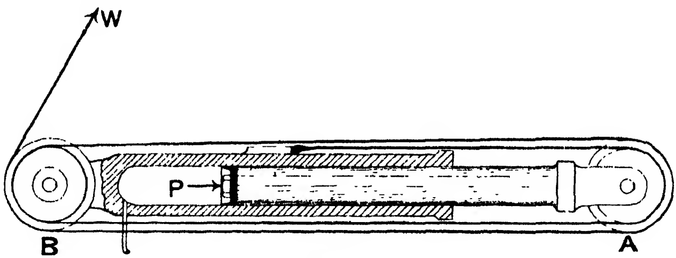


Fig. 50.

cylinder is lengthened by this amount. Hence the load is raised 4×6 or 24 feet. The velocity-ratio is therefore $\frac{1}{4}$, the object in this case being to obtain a large travel of the wire rope for a comparatively small movement of the plunger.

31. Energy. Energy wasted in friction. Suppose that in using a certain machine to raise a load of 200 lbs. to a height of 1 foot, we find that we have to do 300 ft. lbs. of work. The Principle of Work states that in this case 100 ft. lbs. of work are expended in overcoming the resistances in the machine. If no work is done in raising parts of the machine itself, we assume that this quantity of work is equal to the sum of all the different quantities of work which are expended in overcoming the forces of friction at the various rubbing surfaces of the machine. We will try to indicate briefly the broader principle upon which this assumption is based.

Observe that in raising the load, we thereby give it the capacity of doing 200 ft. lbs. of work, for in descending to its original position it is capable of exerting a force of 200 lbs. wt. through 1 foot.

The capacity or capability of doing work we call *energy*. Thus we say that in raising the load of 200 lbs. through 1 foot we give it 200 ft. lbs. of energy.

If no work had to be done in overcoming resistances in the machine, we should only have to do this amount of work; in other words, our capacity of doing work, that is, our energy, would be lessened by 200 ft. lbs. In this case no energy would be wasted. What happens to the additional 100 ft. lbs. of our energy which we have assumed are expended in doing work against friction? This energy is certainly wasted for we do not get back any useful work from it. It has not disappeared however but remains in the machine in such a form that we cannot make use of it. To understand this let us notice what happens when we do work against friction. If we rub two pieces of wood together we find that the rubbing surfaces become hot; that is, for our expenditure of energy we get some heat produced. Now, heat possesses the capacity of doing work. For instance, we realise that the mechanical work done by a steam-engine is derived from the heat given out when the coal burns. Heat we say is a form of energy. Reverting to the above example, you will now be more ready to accept the statement that when we expend 100 ft. lbs. of our energy in doing work against friction in the machine, this energy is converted at the various rubbing surfaces into a certain quantity of heat energy. And further, a long series of experiments has shewn that if we could, so to speak, harness this heat energy and make it do all the mechanical work of which it is capable (for instance, in raising a load) we should then get back exactly our 100 ft. lbs. of mechanical energy. To sum up, none of the energy we expend on a mechanical machine is lost; part of this energy we get back from the machine in a useful form and the remainder is converted into an equivalent quantity of some form of energy (chiefly heat) in the machine. When you become familiar with the various forms that energy can take, and the methods by which these forms can be changed one into the other, you will realise more fully the broad funda-

mental principle which asserts that whatever transformation energy undergoes, no energy can ever be created or destroyed.

This principle is known as the *Conservation of Energy*.

The Principle of Work expresses this fundamental principle in the limited form in which we use it in Statics.

EXAMPLES II.

1. Find the work done when a weight of 10 lbs. is lifted 40 feet.

2. A 50-ton gun is lifted by a crane to a height of 12 feet. What is the work done on the gun?

3. A boy, who weighs 8 stone, mounts two flights of stairs, each 20 feet high. How much work does he do?

4. A force of 35 lbs. wt. acts through a distance of 6 feet 4 inches. Find the work done.

5. What force acting through a distance of 2 feet 8 inches will do 2 ft. tons of work?

6. If 16,000 ft. lbs. of work are done in lifting 2 tons of coal, through what height is the coal lifted?

7. Through what distance in feet must a force of 1200 lbs. wt. be exerted in order to perform 20 inch tons of work?

8. A boy who climbs to a height of 20 feet does 1400 ft. lbs. of work. What is his weight?

9. How much work is done when a bucket, which weighs 2 lbs. and contains 2 gallons of water, is lifted 30 feet from a well? [1 gall. of water weighs 10 lbs.]

10. A horse draws a cart with a steady pull of 90 lbs. wt. How much work does it do in drawing the cart 1 mile?

11. What is the total amount of work done by a man of 15 stone when he carries a load of 50 lbs. up a ladder through a vertical height of 20 feet?

How much work would he do if instead of mounting the ladder, he were to haul the weight up through the same height by means of a rope passed over a pulley? (Neglect the work done in overcoming the friction of the pulley.)

12. What work has to be done to lift 50 tons of coal from a lighter to the deck of a ship if the average distance through which it has to be raised is 30 feet?

13. A man rowing in a boat-race exerts on his oar, during each stroke, a steady horizontal pull of 33 lbs. wt. through a distance of 4 feet. If he rows for 7 minutes at an average rate of 36 strokes per minute, find the total amount of work he does in the time.

14. If 220 ft. lbs. of work are expended in pulling a box a distance of 12 feet along the floor, calculate the force of friction.

15. If the resistance to the motion of a cyclist be taken as 5 lbs. wt., how much work does he do when he travels half a mile?

16. During the forward stroke with a jack-plane, in planing a piece of wood, a man exerts a constant force of 4 lbs. wt. through a distance of 3 feet. To bring the plane back again he applies a constant force of $\frac{1}{2}$ lb. wt. Find how much work he does in making 20 complete strokes.

17. A punch exerts a uniform force of 30 tons wt. in punching a hole through a $\frac{1}{2}$ inch iron plate. What is the work done?

18. An anchor whose weight is 5 cwt. is 40 feet below the surface of the water. Find the work which has to be done on it to raise it to the surface, and then lift it a further height of 20 feet, supposing that the apparent weight of the anchor in water is $\frac{4}{5}$ths of its weight in air.

19. A man in turning a winch exerts a constant force of 35 lbs. wt. on the handle of a crank, 10 inches long. If the direction of this force is always at right angles to the crank, find the work done in 40 revolutions of the handle.

20. Suppose that a cyclist applies an average driving force of 10 lbs. wt. continuously in a direction always at right angles to the cranks, which are 7 inches long. If for one revolution of the pedals he travels a distance of 19 feet, how much work does he do on the bicycle when he rides half a mile?

21. Calculate the work done in pulling a roller weighing 3 cwt. a distance of 40 yards up a slope of 1 in 20 (the slope rises 1 foot vertically for every 20 feet along the slope). The pull is exerted in a direction parallel to the slope. Neglect the effect of friction.

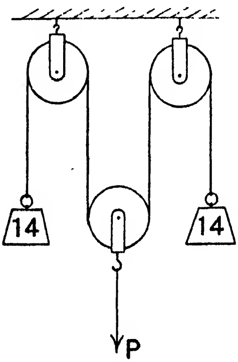
Hence, by the Principle of Work, calculate the steady pull required.

22. What work does a boy do when he rides a bicycle a hundred yards on the level? The force to be overcome is chiefly the resistance of the air which may be taken to be 3 lbs. wt.

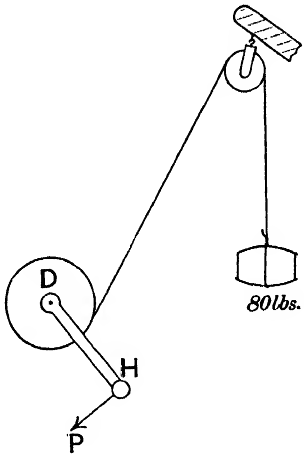
If the boy and bicycle together weigh 140 lbs., how much work will the boy do in riding up a hill, a quarter of a mile long, if the road rises 1 foot vertically for every 18 feet along the surface, assuming that the resistance of the air remains the same?

23. In a wheel and drum (as shewn in Fig. 45 on page 41) the diameters of the wheel and drum are 10 inches and 3 inches respectively. Neglecting friction, calculate by the Principle of Work what effort is required to balance a load of 48 lbs.

24. Referring to the arrangement in Fig. (*a*), the movable pulley is pulled down by a force P. Neglecting friction and the weight of the movable pulley, calculate the work done in pulling the movable pulley down 1 foot. Hence find the value of the force P required.



(*a*)



(*b*)

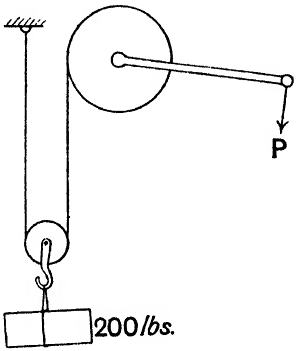
25. (See Fig. 43 on page 39.) Neglecting friction, find the work done in pulling a roller weighing 5 lbs. a distance of 2 feet up a plane inclined at an angle of 30° to the horizontal. What is the pull required?

26. To pull a roller, weighing 10 lbs., up an inclined plane a force of 3 lbs. wt. is required, when applied in a direction parallel to the plane. If the friction is negligible, find the inclination of the plane.

27. (Fig. *b*.) A simple winch consists of a drum D turned by the handle H. The rope attached to the load of 80 lbs. passes over a fixed pulley and is wound on the drum. The diameter of the drum is 6 inches, and the length of the crank is 14 inches. Calculate the work done on the load when the handle makes one revolution. Hence, neglecting friction, find what force P must be applied to the handle at right angles to the crank in order to raise this load.

28. To draw a cart weighing 1 ton up a slope of 1 in 30 requires a force of 112 lbs. wt. Calculate the work done against friction in pulling the cart 100 yards up the slope.

29. To drag a box weighing 1 cwt. up an incline of 1 in 4 requires a force of 80 lbs. wt. Calculate the force of friction.



(*c*)

30. (Fig. *c*.) A load of 200 lbs. is raised by means of a simple winch and a movable pulley. The diameter of the drum of the winch is 7 inches and the length of the crank is 15 inches. Calculate the work done on the load when the winch makes one turn.

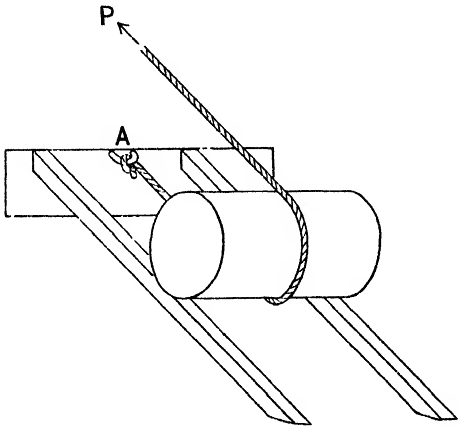
Hence, neglecting friction and the weight of the movable pulley, calculate what force must be applied to the handle of the winch to raise the load with steady motion.

31. A roller weighing 2 cwt. is pushed up a slope of 10° by applying a *horizontal* force to it. Find the work done on the roller in moving it 10 feet up the slope, and find the distance through which the horizontal force has been exerted in its own direction. Hence, neglecting friction, calculate by the Principle of Work the horizontal force required.

32. Fig. (*d*) shews a method by which a heavy cylinder may be raised. A rope is fixed at A and pulled in the direction P so as to cause the cylinder to roll up the sloping rails. These rails are 8 feet long and rise to a height of 3 feet.

Calculate the work done on the cylinder in raising it vertically 3 feet, and find through what distance the force P has to be exerted to do this.

Hence, by the Principle of Work, calculate the magnitude of the pull P required.



(*d*)

33. What is meant by saying that the *velocity-ratio* of a certain machine is 16? What load could be overcome by applying an effort of 20 lbs. wt. to this machine if there were no resistances to be overcome in the machine itself?

34. A lifting machine has a velocity-ratio of 20, and a weight of 100 lbs. is lifted by an effort of 12 lbs. wt. Find the mechanical advantage and the efficiency of the machine in this case.

35. With the aid of a certain arrangement of pulleys or 'tackle,' whose velocity-ratio is 6, a load of 160 lbs. is raised by an effort of 44 lbs. wt. After the pulleys have been oiled, the effort required to lift the same weight is only 37 lbs. wt. Find the efficiency of the machine in each case.

36. By means of a certain lifting machine it is found that a load of 500 lbs. can be raised 1 foot by exerting an effort of 60 lbs. wt. through a distance of 20 feet. What is the velocity-ratio of this machine? What are the mechanical advantage and the efficiency of the machine in this particular case?

37. A certain machine has a velocity-ratio of b. For loads of 50 lbs., 100 lbs., and 150 lbs. the corresponding efforts are found by experiment to be 24 lbs. wt., 43 lbs. wt., and 53 lbs. wt. respectively. Find the efficiency for each of these loads.

38. In Fig. (*b*), if the diameter of the drum of the winch is 4 inches and the length of the crank 1 foot, calculate the velocity-ratio.

39. A Bramah Press has a velocity-ratio of 144. Assuming its efficiency to be 80 %, find what load or resistance can be overcome by an effort of 50 lbs. wt.

40. If a lifting machine having a velocity-ratio of 28 lifts a load of 250 lbs. with an efficiency of 42·5 %, what effort is required and what is the mechanical advantage?

41. (Fig. *c*.) If the diameter of the drum of the winch is 5 inches and the length of the crank is 14 inches, calculate the velocity-ratio of the machine. If it is found that a load of 2 cwt. can be raised by exerting an effort of 48 lbs. wt., calculate the efficiency in this case.

42. A machine is contrived by means of which a weight of 125 lbs. in falling 12 feet is able to lift a weight of 15 cwt. to a height of 4 inches. Find the work done by the falling weight, and what part of the work is used in overcoming the friction of the machine.

43. With a system of pulley-blocks such as is shewn in Fig. 47 on page 47 it was found that an effort of 12 lbs. wt. was required to raise a load of 19 lbs. Calculate the mechanical advantage and the efficiency in this case.

If the movable pulley weighed 1 lb., calculate how much of the effort was employed in overcoming the friction of the machine.

44. With a machine having a velocity-ratio of 28, it is found that a load of 5 cwt. can be raised by an effort of 42 lbs. wt. What is the friction-effect, that is, what part of the effort is used in overcoming the resistances in the machine?

How much work is done against friction in raising this load 2 feet?

45. Calculate the velocity-ratios of the tackles shewn in Figs. (*e*), (*f*), (*g*) and (*h*), giving in each case the reasoning on which you base your answer.

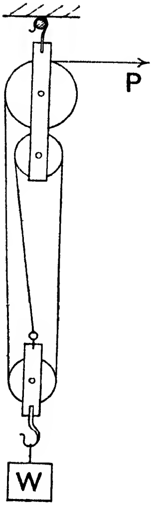
46. Find what load can be lifted by each of the tackles in Example 45 by an effort of 50 lbs. wt., assuming them all to have an efficiency of 60 %.

47. Assuming all the tackles in Example 45 to have an efficiency of 64 %, find what effort must be applied in each case to lift a load of 200 lbs. wt.

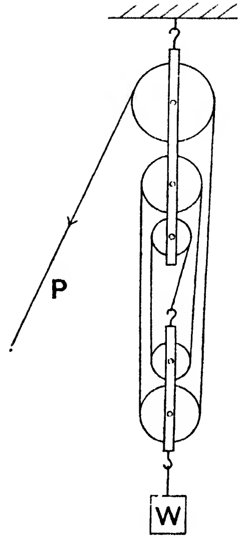
48. Draw diagrams to shew how you would construct tackles with velocity-ratios of 7 and 8 respectively.

How would you find the efficiency of one of these for a load of 2 cwt.?

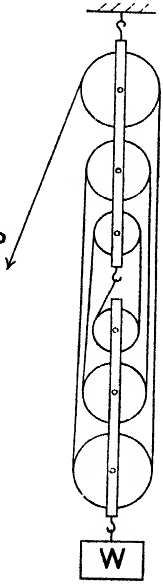
(e)



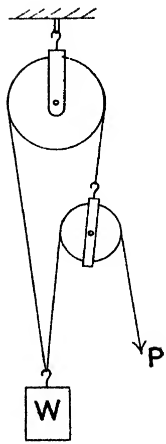
(f)



(g)



(h)



49. Sketch a worm and worm-wheel, and describe its action.

50. A worm and worm-wheel are used for applying the twist to a bar of iron under a torsion test. The worm is turned by a hand-wheel, and the bar under test is connected to the shaft on which the worm-wheel is mounted. If the worm-wheel has 80 teeth and the worm is single-threaded, how many degrees of twist will be given to the iron bar by turning the hand-wheel through 160 revolutions?

51. A bicycle fitted with 7-inch cranks has a back-wheel whose diameter is 28 inches, and is geared so that this wheel turns $2\frac{1}{2}$ times for each revolution of the pedals. Calculate the velocity-ratio of the bicycle.

If it be supposed that the cyclist exerts a steady force of 20 lbs. wt. on the driving pedal always at right angles to the crank, and if the external resistance to motion be reckoned at $3\frac{1}{2}$ lbs. wt., find the efficiency.

52. What is meant by saying that the gear of a bicycle is 72?

The number of teeth in the sprocket wheels of a bicycle are 45 and 18 respectively. If the diameter of the wheel is 28 inches, find the gear of the bicycle.

If the cranks are 7 inches long, what is the velocity-ratio?

53. You wish to design a machine to give a mechanical advantage of 20. If the maximum efficiency which you can expect is 40 %, what is the least value you could take for the velocity-ratio?

54. The mechanism of a machine is concealed in a closed case, from which hang down two chains. You find that if one chain is pulled down the other rises. How would you find by experiment the efficiency of the machine when employed to lift a given load?

55. Shew that with all machines the efficiency is given by $\frac{W}{P \cdot V}$, where W = the load, P = the corresponding effort, and V = the velocity-ratio.

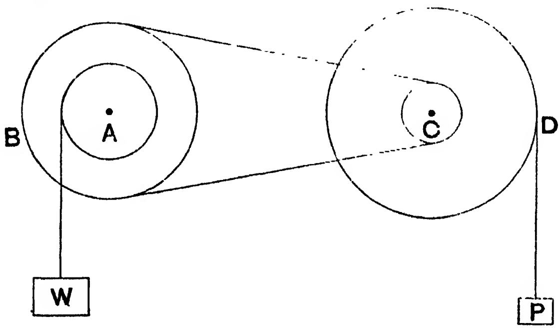
56. With a certain machine whose velocity-ratio is 16, it is found that an effort of 18 lbs. wt. is required to make the machine overcome a resistance of 1 cwt. How much of this effort is instrumental in making the machine overcome the resistance, and how much is employed in overcoming the friction of the machine?

57. Calculate the velocity-ratio of a wheel and drum in which the wheel has a diameter of 2 feet 9 inches and the drum has a diameter of 5 inches; and neglect the size of the ropes.

If the rope on the wheel is $\frac{1}{4}$ inch in diameter and that on the drum $\frac{1}{2}$ inch in diameter, what is the true velocity-ratio?

58. An 18″ pulley on an engine shaft drives by a belt a 32″ pulley on another shaft. This second shaft has upon it a single-threaded worm which gears with a worm-wheel, having 80 teeth, on a third shaft.

How many times must the shaft of the engine rotate in order to rotate this third shaft once?



(*i*)

59. In Fig. (*i*) *A* and *B* represent two pulleys keyed to the same spindle, and *C* and *D* two pulleys keyed to a parallel spindle. A band or belt passes round *B* and *C*. A weight *W* is attached to a cord wound on *A*, and is balanced by a weight *P* on the end of a cord wound on *D*.

The diameters of *A*, *B*, *C*, and *D* are 5, 9, 3 and 11 inches respectively.

Neglecting friction, calculate the ratio of *W* to *P*.

60. The ram or plunger of the cylinder which works a hydraulic crane in Fig. 50 on p. 52, has a diameter of 6 inches. If the pressure of the water is 1800 lbs. wt. per square inch, and the efficiency of the machine is 80 %, calculate the greatest load which can be raised at a uniform speed.

CHAPTER III

LAWS OF MACHINES

32. Experiments on machines. In the preceding chapter we have learnt that, if there were no internal resistances to be overcome, we should get back as much work from a machine as we expend upon it; that is, the efficiency would be 1 or 100 %; and the mechanical advantage would equal the velocity-ratio. If we were to increase the load on such a machine, it is clear that the necessary effort would increase in the same ratio, while the mechanical advantage and efficiency would remain constant. We know, however, that the effects of friction are generally considerable and (i) increase the effort necessary to raise any given load, and (ii) decrease the mechanical advantage and efficiency.

The next question which arises is, How do these effects change as the load on the machine increases?

To answer this question, we now proceed to test machines under various loads to find out if there is any simple law connecting the friction-effect of the machine and the load upon it, and to ascertain how the effort, efficiency and mechanical advantage are thereby affected.

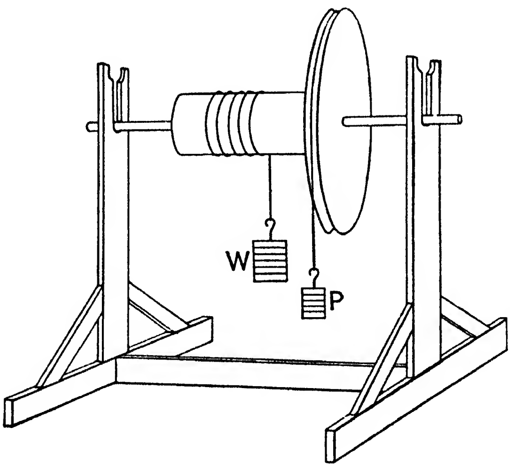
These tests are carried out as follows:—(1) The velocity-ratio is calculated from the dimensions of the machine. (2) This value of the velocity-ratio is checked by direct measurement. (3) The load is increased by equal steps up to the maximum the machine can safely carry, and the corresponding efforts required to raise these loads are determined. These values are tabulated, and from them the friction-effect and the efficiency are calculated for each load.

The values of these quantities are then plotted on squared

paper for corresponding values of the load, to shew the relations existing between the

Effort and Load,
Friction-effect and Load,
Efficiency and Load.

33. Test of a Wheel and Drum. A common type of this machine is a simple winch, illustrated in Fig. (*b*) on p. 56. The effort is exerted on the handle of a crank which is keyed to the end of the spindle. In order to experiment upon the machine, we remove the crank and substitute a wheel having a radius equal to the length of the crank, and apply the effort to the end of a cord wound round it. We then have the machine in the form shewn in Fig. 45 on p. 41, which for convenience is printed again here.



(1) *Velocity-ratio by calculation.* When the spindle revolves once, the load W is raised a distance equal to the circumference of the drum, and the effort P descends a distance equal to the circumference of the wheel.

Hence the Velocity-ratio, which is

$$= \frac{\text{Displacement of effort}}{\text{Corresponding displacement of load}}$$

$$= \frac{\text{Circumference of Wheel}}{\text{Circumference of Drum}}.$$

These circumferences are found by winding the cords used once round the wheel and drum, and then measuring on a scale the lengths required.

With this particular machine, these measurements were found to be

Circumference of Wheel = 37·8 inches,

Circumference of Drum = 9·4 inches,

$$\text{Velocity-ratio} = \frac{37{\cdot}8 \text{ ins.}}{9{\cdot}4 \text{ ins.}} = \mathbf{4{\cdot}02}.$$

(2) *Velocity-ratio by experiment.* Two metre scales are clamped vertically close beside the weights W and P, and the position of W is adjusted so that its lower edge is opposite to a convenient division of the scale. The height of the lower edge of P is then measured. P is now pulled down till W has risen a convenient distance, such as 20 cms., and the height of the lower edge of P is again measured. The difference between the initial and final heights of P gives the distance that P descends. In this way it was found with this machine that, to raise W through 20 cms., P had to be pulled down 79·4 cms.

Hence $$\text{Velocity-ratio} = \frac{79{\cdot}4 \text{ cms.}}{20 \text{ cms.}} = \mathbf{3{\cdot}97}.$$

In our calculations we shall take the velocity-ratio of this machine as 4.

(3) We have next to find the efforts necessary to raise a series of loads. To do this we increase, step by step, the load on the end of the cord wound on the drum and at each step add weights to the end of the string wound on the wheel, until, on being given a start, these weights descend steadily ($\frac{1}{10}$ lb. and $\frac{1}{100}$ lb. weights are used).

The results of an experiment with this machine are given in the first two columns of Table I.

TABLE I. *Wheel and Drum.*

Velocity-ratio 4.

Forces exerted *on*, and *by*, the machine.

Load lbs. wt	Effort lbs. wt.	*Useful* effort lbs. wt.	Friction-effect or *wasted* effort lbs. wt.
0	0·50	0	0·50
2	1·02	0·50	0·52
5	1·86	1·25	0·61
10	3·29	2·50	0·79
15	4·67	3·75	0·92
20	6·05	5·00	1·05
25	7·50	6·25	1·25
30	8·86	7·50	1·36
35	10·30	8·75	1·55
40	11·70	10·00	1·70

The *useful* effort (col. 3) is that part of the actual effort which is instrumental in raising the load only, and is found by dividing the load by 4, the velocity-ratio (Art. 26). By subtracting the useful effort from the actual effort we obtain the wasted effort or friction-effect (col. 4).

34. Relation between Effort and Load. To illustrate this relation, we plot on squared paper the values of the efforts in col. 2 against the corresponding values of the loads in col. 1 (Fig. 51). The latter are represented by distances measured parallel to the horizontal axis (*abscissae*), and the former by distances measured parallel to the vertical axis (*ordinates*).

Now, if there is any simple law connecting the effort and load, we should expect these points to lie on a straight line or a simple curve. We find that the points lie very nearly on a straight line, so we draw the straight line which lies most evenly

among all the points. This we do by stretching a piece of fine cotton along the points, shifting it about until it appears to occupy the best position, then, pricking through the cotton at each end, we draw a straight line between the two points thus obtained.

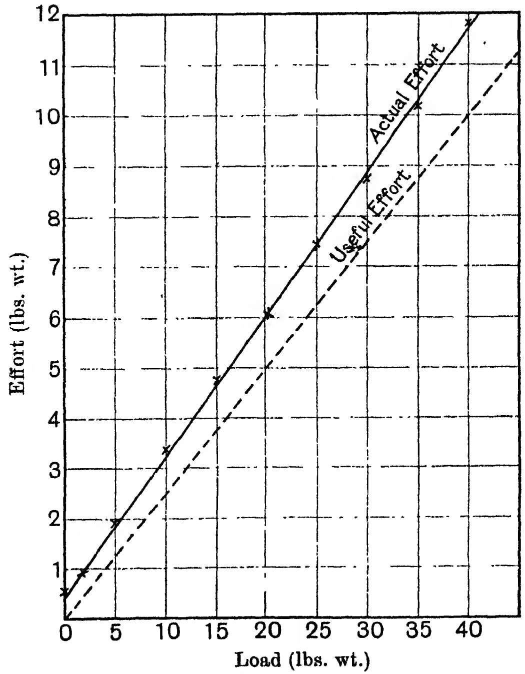


Fig. 51. **Relation of Effort to Load in Wheel and Drum.**

What is the simple law here represented? We see that if we increase any load by 5 lbs. wt., the corresponding increase in the effort is 1·4 lbs. wt.; if we increase any load by 10 lbs. wt., the increase in the effort is also twice as much as before, namely 2·8 lbs. wt.; and so on. That is, the *increase* in the effort is directly proportional to the *increase* in the load.

The straight line does not pass through the origin but cuts the vertical axis at a point representing about 0·5 lb. wt. This is the effort required to work the unloaded machine and is therefore the friction-effect of the machine alone.

A similar simple law will be found to apply to all machines. The graph shewn by the dotted line is that obtained by plotting the values in col. 3 against the corresponding loads. This shews us at a glance the value of the effort which would be required to raise any given load if there were no resistances to be overcome in the machine itself, that is, it tells us what part of the actual effort is instrumental in raising the load.

35. Relation between Friction-effect and Load.

To illustrate this relation we plot the values of the friction-effect

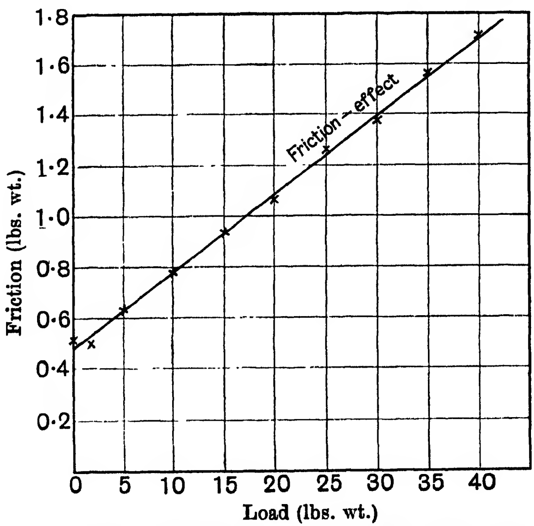


Fig. 52. Relation of Friction-effect to Load in Wheel and Drum.

against the loads (Fig. 52). A larger scale, however, is here taken for the friction than that used for the effort in Fig. 51. Here again, we observe that the points lie approximately on

a straight line, shewing that *any increase of the load produces a proportionate increase in the friction-effect.*

If we alter the bearings of our machine in any way, for instance, by cleaning them or by using different lubricants, we obtain a different series of values for the friction-effect, but we find that the law connecting this force and the load remains of the same simple kind. Moreover, if we carry out a similar test with any machine, be it an arrangement of pulleys, a screw-jack, or a worm and wheel gear, we always find a similar law.

Reverting to Fig. 51, observe that the friction-effect, being the difference between the actual and the useful efforts, is represented for any load by that part of the corresponding ordinate which lies between the two graphs. When there is no load, the friction-effect, which is about 0·5 lb. wt., is caused by the weights of parts of the machine itself.

36. Relation between Efficiency and Load. Having considered the forces exerted on, and by, the machine, let us now consider the work done by each of these forces when the load is raised any given distance.

TABLE II. *Wheel and Drum.*

Analysis of work done on the machine.

Work done on Load in raising it 1 foot ft. lbs.	Corresponding work done by Effort ft. lbs.	Work done against Friction ft. lbs.	Efficiency per cent.
0	2·00	2·00	0
2	4·08	2·08	49·0
5	7·44	2·44	67·2
10	13·16	3·16	76·0
15	18·68	3·68	80·3
20	24·20	4·20	82·6
25	30·00	5·00	83·3
30	35·44	5·44	84·7
35	41·20	6·20	85·0
40	46·80	6·80	85·5

The values in Table II are calculated from our experimental results in the first two columns of Table I as follows. To take an example; when a load of 10 lbs. is raised through 1 ft., the work done upon it is 10 ft. lbs. (col. 1). Since the corresponding effort, namely 3·29 lbs. wt., is exerted through 4 ft., the work done on the machine is $4 \times 3{\cdot}29$ or 13·16 ft. lbs. (col. 2). The difference, namely $13{\cdot}16 - 10$ or 3·16 ft. lbs. is the work done

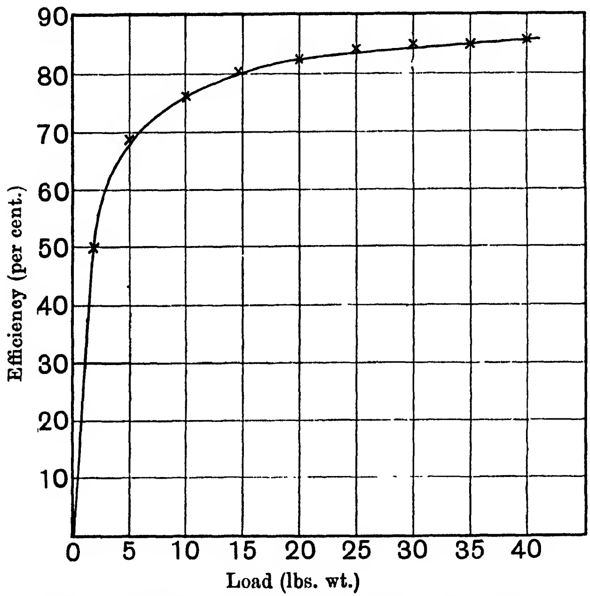


Fig. 53. **Relation of Efficiency to Load in Wheel and Drum.**

against friction (col. 3). The same result is obtained by multiplying the corresponding friction-effect by 4; hence these values in col. 3 increase with the load according to the same law as the friction-effect. That is, we can express the friction law of the machine by saying that *any increase in the load causes a proportionate increase in the work done against friction, during a given movement.*

Let us now see how the efficiency of the machine changes as the load is increased. Since the efficiency is the ratio

$$\frac{\text{Useful work done on load}}{\text{Corresponding work done by effort}},$$

the values in col. 4 are obtained by dividing those in col. 1 by those in col. 2. Thus, when the load is 10 lbs. the efficiency $= \frac{10 \text{ ft. lbs.}}{13\cdot16 \text{ ft. lbs.}} = \cdot76$; or, for every 100 ft. lbs. of work done on the machine, we get back in a useful form $100 \times \cdot76 = 76$ ft. lbs. That is, the efficiency per cent. $= 76$. The efficiencies are now calculated and plotted to a base of loads (Fig. 53).

Here we find the points lie on a curve which is at first steep, but becomes less steep as the load increases and finally becomes nearly horizontal. The efficiency is low for small loads and rapidly increases up to a certain point beyond which the increase becomes very slow.

It would obviously be uneconomical, as far as work is concerned, to use this machine for raising loads or overcoming other resistances if the values of these were less than about 15 lbs. wt.

We notice with this machine that if the effort is removed, the load runs down, *i.e.* the machine 'overhauls.' This is prevented in practice by using a brake or by fitting to the machine a *pawl* and *ratchet wheel* (Fig. 54). When the pawl engages the teeth, the spindle can only turn in the direction necessary to raise the load. Compare the winch used for pulling up a tennis net.

37. The Screw-Jack. This machine is very extensively used in workshops, and for building operations, to lift very heavy weights through short distances. One common form of it is illustrated in Fig. 55. S is a square-threaded iron screw, which works in a stout nut C mounted on a strong tripod support. The screw terminates in a spherical iron head, which is pierced by two sockets at right angles to each other. An iron cap is fitted to the top of the head, being held in position by a vertical

pin about which it can turn freely. To use the screw-jack it is placed beneath the object to be lifted, a bar is inserted in one of the sockets in the head and, by applying a force to the end of this bar, the screw is turned and the body thereby raised. The cap turns on the head, so no rotation is given to the object.

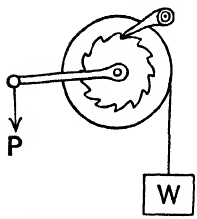


Fig. 54. Pawl and ratchet wheel.

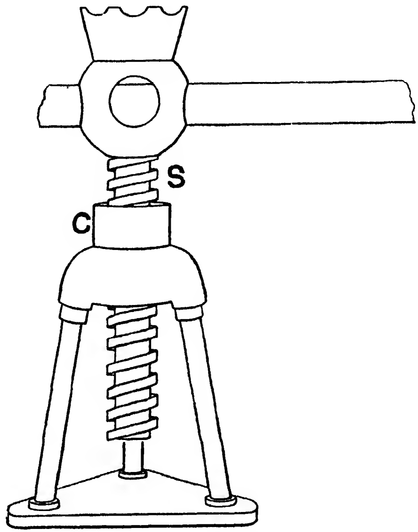


Fig. 55. Screw-jack.

One very convenient property of this machine is that it does not 'overhaul,' a property which we found was not possessed by the Wheel and Drum. The screw when left to itself continues to support the load, and does not rotate backwards.

38. Velocity-ratio of a Screw-Jack. When the screw-head, which is single-threaded, makes one complete revolution, the load is lifted through a distance equal to the pitch of the screw, that is, the distance between the lower faces of successive threads. The effort is applied to the end of the bar, and most

effectively in a direction always at right angles to it. Therefore, during one revolution, the effort moves through the circumference of a circle, whose radius is the distance from the axis of the screw to the point of the bar at which the effort acts. Hence the velocity-ratio is the ratio of this circumference to the pitch of the screw.

For instance, if the distance from the central axis of the screw to the point of the bar at which the effort acts is 2 feet, and the pitch of the screw is $\frac{1}{2}$ inch, then

$$\text{Velocity-ratio} = \frac{48\pi \text{ inches}}{\frac{1}{2} \text{ inch}} = \mathbf{302} \text{ approx.}$$

The Screw-Jack has a very high velocity-ratio.

39. Test of a Screw-Jack. A Screw-Jack adapted for experimental purposes is shewn in Fig. 56. The spherical head of the screw is replaced by a flat horizontal table, on which to

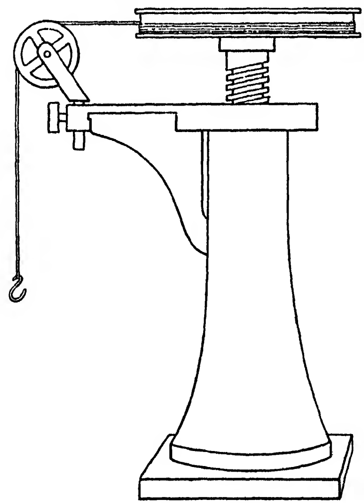


Fig. 56. Bottle Screw-Jack, adapted for testing purposes.

place the weights representing the load. The table is circular, and has a broad flat groove round its edge. A cord, one of whose ends is made fast to a stud, is coiled round the groove and passed over a small pulley carried on a projecting arm. Weights attached to the free end of the cord cause the screw to turn and lift the load. These weights measure the effort.

To determine the velocity-ratio by experiment, a wooden scale is clamped vertically close to the table of the jack, and the height of the latter adjusted so that its upper edge is opposite to a convenient division of the scale. A pencil mark is then made on the edge of the table opposite the scale. The table is now raised by rotating it exactly 10 times, and the distance it has risen is measured on the scale. By dividing this distance by 10 we obtain the pitch of the screw. The circumference of the table is now determined by winding the cord once round it and measuring on a scale the length required. This is the distance through which the effort is exerted to raise the load through the pitch of the screw, and therefore by dividing the former distance by the latter we obtain the velocity-ratio.

With the particular Screw-Jack we are testing, the pitch of the screw is found to be exactly $\frac{1}{2}$ inch, and the mean of three measurements of the effective circumference of the table gives a length of 25·5 inches.

$$\text{Hence, the velocity-ratio} = \frac{25{\cdot}5 \text{ ins.}}{\frac{1}{2} \text{ in.}} = \mathbf{51}.$$

In testing this machine we increase the loads by 28 lbs. at a time up to a total load of 168 lbs. The effort required to raise each load is found by adding sufficient weights to the end of the cord to make it descend steadily when once it has been started with a slight pull.

The results are given in the first two columns of Table III.

TABLE III. *Test of a Screw-Jack.*

Velocity-ratio = 51.

Load lbs. wt.	Effort lbs. wt.	Useful effort lbs. wt.	Friction-effect lbs. wt.	Efficiency per cent.
28	1·85	·55	1·30	29·7
56	3·20	1·10	2·10	34·3
84	4·60	1·65	2·95	36·6
112	5·85	2·20	3·65	37·5
140	7·30	2·75	4·55	38·1
168	8·50	3·30	5·20	38·6

How are the values in the last 3 columns obtained? Fig. 57 shews the results of plotting on squared paper the effort, friction-effect, and efficiency, to the same base of loads. The same vertical scale is used in this case for both the effort and the friction-effect, but a different scale is taken for the efficiency.

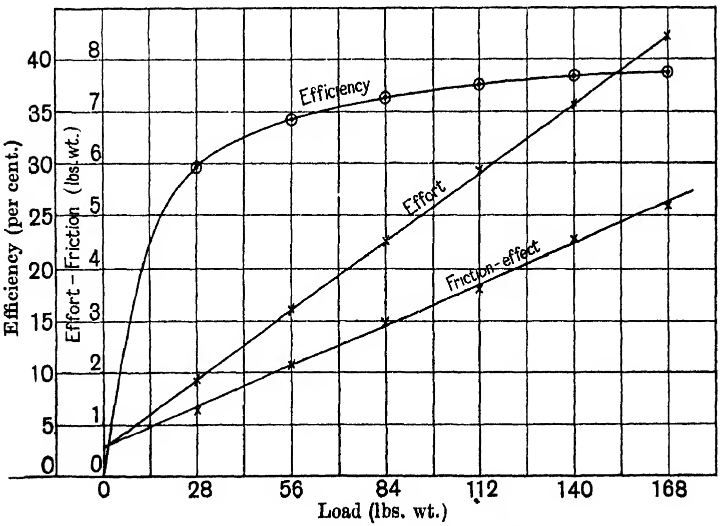


Fig. 57. Results of test of Screw-Jack.

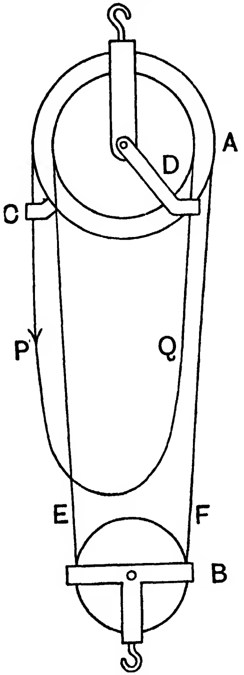
Although the values obtained for these different quantities differ very much from those obtained with the Wheel and Drum —the greatest measured efficiency of this machine being less than half that of the Wheel and Drum—the graphs shew that the Screw-Jack obeys similar laws. When you have performed this test for yourself, ask someone who has weighed himself to tell you his weight. Find from your Effort-Load graph the effort which should be required to raise him with the jack. Then while he stands on the table of the jack, test your result by finding what weight on the end of the cord is required to raise him.

Note that the values given for the friction-effect include in each case that constant part of the effort which is used in raising the screw and table. The screw and table together are found to weigh 10·5 lbs., and hence the force used in raising them is $\frac{10 \cdot 5}{51}$ or 0·2 lb. wt. By subtracting this force from each of the values in the third column of Table III we could obtain a force in each experiment which is effective in turning the screw against the frictional force alone. We shall commonly use the name 'friction-effect' in the wider and not strictly accurate sense.

40. Weston's Differential Pulley Block. The Differential Pulley Block is another machine which is commonly used for lifting heavy weights.

Its construction is explained by the diagram of Fig. 58. *A* is an iron block which has two sheaves of unequal diameters. These sheaves are cast in one piece and consequently turn together. An endless chain *PQEF* passes over each of the sheaves in turn, and hangs down in two loops. As it is essential that the chain should not slip when it is in contact with the upper block, pairs of lugs which project inwards are cast in the grooves of the sheaves, so spaced that there is just sufficient room for one link of chain to lie flat between consecutive pairs of them (Fig. 59). In addition, the chain passes through apertures at the ends of the arms *C* and *D*, which act as guides and ensure

that the chain is presented to the sheaves in such a way that it will be properly gripped by the lugs.



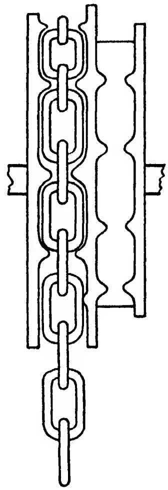


Fig. 59. Side view of the upper block of a Weston's Differential Pulley Block, shewing how the chain is gripped by the lugs.

Fig. 58. Weston's Differential Pulley Block.

One of the loops carries a movable block B which has no lugs. The load is suspended from the hook below B, and is raised by pulling on the chain at P. We find that this machine, like the screw-jack, does not overhaul.

41. Velocity-ratio of the Differential Pulley Block.

A Differential Pulley Block, designed to lift $\frac{1}{4}$ ton, has 8 pairs of lugs on the larger sheave and 7 pairs on the smaller one, and, on examining the chain on the block, we notice that for each pair of lugs we have two links of chain. Hence the effective circum-

ferences of these sheaves are equal in length to 16 and 14 links of chain respectively.

Now suppose we pull on the chain at P until the upper block has made one complete revolution. During this movement 16 links of chain pass from the loop EF to the loop PQ. But the smaller sheave has also made one complete revolution, and in so doing has wound 14 links of chain from the loop PQ on to the loop EF. On the whole, then, the loop EF has been shortened by two links of chain. It follows that each side of the loop of the chain supporting the movable block and the load is shortened by one link, which is therefore the distance through which the load has been raised. Thus, when the effort is exerted through a distance equal to the length of 16 links of chain, the load is raised a distance of 1 link. The velocity-ratio is therefore 16. How would you test this value of the velocity-ratio by direct measurement?

42. Results of a test of a Differential Pulley Block. To find the efforts required to raise the same series of loads as we used in the case of the screw-jack, sufficient weights are hung from links on the chain P to make it descend steadily when once it has been given a start. These weights are conveniently attached by light steel hooks inserted into the links of the chain.

The experimental results obtained with a Differential Pulley Block are given in the first two columns of Table IV.

TABLE IV. *Test of a Weston's Differential Pulley Block.*
Velocity-ratio = 16.

Load lbs. wt.	Effort lbs. wt.	Friction-effect lbs. wt.	Efficiency per cent.
28	6·00	4·25	29·2
56	10·50	7·00	33·3
84	15·00	9·75	35·0
112	18·50	11·50	35·9
140	23·75	15·00	36·7
168	28·50	18·00	36·8

How are the values of the friction-effect and the efficiency calculated? Fig. 60 shews the results obtained by plotting these values in the same way as with the foregoing machines. As in the test of a screw-jack, the same scale can conveniently be used for the effort and the friction, a different scale being used for the efficiency.

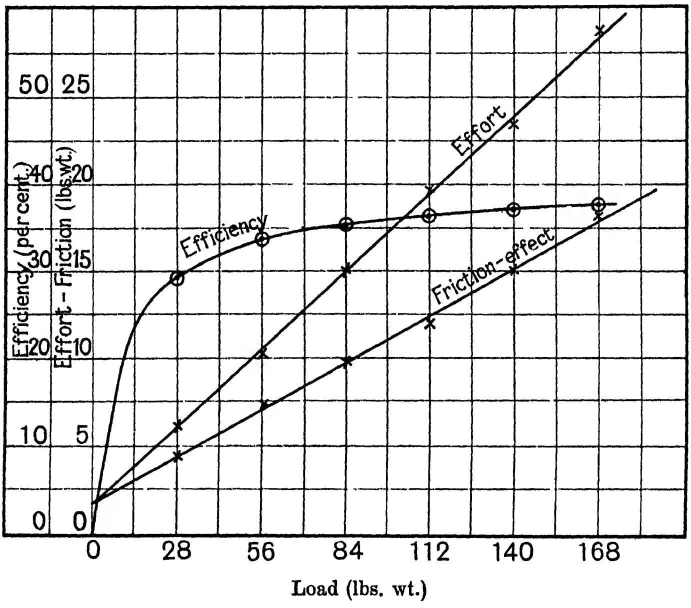


Fig. 60. Results of test of Weston's Differential Pulley Block.

We again find similar laws governing the behaviour of this machine.

43. Relation between Mechanical Advantage and Load. The load-efficiency graph may be used to illustrate the relation between the mechanical advantage and the load.

Efficiency may be expressed as a percentage or as a fraction. In the graphs of Figs. 53, 57 and 60 we have expressed it as a percentage, but in each instance the corresponding fraction may be read; *e.g.* an efficiency of 30 °/₀ is an efficiency of 0·3.

In calculating the efficiency at any load, we have:

$$\text{Work done in raising the load 1 ft.} = W \text{ ft. lbs.}$$

While the load rises 1 foot, the effort acts through V feet, V being the velocity-ratio, so if P is the effort corresponding to a load W,

$$\text{Work done by the effort} = P.V \text{ ft. lbs.},$$

and the efficiency (as a fraction) is $W/P.V$, that is W/P, the mechanical advantage $= V \times$ efficiency, and V is constant for any particular machine.

The reader may now in each of Figs. 53, 57 and 60 set up (for convenience on the right-hand side) a scale of mechanical advantage, and, with this scale, the efficiency graph will illustrate the change of mechanical advantage with changing load.

Like the efficiency, the mechanical advantage becomes nearly constant for loads approaching the magnitude of that for which the machine is designed.

Further, since the effect of friction is to reduce the mechanical advantage of a machine to the same extent as it reduces the efficiency, we see from our results how great this effect is. For example, with the Differential Pulley Block, the highest value of the mechanical advantage we obtained was $\frac{168}{28 \cdot 5} = 5 \cdot 9$. If there had been no friction in the machine, this value would have been equal to the velocity-ratio, namely, 16. If we want a machine with a large mechanical advantage, we must design it to have not only a large velocity-ratio, but at the same time little friction.

43*a*. Comparison of the characteristics and useful applications of the foregoing machines. The chief

characteristics of the three foregoing machines are summed up as follows:

Machine	Velocity-ratio	Highest values obtained for	
		Mech. Adv.	Efficiency
Wheel and Drum	4	3·4	86 %
Differential Pulley Block	16	5·9	37 %
Screw-Jack	51*	19·7*	39 %

* *These values are considerably greater in practice when using a bar to turn the Jack* (Art. 38).

In selecting a machine for any given purpose, especially when we want to use manual labour to the best advantage, the efficiency of the machine may be of secondary importance, and other considerations of greater consequence. For example, contrasting the Wheel and Drum with the other two machines we see that its advantage lies not only in its greater efficiency but in the fact that the load can be raised a greater distance, since a considerable length of rope can be wound on the drum. Thus, if we want to raise a sack of coals through 20 feet, we use a machine of this type. Its mechanical advantage however is low, and, although we can increase it by reducing the diameter of the drum, the result of this will be to diminish the amount of rope which can be effectively wound upon it. We can, however, increase the mechanical advantage by combining this machine with a screw, as in the Worm and Wormwheel (Art. 28), or with a train of toothed wheels, as in a Crab Winch (Fig. (*e*) on p. 92).

Turning to the Differential Pulley Block and the Screw-Jack, we find their advantage over the Wheel and Drum lies in their greater compactness, the ease with which they are applied, and their convenient property of not overhauling. The screw-jack is specially remarkable for its high mechanical advantage. We use this machine when we want to overcome

a very large force through a very short distance. For example, to raise the axle of a motor car for the purpose of changing a tyre or a wheel, a screw-jack is obviously the proper machine to employ. Here the main consideration is to produce a large enough force without undue muscular effort; the low efficiency of the machine is of secondary importance, especially as the total work required for such an operation is not large.

On the other hand, with machines which are worked continuously and where much larger quantities of work are involved, the efficiency is generally the most important consideration. For instance, no one would buy a bicycle unless its efficiency were very high.

Efficiency is specially important in machines which are driven, not by muscular effort, but by steam, gas, or water pressure. These machines as a rule run continuously and do great quantities of work. Clearly the cost of running such machines is smaller the greater their efficiency. Hence, especially with large machines, the aim of the designer is chiefly directed towards securing a high efficiency.

44. Equations of Machines. Referring to the results we obtained in our tests of machines, let us consider the numerical relation between the effort and load (and also the friction and load) which is graphically represented on squared paper by a straight line.

This relation for any particular machine we can express algebraically by a simple equation of the form $P = a + b \, . \, W$, which will give us the value of the effort P required to raise any given load W. a and b are two constants for the machine, the values of which depend on the velocity-ratio and the friction.

To understand this let us examine these graphs, which are of the form represented in Fig. 61.

Selecting any point A on the horizontal axis, we see that OA represents a certain load W. The ordinate AB, *i.e.* the vertical

line drawn from A to the graph, represents the corresponding value of the effort P.

Through C, the point where the graph meets the vertical axis, draw CE parallel to OX to cut AB in E.

Now CE equals OA and represents W lbs. wt.

And AE equals OC and represents a constant effort which we will call a lbs. wt.

Also EC equals $(AB - AE)$, and therefore represents $(P - a)$ lbs. wt.

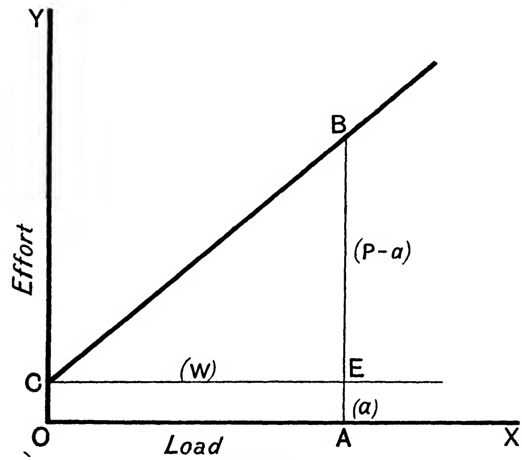


Fig. 61.

Further, we observe that $\frac{EB}{EC}$ is always a constant ratio, *i.e.*

$$\left(\frac{EB}{EC} = \tan BCE\right).$$

Hence $\frac{(P-a) \text{ lbs. wt.}}{W \text{ lbs. wt.}}$ is a constant ratio.

If we call this constant ratio b, we have

$$\frac{P-a}{W} = b, \quad \text{or} \quad P = a + b \,.\, W.$$

This is called the *Equation of the Machine.* It expresses the fact that the effort required to raise any load W is equal to a constant force a lbs. wt., which is the effort required to work the unloaded machine, plus a constant fraction b of the load. The equation enables us to calculate the effort required to raise any given load when the values of the constants a and b for the particular machine are known. Let us find the values of these constants in the case of the Wheel and Drum. Referring to our Effort-load graph we see that when $W=0$, the effort is approximately 0·5 lb. wt. That is, $a=0{\cdot}5$ lb. wt.

Now, taking corresponding values of W and P from the graph we find, for example, that to overcome a resistance of 40 lbs. wt., an effort of 11·7 lbs. wt. is required. Substituting these values in the equation $P=a+b\,.\,W$, we have

$$11{\cdot}7=0{\cdot}5+b\times 40, \quad \text{or} \quad b=0{\cdot}28.$$

The equation of the machine is therefore given by

$$P=0{\cdot}05+0{\cdot}28\,W.$$

Test this equation by calculating from it what effort is necessary to raise a load of 22·5 lbs. wt., and then comparing this result with that read off from the graph.

Shew that the equation for the Screw-Jack (Art. 39) is given by $P=0{\cdot}5+0{\cdot}047\,W$, and that for the Differential Pulley Block (Art. 42) by $P=1{\cdot}5+0{\cdot}16\,W$.

Since the relation between the friction-effect and the load is graphically represented by a similar straight line, it follows that this relation can also be expressed for each machine by an equation of the form

$$\text{Friction-effect}=a+c\,.\,W.$$

We see that a here is the same constant force as that in the equation of the machine, but c is a new constant.

This equation expresses the fact that the friction-effect is equal to that of the unloaded machine, plus a constant fraction c of the load.

Note. The connection between this friction law and that governing the effort, which we must regard as dependent upon it, may be shewn thus:

Actual effort = Useful effort + Friction-effect.

That is,
$$P = \frac{W}{V} + a + c \,.\, W,$$
where V = velocity-ratio, or
$$P = a + \left(\frac{1}{V} + c\right) W.$$

The expression $\left(\frac{1}{V} + c\right)$ is constant for the machine, and is represented by b in the equation of the machine.

44 *a.* **Overhauling.** Let us now enquire why it is that those machines, which (like the Screw-Jack) have a low efficiency, do not *overhaul, i.e.* do not allow the load to run back when the effort is removed, whereas this property is not possessed by machines (like the Wheel and Drum) whose efficiency is high.

Suppose that, in using a certain machine to raise a load of 100 lbs. through 1 ft., we have to do 200 ft. lbs. of work upon the machine. Since only half of this work is usefully employed in raising the load, the other half must be expended in overcoming resistances in the machine, which we will assume are caused by friction only. Thus, the efficiency in this case is $\frac{1}{2}$ or 50 °/。.

Now, if we assume (as is generally the case)* that the removal of the effort does not materially alter the friction between the various rubbing surfaces of the machine, then, in order that the load may descend through 1 ft., it must be capable of doing 100 ft. lbs. of work in overcoming the friction during this movement. We observe that the load of 100 lbs. in descending 1 ft. is just capable of doing this amount of work; hence, if the machine is once set in motion, the load will descend.

If, however, the efficiency of the machine is less than 50 °/。, it means that in raising the load through 1 ft., we have to do more than 200 ft. lbs. of work, and hence more than 100 ft. lbs. in overcoming friction. Consequently, more than 100 ft. lbs. of

* The common pulley block tackle is an exception.

work must be available for overcoming friction before the load can run back.

Since, in descending 1 ft., the load is incapable of doing the work against friction which is now necessary, the machine will not overhaul.

It should be clear from this example that a machine will, or will not, overhaul according as its efficiency is greater, or less than, 50 %.

We can further illustrate this general principle by shewing how it applies in the case of a simple machine such as an inclined plane. Consider the case of a load W being pulled up a plane by an effort exerted parallel to the plane. Let the inclination of the plane be such that when the effort is removed, the load is just supported by the friction between it and the plane, that is, overhauling is on the point of taking place. This means that the load when given a start will move steadily down the plane again.

Observe that when the load has moved down the plane a distance corresponding to a vertical drop of 1 ft., the load has done W ft. lbs. of work against friction. Now, to pull the load up the plane again through the same distance, the same amount of work has to be done against friction, and also a further W ft. lbs. of work are necessary to raise the load through the vertical height of 1 ft. Hence, to do W ft. lbs. of useful work on the load, the effort must do $2W$ ft. lbs. That is, the efficiency is $\frac{1}{2}$ or 50 %.

EXAMPLES III.

1. The following results were obtained in a test of a Wheel and Drum.

Effective circumference of wheel = 90 inches.

Effective circumference of drum = 15 inches.

Load W lbs. wt.	Effort P lbs. wt.
20	4·13
40	7·71
60	11·34
80	14·90
100	18·52
120	22·14

Plot on the same sheet of squared paper to the same base of loads the values of

(*a*) the actual effort P,

(*b*) the effort if there were no friction.

[Use the following scales: Load, 1″=20 lbs. wt.; Effort, 1″=5 lbs. wt.]

Find from your graphs

(i) the effort required to overcome a load of 90 lbs. wt., and hence the mechanical advantage in this case,

(ii) the effort required to turn the machine when unloaded,

(iii) the friction-effect when the load is 85 lbs. wt.

2. From the results given for the Wheel and Drum in Ex. 1, calculate the values of the friction-effect for each load, and plot these values on squared paper to a base of loads.

[Take the following scales: Load, 20 lbs. wt. = 1″; Friction ·5 lb. wt. = 1″.]

What do you learn from the graph thus obtained?

3. From the results given for the Wheel and Drum in Ex. 1, calculate the efficiencies (per cent.) of the machine when the given loads are raised. Plot these values to a base of loads.

[Scales: Load, 20 lbs. wt. = 1″; Efficiency, 20 °/₀ = 1″.]

What do you learn from this curve?

(i) What is the efficiency corresponding to a load of 25 lbs. wt.?

(ii) What is the maximum efficiency of this machine, approximately?

(iii) In overcoming a load of 100 lbs. wt. through a distance of 12 ft., how much work is done against friction?

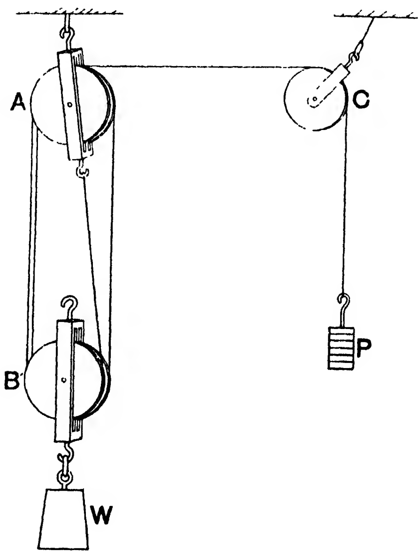
4. A simple lifting screw-jack has a screw whose pitch is $\frac{1}{3}$ inch. How many revolutions must the screw make in order to raise a load through 5 inches?

What must be the effective length of the lever used to turn it, in order that the velocity-ratio may be 132?

What would this ratio be if the length of the lever were 2 ft. 4 ins.?

5. A screw-jack is used to lift a load of 5 cwt. The pitch of the screw is $\frac{1}{4}$ in. If the efficiency of the machine for this load is 40 °/₀, and you did not wish to exert a force of more than 5 lbs. wt., what length of lever would you use?

6. Fig. (*a*) shews the construction of a Laboratory form of a Double-sheaved Pulley Block Tackle. *A* and *B* are two blocks, each with two sheaves. A cord is attached to a hook at the bottom of the upper block, and is passed round the sheaves in turn and then over a fixed block *C*. *W*, the load to be raised, is hung from the lower block *B*, and is lifted by applying to the free end of the cord the effort *P*.



(*a*) Double-sheaved Pulley Block Tackle.

The following results were obtained in a test of this machine:

Load *W* lbs. wt.	Effort *P* lbs. wt.
0·5	·28
1·0	·41
1·5	·55
2	·67
3	·94
4	1·21
6	1·73
8	2·27
10	2·8

(*a*) Calculate the velocity-ratio.

(*b*) Tabulate the values of the efficiency for each load.

(*c*) Plot on the same sheet of squared paper the effort and the efficiency to the same base of loads.

(i) What effort is required to raise a load of 2·5 lbs. and what is the efficiency in this case?

(ii) What is the maximum mechanical advantage, and what is the maximum efficiency of this machine, approximately?

(iii) If the weight of the lower block is $\frac{1}{2}$ lb., find how much of the effort is used in overcoming friction only, when the load is 10 lbs.

7. The pitch of the screw of a jack is $\frac{3}{8}$ in. and the length of the lever used is 2 ft. Find the work that must be done to lift a load of 2 tons to a height of 10 ins., assuming that the efficiency of the machine for this load is 35 °/ₒ.

How much work is done against friction in this case?

8. The following results were obtained in a test of a screw-jack:

Pitch, 2 threads to the inch.

Length of arm, 30 inches.

W cwt.	*P* lbs. wt.
1	1·4
2	2·3
3	3·2
4	4·1
5	4·9
6	5·6
7	6·5
8	7·4

Calculate and tabulate the values of the friction-effect and efficiency (per cent.).

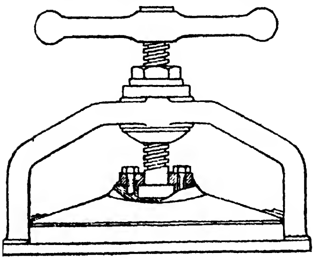
Plot on the same sheet of squared paper, the effort, friction-effect and efficiency to the same base of loads. Use the same vertical scale for the friction as for the effort.

(i) How would you change the scale on which you have plotted the efficiency so that this curve may represent the mechanical advantage in terms of the load?

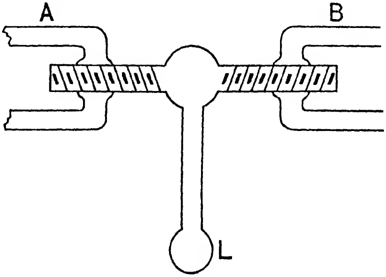
(ii) What is the mechanical advantage when raising a load of $3\frac{1}{2}$ cwt.?

(iii) In raising a load of $6\frac{1}{2}$ cwt. through 6 ins., how much work is done against friction?

9. The screw of a book-press (Fig. *b*) has a pitch of $\frac{3}{8}$ in. Two forces, each of 20 lbs. wt., are applied in opposite directions to the ends of the handle and at right angles to it, at distances 10 ins. from its axis. Calculate the thrust produced on the book, assuming an efficiency of 40 %.



(*b*)



(*c*)

10. Fig. (*c*) represents the screw-coupling used to connect together the carriages of a railway train. The links *A* and *B* are connected by a bar, one end of which has a right-handed, while the other end has a left-handed, screw. The pitch of each screw is $\frac{1}{8}$ in. The efficiency is 45 %.

Calculate the greatest force which can be exerted on each carriage by applying a force of 40 lbs. wt. to the lever *L* in a plane at right angles to the axis of the screw, and at a distance of 16 ins. from this axis.

11. Make a rough drawing of Weston's *Differential Purchase.*

If the effective diameters of the sheaves in the upper block are $3\frac{1}{2}$ and $2\frac{1}{2}$ ins. respectively, find the velocity-ratio of the machine.

If the efficiency for a load of 12 cwt. is 25 %, what force is required to lift this load?

12. In two experiments with a Differential Chain Pulley Block having a velocity-ratio of 16, it was found that, to raise a load of 40 lbs. required an effort of 9·6 lbs. wt.; to raise a load of 100 lbs. required an effort of 20·4 lbs. wt.

What will be the effort required to raise a load of 80 lbs.?

How much work will be used in overcoming friction when a load of 120 lbs. is raised through 3 ft.?

13. In a Weston's *Differential Purchase*, the differential block has sheaves whose effective diameters are in the ratio of 12 to 11.

Calculate the velocity-ratio.

Describe how you would carry out a test with this machine.

14. The following data were obtained with a Worm and Worm-wheel (see Fig. 48 on p. 49): Diameter of wheel $B=12''$. Diameter of drum $D=4''$.

Worm single-threaded. Number of teeth on Worm-wheel $=90$.

W lbs. wt.	50	100	150	200	250	300
P lbs. wt.	1·15	2·00	2·94	3·95	4·86	5·80

(*a*) Calculate the velocity-ratio.

(*b*) Plot on squared paper the actual effort, the useful effort, and the friction-effect to the same base of loads.

(*c*) Plot on squared paper the mechanical advantage and the efficiency to the same base of loads.

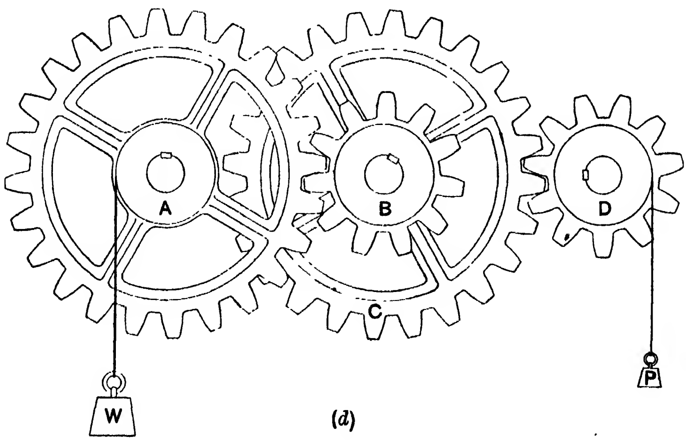
State the chief characteristics of this machine which your results demonstrate.

15. Fig. (*d*) represents a Train of Wheels. A pinion D (11 teeth) gears with a wheel C (24 teeth). On the same spindle as C is fixed a pinion B (11 teeth) which gears with the wheel A (24 teeth).

How many revolutions does A make when D makes 120?

If a load W on the end of a cord wound on a drum which is fixed to the spindle of A is balanced by a weight P on the end of a cord wound on a drum of the same diameter, fixed to the spindle of D, what is the ratio of W to P supposing that there is no friction?

If an effort P of 18 lbs. wt. is capable of keeping a load W of 80 lbs. moving upwards at a steady speed, what is the efficiency of the machine?



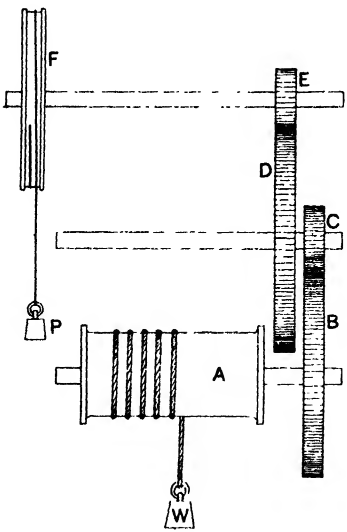
(*d*)

16. With a certain machine having a velocity-ratio of 216 the following results were obtained:

W lbs. wt.	40	80	120	160	200	240
P lbs. wt.	2·24	3·42	4·55	5·73	6·9	8·02

Plot the friction-effect of the machine in terms of the load.
Express in words the law which your graph represents.
What is the friction-effect of the machine, when unloaded?

17. Fig. (e) represents in diagrammatic form a Crab Winch having a double gear. As shewn in the diagram, this machine has been adapted for experimental purposes by removing the crank-handles from the axle FE and substituting the wheel F, in order that we may conveniently apply the required effort P by means of a weight hung from a cord. The pinion E on the same axle as F gears with the wheel D. On the same axle as D is another pinion C gearing with the wheel B. On the axle of B is fixed the drum A round which is wound a rope carrying the load W.



(e)

Calculate the velocity-ratio of this machine from the following data:

Effective circumference of drum $A = 17{\cdot}6''$.

Effective circumference of wheel $F = 32{\cdot}2''$.

Number of teeth on $B = 45$, $C = 10$, $D = 23$, $E = 10$.

18. The following results were obtained when testing the Crab Winch described in Ex. 17.

When finding the velocity-ratio by direct measurement, W was lifted 6·4 cms. while P was pulled down through 121 cms.

W lbs. wt.	0	56	112	168	224	280
P lbs. wt.	5·0	7·9	11·0	14·0	17·0	19·9

Calculate and tabulate the values of the efficiency for each load.

Draw the effort-load and efficiency-load curves on the same sheet of squared paper.

Determine the equation of the machine.

What is the least load at which the machine will overhaul?

19. Shew that the relation between the effort and load for any machine can be expressed by an equation of the form $P = r + s \, . \, W$.

What does the constant r signify? Upon what does the constant s depend?

Find the equation for the machine in Ex. 1.

20. Find the equation for the screw-jack in Ex. 8.

Determine from this equation what effort will be required to raise 1 ton with the jack.

21. In two experiments with a certain machine it was found that, to overcome a resistance of 100 lbs. wt. an effort of 11·2 lbs. wt. was required; and to overcome a resistance of 200 lbs. wt. an effort of 17·2 lbs. wt. was required.

What is the probable equation of the machine?

22. The equation of a certain machine is given by

$$P = 3{\cdot}3 + 0{\cdot}42 W.$$

If the velocity-ratio of the machine is 4, determine the equation connecting the friction-effect and the load.

23. When is a machine said to 'overhaul'? If a machine is on the point of overhauling with a certain load, what is the efficiency of the machine for this load?

The equation of a certain machine is $P = 4{\cdot}02 + 0{\cdot}33W$. If the velocity-ratio is 12, will this machine overhaul?

24. To raise 15 cwt. with a certain screw-jack whose velocity-ratio is 300, an effort of 28 lbs. wt. is required. What effort will be required to lower this load?

25. Explain why a machine will not overhaul if its efficiency is less than 50 %.

26. To drag a load of 80 lbs. up an inclined plane, a force of 30 lbs. wt., acting along the plane, is required. If, on removing the applied force, the load is on the point of sliding down the plane, find the inclination of the plane.

CHAPTER IV

LAWS OF LIMITING FRICTION

45. Friction. Let us first sum up the conclusions we arrived at in our preliminary study of friction in Chapter I (Art. 12).

When we make, or try to make, one body slide over another against which it is pressed, a frictional stress is set up (owing to the inevitable roughness of the contact surfaces), each body exerting a force on the other along the surfaces of contact. This force, with which one surface opposes the sliding of another over it, is called the Force of Friction (or simply the Friction).

As long as no sliding takes place, the force of friction on a surface always adjusts itself to balance the force tending to slide that surface over the other, and hence, when the latter force is increased, the force of friction likewise increases until a certain limiting value is reached.

This limiting value is greater when the surfaces are on the point of sliding over one another than when sliding is actually taking place, that is, the Starting Friction is greater than the Sliding Friction.

We now propose to experiment upon this limiting friction which is called into play when one surface slides, or is about to slide, over another under various conditions.

Recollect that, when dealing with machines, we did not seek to discover the separate forces of sliding friction at the various rubbing surfaces in a machine, but only examined the total effect which these forces produced. This effect we called the friction-effect, and defined it as that part of the effort which is instrumental in overcoming all the resistances within the machine. The simple nature of the law which we found to exist between this effect of friction of a machine and the load upon it, leads us

to expect that experiment will shew that the force of sliding friction of one surface on another also obeys laws of a simple kind.

46. Friction between dry surfaces. We will first investigate the limiting friction of smooth surfaces which are dry, *i.e.* not lubricated.

The following simple but somewhat rough experiments are intended as an introduction to more accurate ones.

A well-seasoned oak board, the surface of which has been planed smooth, is placed on a table. On this board is placed a smooth flat slide (also of some hard well-seasoned wood) to which a spring-balance is connected by a string, as illustrated in Fig. 30 on page 17.

(i) We place a weight of 14 lbs. on the slide, and increase the horizontal pull on the ring of the spring-balance until the slide starts to move. The reading of the balance when the slide is just on the point of starting, gives us the value of the Starting Friction. The reading of the balance while we pull the slide with steady motion, gives us the value of the Sliding Friction. We notice that the starting friction is greater than the sliding friction. Moreover, if we repeat this experiment several times, we observe that whereas the values obtained for the sliding friction are remarkably constant, those for the starting friction vary between rather wide limits, shewing that the starting friction for these surfaces is a somewhat uncertain quantity.

(ii) We next place some other smooth material, such as a plate of glass, between the slide and the board, and again determine the starting and sliding frictions. We now get values differing from those obtained in the first case, shewing that the frictions depend on the materials of the surfaces.

This experiment should also be carried out with a smooth metal slide on a smooth metal plane. In this case it will be found that the values of the starting friction agree more closely among themselves and at the same time are only slightly greater than the values of the sliding friction.

(iii) We again place the slide on the board and determine the values of the starting and sliding frictions when the slide is loaded successively with 7, 14, and 21 lbs.

We first remark that the friction increases with the load. Our results, however, indicate more than this, for we find that when the load is doubled, and then trebled, the friction also becomes approximately first doubled, and then trebled. This leads us to predict that more careful measurements will shew that the limiting friction is directly proportional to the force with which one surface is pressed against the other. That this most important law is true, we shall verify presently by a more accurate method of experiment.

(iv) We now try the effect of altering the area of the rubbing surface. To do so, we determine, as before, the starting and sliding frictions for a certain load, and then reduce the area of contact by splitting off about half the slide. On now repeating our observations with the same load on the smaller slide, we find that our results do not differ to any appreciable extent from those obtained with the larger one. It would appear therefore that the friction is independent of the area of contact. This also we shall verify by more careful experiment.

(v) We now try the effect on the sliding friction of altering the speed with which we pull the slide along the board. We find that the readings of the spring-balance are approximately the same for different steady speeds, shewing that the friction is nearly independent of the speed of rubbing within the moderate range of speeds we can employ.

47. To shew that the Force of Friction on a rubbing surface is directly proportional to the Normal Reaction upon it. Instead of using a spring-balance, we now resort to a more accurate method of measuring the force of friction. The string attached to the wooden slide is passed over a pulley mounted on a bracket which is screwed to the end of

the board, and weights are added to the end of this string until the slide, when given a start, moves with constant speed in the direction of the pulley (Fig. 62).

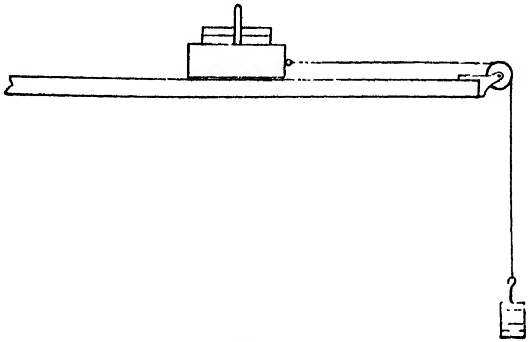


Fig. 62.

The *horizontal* forces now acting on the slide are the pull of the string, and the equal and opposite force of friction. Since we use a pulley which is very nearly free from friction, the pull of the string and hence the friction are each very nearly equal to the weight of the load on the end of the string.

Considering now the *vertical* forces on the slide, we see that the weight downwards is balanced by the upward reaction of the plane. This reaction, being at right angles to the rubbing surface, is called the *normal reaction* upon it. To ascertain how the force of friction depends on the normal reaction, we increase this reaction by increasing the load on the slide, step by step, and determine the corresponding values of the friction by finding what weight on the end of the string will cause the slide, when given a start, to continue to move steadily. The results in Table I below were obtained with an oak slide on an oak board, the grain of the wood of both being parallel to the direction of motion. The weight of the slide itself was 1 lb. and the load upon it was increased by 1 lb. at a time.

TABLE I. *Results of experiment to determine the connection between Sliding Friction and Normal Reaction.*

Oak slide on oak board. Weight of slide, 1 lb.

Total normal reaction on surface of slide (R) lbs. wt.	Force of Sliding Friction (F) lbs. wt.	Ratio of Sliding Friction to normal reaction (F/R)	
1	0·21	·210	Mean value ·210
2	0·42	·210	
3	0·62	·206	
4	0·83	·207	
5	1·07	·214	
6	1·29	·215	

The numbers in the third column are obtained by dividing the total normal reaction into the corresponding value of the

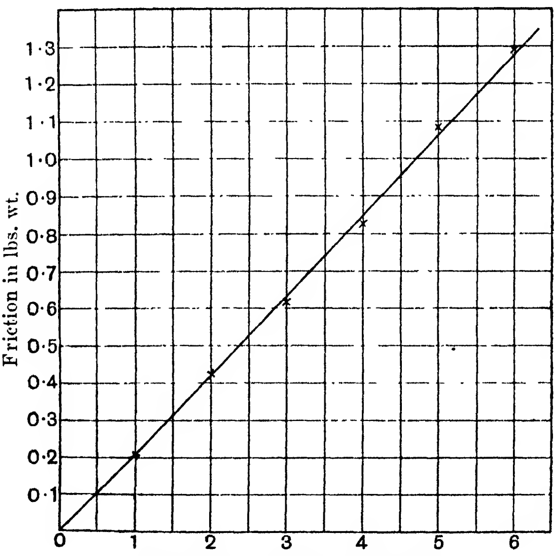


Fig. 63.

sliding friction in each case. The numbers thus obtained are so nearly equal that we are justified in saying that the **sliding friction is directly proportional to the normal reaction.**

Let us also illustrate this law graphically by plotting the results on squared paper (Fig. 63), the values of the normal reaction being taken as abscissae, and the corresponding values of the sliding friction as ordinates.

The line which lies most evenly among the points is a straight line passing through the origin *O*. From this graph we can read off the sliding friction corresponding to any given normal reaction. For instance, we see that the friction corresponding to a normal reaction of 4·5 lbs. wt. is 0·95 lb. wt.

If we now repeat our experiments with slides and planes made of different materials, we find the same simple law always holds good. However, the value of the ratio $\dfrac{\text{friction}}{\text{normal reaction}}$, although constant for any one pair of surfaces, is found to be different for each different pair of surfaces we employ.

48. To shew that the force of sliding friction on a body is independent of the area of the rubbing surface. A rectangular block of some close grained wood, such as boxwood, is prepared, for use as a slide, by planing as smooth as possible the two opposite faces. The area for contact for one face is reduced to about half its former area by cutting a channel from it with a chisel. The block is first placed with its larger face in contact with the plane and the sliding friction measured as in the foregoing experiments. The block is then turned so that its smaller face is in contact with the plane and the friction again measured. The block should have two eyes screwed into its end at points such that, when the string is transferred from one to the other on turning the block over, this string is still parallel to the plane. Additional measurements should be made with each face, increasing the load on the block step by step as in the foregoing

experiment. The results in Table II below were obtained with a pear wood block sliding on a polished steel plane.

TABLE II. *Results of experiments to illustrate the effect of altering the area of the rubbing surface.*

Pear wood block on steel plane. Weight of block, 0·7 lb.

Total normal reaction	Force of Sliding Friction in lb. wt.	
lbs. wt.	Large face	Small face
0·7	0·17	0·15
1·7	0·40	0·38
2·7	0·59	0·60

We see that the close agreement of the values in the 2nd and 3rd columns justifies the statement that *the sliding friction is independent of the area of the rubbing surface.*

You will more readily accept the truth of this law when you perceive that it can be deduced, as follows, from the law which we have already verified in the preceding article. The total force of friction on the block in each case is clearly the sum of all the forces of friction on every square inch of its rubbing surface. Let us suppose the area of this surface to be doubled while the total load remains the same. Assuming this load to be equally distributed over the contact surface in each case, we observe that the effect of doubling the area is to halve the normal reaction on each square inch, for the same load is now supported by twice the number of square inches. Hence the friction on each square inch is now only half that in the first case (for we already know that the friction for each square inch is proportional to the normal reaction upon it). Thus we see that, when the area is doubled, we have twice the number of square inches but the force of friction on each is only half what it was before; hence the total force of friction remains the same.

This is the traditional argument. It is, however, open to question whether, in the experiment we have described, we have succeeded in varying the area of contact in the way suggested. We cut away half one face of the slide and we left it to be understood that the surface of contact of one face was double that of the other. Now rub the surface of the plane, that on which the slide moves, lightly with powdered graphite; then, with the faces of the slide clean, move the slide up and down the plane, first on one face and then on the other, and from the black marks left on the faces of the slide observe the actual areas of contact. The areas are not sharply defined or easily measured, and it is even possible that the surface which we attempted to reduce by channelling may in fact make the better contact; this will depend more upon the quality of the planing than upon the whole areas of the two faces. Contact over a whole pair of faces is difficult to secure, so difficult that perhaps the experiment is beyond the scope of the work which we intend to cover in this book; still, if the resources of the school laboratory permit, the experiment should prove interesting. In most machines the surfaces of the moving parts are not dry but lubricated; also it is likely that the area of contact, whatever it may be, will remain fairly constant.

49. Summary of the Laws of Limiting Friction for Dry Surfaces. We can now sum up the laws of sliding friction as follows: *the Force of Sliding Friction on a rubbing surface is*

(1) *directly proportional to the total normal reaction upon it (or the total force with which one surface is pressed against the other),*

(2) *independent of the area of the rubbing surface,*

(3) *independent of the speed of rubbing (for moderate speeds).*

The first two laws also hold good for Starting Friction, although, as we have demonstrated, the values obtained for this force under the same given conditions may not agree very closely

among themselves for certain surfaces. For hard polished metal surfaces, however, the agreement is very close.

50. Coefficient of Friction. You will perceive that the foregoing laws are embodied in this one expression 'the ratio of the sliding friction to the normal reaction is, for two given surfaces, a constant whose value depends solely on the nature of the surfaces, *i.e.* on the materials of which they are made and their degree of smoothness.'

This ratio $\dfrac{\text{Sliding Friction } (F)}{\text{Normal Reaction } (R)}$ is called the *Coefficient of sliding friction* for the given surfaces, and is generally denoted by the Greek letter μ.

That is, $\dfrac{F}{R} = \mu$ or $F = \mu \, . \, R$.

For example, taking the mean of the numbers in the third column of Table I, we see that the coefficient of sliding friction for the two oak surfaces has the value 0·21. Thus the relation between the force required to keep one of these surfaces sliding over the other, and the total normal reaction upon it, is given by $\dfrac{F \text{ (lbs. wt.)}}{R \text{ (lbs. wt.)}} = 0{\cdot}21$ or $F = 0{\cdot}21\,R$. Similarly, the ratio of the starting friction to the normal reaction we call the 'coefficient of starting friction.' The value of this coefficient is somewhat uncertain in many cases, but it is always greater than the coefficient of sliding friction.

As a rule we shall use simply the expression 'coefficient of friction,' and in doing so we shall generally refer to sliding friction. However, when using this expression in any example you will find that there is never any doubt as to which of the above coefficients is implied.

51. Examples on the use of Coefficient of Friction.

Ex. 1. A man pulls a block of wood, weighing 400 lbs., along a horizontal floor. What horizontal force must he exert to keep the block moving at a steady speed, if the coefficient of friction for the rubbing surfaces of the block and the floor is 0·28?

Here the total normal reaction of the floor on the block is equal to the weight of the block, namely, 400 lbs. wt.

The force of sliding friction opposing the steady motion of the block will therefore be $0\cdot28 \times 400 = \mathbf{112}$ **lbs. wt.,** and this equals the horizontal force which the man must exert.

Ex. 2. If the man in Ex. 1 weighs 13 stone, what must be the least value of the coefficient of starting friction between the soles of his boots and the floor to enable him to exert the necessary horizontal force?

To exert the necessary horizontal force, the least force of friction of the floor on the man's boots must be 112 lbs. wt. The normal reaction of the floor on his boots is equal to his weight, namely, 13×14 lbs. wt. If μ is the necessary coefficient of starting friction, then the greatest force he can exert is $\mu \times 13 \times 14$ lbs. wt. Hence, we have

$$\mu \times 13 \times 14 = 112, \text{ or } \mu = \frac{112}{13 \times 14} = \mathbf{0\cdot61}.$$

Ex. 3. The weight on the driving wheels of a locomotive is 30 tons, and the coefficient of starting friction for the wheels and the rails is 0·18. What is the greatest pull the locomotive can exert?

The greatest pull the locomotive can exert is equal to the greatest horizontal force which the driving wheels can exert on the rails, which again is equal to the total limiting friction of the rails on the wheels.

Now the limiting friction $= \mu . R = 0\cdot18 \times 30$ tons wt. $= \mathbf{5\cdot4}$ **tons wt.,** which is therefore equal to the greatest pull the locomotive can exert.

Ex. 4. (Fig. 64.) A ring, weighing $1\frac{1}{4}$ lbs., on a uniform vertical mast is pulled horizontally by a force of P lbs. wt. applied to the end of a long cord fastened to the ring. If the coefficient of starting friction for the surfaces of the ring and the mast is 0·2, what is the least value of the force P necessary to prevent the ring from starting to slip downwards?

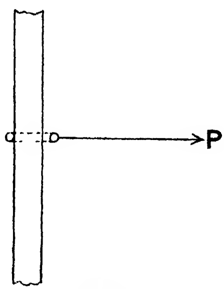


Fig. 64.

The horizontal forces acting on the ring are the pull P and the normal reaction of the mast, and these forces are equal and opposite. The vertical forces acting on the ring are its weight downwards and the force of limiting friction upwards. Now this force of friction $= 0\cdot2 \times$ normal reaction $= 0\cdot2\,P$ lbs. wt., and since, to prevent the ring from slipping, this force must equal the weight of the ring, we have

$$0\cdot2P = 1\tfrac{1}{4} \text{ lbs. wt. or } P = \frac{1\cdot25 \text{ lbs. wt.}}{0\cdot2} = \mathbf{6\cdot25 \text{ lbs. wt.}}$$

52. Friction of bearings. Lubrication. To experiment upon the frictional force which the bearings of a shaft, or axle, exert upon it when rotating, we use a hollow metal tube supported horizontally in two metal bearings (Fig. 65). Pins, projecting axially from the tube, are fitted at its ends, so that these ends may be loaded by sliding on to them uniform circular brass weights. In the apparatus we use, the tube weighs $\frac{3}{4}$ lb. and has an external diameter of about $1\frac{1}{2}$ inches; the bearings are about 6 inches apart, and the length of tube resting on each is about $\frac{1}{2}$ inch.

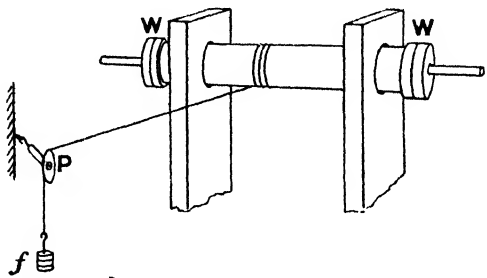


Fig. 65. Apparatus for experiment on friction of bearings.

To measure the sliding friction of the bearings, a piece of thread is fastened to the middle of the axle, wound round it and then passed horizontally from beneath over a pulley P (having ball-bearings); weights (f) are then added to the end of this thread until, on giving the axle a starting twist, it continues to turn steadily. The total vertical reaction of the bearings in this case is equal to the weight of the axle alone (w). This can be increased by adding equal weights (each W) to the pins at the ends of the axle. In this case the forces acting on the axle, when turning steadily, are shewn in Fig. 66. Here, for simplicity, the two bearings are considered as one. The total reaction of the bearings (R) is equal to the weight of the axle plus the load upon

it, namely $(w+2W)$, and the total friction of the bearings is equal to f, the weight of the load on the end of the string.

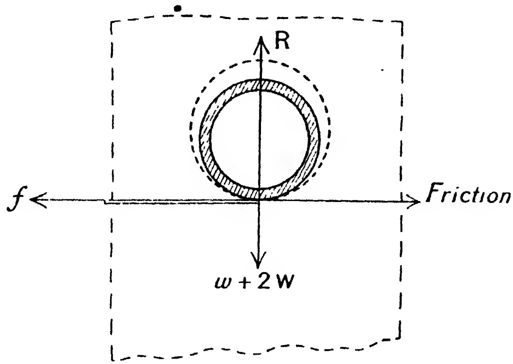


Fig. 66. Forces acting on axle.

Experiment 1. *Bearings not lubricated.*

The following results in Table III were obtained by increasing each of the weights W by 1 lb. at a time, the friction of the dry bearings being determined by finding the weight f necessary to keep the axle turning steadily when given a start. For this purpose $\frac{1}{10}$ lb. and $\frac{1}{100}$ lb. wts. were employed.

TABLE III. *Friction of dry bearings.*

Weight of axle alone = 0·75 lb.

Total normal reaction on axle = Total load $(0{\cdot}75+2W)$ lbs. wt.	Friction of bearings on axle = f lbs. wt.
0·75 (axle alone)	0·17
2·75	0·70
4·75	1·20
6·75	1·71
8·75	2·25
10·75	2·80

In Fig. 67 is shewn the result of plotting the values of the friction in terms of the corresponding reactions or loads, as we did in Art. 47 (Fig. 63). We find, as before, that for dry surfaces the friction is directly proportional to the normal reaction. To calculate the coefficient of friction, we have only to take from the

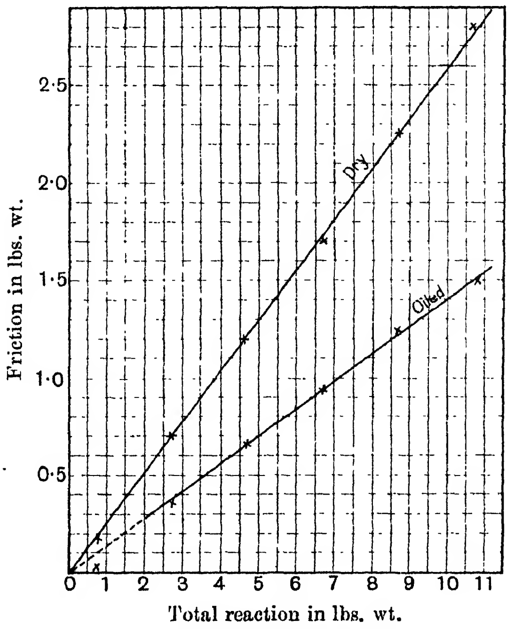


Fig. 67. Friction of bearings.

graph corresponding values of the friction and the reaction of the bearings, and divide the former by the latter. Thus, we see that the friction corresponding to a total reaction of 10 lbs. wt. is 2·57 lbs. wt.; hence

$$\mu = \frac{2{\cdot}57 \text{ lbs. wt.}}{10 \text{ lbs. wt.}} = 0{\cdot}257.$$

Experiment 2. Bearings lubricated.

After lubricating the bearings with machine oil, the above experiments were repeated, the results obtained being given in Table IV.

TABLE IV. *Friction of ordinary lubricated bearings.*

Weight of axle alone = 0·75 lb.

Total normal reaction on axle = Total load $(0{\cdot}75+2W)$ lbs. wt.	Friction of bearings on axle = f lbs. wt.
0·75 (axle alone)	0·03
2·75	0·36
4·75	0·66
6·75	0·94
8·75	1·24
10·75	1·50

Plotting these results as we did those obtained in the foregoing experiment and on the same sheet of squared paper (Fig. 67), we again get a straight line which, if produced, passes through the origin. This shews that the same law holds good for ordinary open lubricated bearings as for dry bearings, namely, the friction is directly proportional to the normal reaction (or total load on the bearings).

The reason for this may be that, for the loads experimented with, the pressure of the axle at the bearings is sufficient to squeeze out the oil to such an extent that the sliding surfaces are not kept completely apart by the oil. These surfaces therefore interact upon one another directly, though to a less extent than when dry. Hence, the effect of the oil in this case is not to alter the law which holds for dry surfaces, but only to reduce the amount of friction. This is shewn by calculating the new coefficient of friction. For instance, we see from the graph that the friction corresponding to a load of 10 lbs. wt. is 1·4 lbs. wt.;

that is, $\mu = \frac{1 \cdot 4 \text{ lbs. wt.}}{10 \text{ lbs. wt.}} = 0 \cdot 14$. We see then that the effect of lubrication in this particular case has been to reduce the coefficient of friction to nearly one-half of that for the same bearings when dry.

Such lubrication as this is said to be imperfect. This is the condition which exists in most machines, such as those already described in the preceding chapters; hence, in these machines the friction on most of the sliding surfaces is directly proportional to the normal reactions upon them.

53. Perfect lubrication. When the pressure of a shaft at its bearings is such that the oil is not squeezed out, but has sufficient 'body' to keep the surface of the shaft quite clear of the surface of the bearings, or when this same condition is attained by supplying enclosed bearings with oil under pressure, the lubrication is said to be perfect. Under these conditions there is evidently no actual rubbing at all between the solid surfaces, with the result that the friction is independent of the nature of these surfaces. Also, the amount of friction is found to be very small and to depend on the quality of the oil. For such bearings the laws of dry friction no longer hold good, as the results of the following experiment will shew.

Experiment 3. *Perfect lubrication of bearings.*

We repeat Experiment 2 with the same apparatus (Fig. 65) but with these differences: the load on the bearings is increased by only 0·4 lb. wt. at a time by adding 0·2 lb. to each end of the string passing round the axle; also, before each measurement the axle and bearings are smeared with a liberal quantity of thick cylinder oil, or Russian tallow. The following results in Table V are obtained in this manner.

TABLE V. *Perfect lubrication of bearings.*

Weight of axle alone = 0·75 lb. Lubricant used, cylinder oil.

Total load on bearings in lbs. wt. $(0{\cdot}75+2W)$	Friction in lbs. wt.
0·75 (axle alone)	·04
1·15	·04
1·55	·05
1·95	·05

These results shew that when the lubrication is perfect, the friction is practically independent of the load on the bearings. Notice also how small the force of friction is in this case. Moreover, it can be shewn by experiment that the friction of lubricated surfaces such as these, differs from that of dry surfaces in that the former increases with the speed of sliding and also with the area of the sliding surfaces wetted by the lubricant.

54*. *Note on Arts.* 52 *and* 53.

It may be asked whether, with the thread arranged as we have drawn it in Figs. 65 and 66, we are justified in stating that the reaction of the bearing upon the tube is vertical, for it seems possible that when the thread is pulled, the tube will roll on the surface of the bearing until it comes into equilibrium on a new line of contact above the lowest point of the bearing, so that the reaction will be inclined to the vertical, that is to say in one or other of the positions shewn in Fig. 67 *a*.

Suppose the tube rolls into a position of equilibrium with the point of contact C displaced forward in the direction of the pull, as shewn in Fig. (i). Consider the moments about the point C of all the forces which act upon the tube. The forces are: the weight of the tube (W), the reaction of the bearing (R), the force of friction (F), and the pull of the string (P). R and F have no moment about C, for they pass through C. W and P have

* To be taken after Chapter V.

moments about C, but both moments are in the same direction, clockwise about C, so there is a resultant moment about C, the tube is not in equilibrium, and this position is therefore impossible. The position of Fig. (ii) is also impossible, for there is a resultant moment about L. In each case the resultant moment is such that if it were set up, by forcibly displacing the tube in its bearing, it would tend to restore the tube to its original position. It follows that if the thread is pulled off the tube, as shewn in

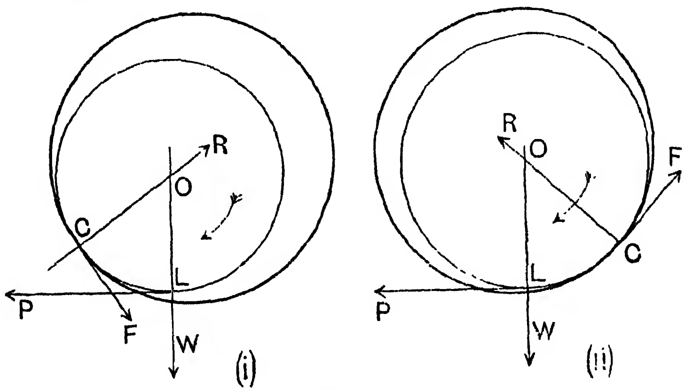


Fig. 67 *a*.

Fig. 65, contact will remain at the lowest point while the tube rotates, the reaction will be vertical and equal to the weight, and the friction will be equal to the pull of the thread.

The reader should draw figures for other positions, shewing the pull of the thread vertically downward, vertically upward and horizontal from the top of the tube, and should prove that in each of these three cases, though the force of friction is always equal to the pull of the thread, in only one of them is the reaction equal to the weight, and in none of them is the reaction vertical.

55. Energy wasted in bearings. Heating of bearings. When a shaft is rotated, work is done in overcoming the frictional resistance of its bearings. Since this

resistance always acts along a tangent to the surface of the shaft, the work done against it, during one revolution, is equal to the product of this force and the circumference of the shaft at the bearings.

Let the total load on the bearings, which is equal and opposite to the total reaction of the bearings on the shaft, be denoted by R lbs. wt., and let the coefficient of friction for the surfaces of the bearings and shaft be denoted by μ. Then the frictional resistance is μR lbs. wt.

If d be the diameter of the shaft in feet, then the circumference of the shaft is πd feet.

Hence the work done against friction during one revolution is $\mu R \pi d$ ft. lbs.

Now the coefficient of friction is independent of the diameter of the shaft. It follows that the work done against friction during one revolution varies directly with the diameter, which should therefore be made as small as possible, consistent with safety.

The wheel of a vehicle is made large, and the axle, where rubbing takes place, is made as small as possible, because in this way the vehicle travels a considerable distance for a small amount of rubbing and, hence, for a small waste of work.

As already stated in Art. 31, the mechanical energy expended in overcoming frictional resistance is transformed into heat. The rubbing bodies therefore rise in temperature until the rate at which they lose heat by conduction and radiation is equal to that at which they gain heat by the continued rubbing. The temperature of the bodies then remains constant. To ascertain if the friction at a bearing of a continuously running shaft is excessive, we feel it; for a high temperature at once results if the lubrication is inefficient.

Experiments, first conducted by Dr Joule of Manchester, have demonstrated that for every 778 ft. lbs. of work which are done against friction, a quantity of heat is produced sufficient to raise 1 lb. of water through one degree Fahrenheit. This quantity of heat is called a British thermal unit. By dividing the work done

(in ft.-lbs.) against friction in any operation by 778 we obtain the resulting heat in British thermal units. 778 ft. lbs. is called the Mechanical Equivalent of one British Thermal Unit of Heat.

Ex. 1. A shaft, of 4″ diameter, makes 120 revolutions per minute. The weight of the shaft and the load upon it amount to 2 tons, and the coefficient of friction for the rubbing surfaces of the shaft and bearings is 0·03. Calculate the work done against friction per minute, and also the heat generated per minute.

The total reaction of the bearings = total load upon them $= 3 \times 2240$ lbs. wt.

Force of friction on shaft $= 0 \cdot 03 \times 3 \times 2240$ lbs. wt.

Circumference of shaft $= 4\pi$ inches $= \dfrac{4\pi}{12}$ feet.

In one minute this force of friction is overcome through a distance of

$$120 \times \frac{4\pi}{12} \text{ feet.}$$

Hence, work done against friction per minute

$$= 0 \cdot 03 \times 3 \times 2240 \times 120 \times \frac{4\pi}{12} = \mathbf{25{,}344} \text{ ft. lbs.}$$

Since 1 British thermal unit of heat is produced by the expenditure of 778 ft. lbs.,

$$\text{Heat generated} = \frac{25344}{778} = \mathbf{32 \cdot 6} \text{ British thermal units.}$$

Ex. 2. Equal weights, each of 20 lbs., hang on the ends of a cord over a pulley (Fig. 68). The diameter of the pulley wheel is 6″ and the diameter of the axle is $\frac{1}{2}$″. If the weight of the pulley wheel alone is 4 lbs. and the coefficient of friction for the surfaces at the bearing is 0·09, calculate the downward force which must be applied to either end of the cord to maintain steady motion. Let f lbs. wt. be the required force. The work done by this force during 1 rev. $= f \times 6\pi$ in. lbs. and this, by the Principle of Work, equals the work done against friction at the bearing. That is,

$$f \times 6\pi \text{ in.-lbs.} = 0 \cdot 09 \times 44 \times \pi \times \tfrac{1}{2} \text{ in. lbs.}$$

or
$$f = \frac{0 \cdot 09 \times 44 \times \pi}{6\pi \times 2} = \mathbf{0 \cdot 33 \text{ lb. wt.}}$$

Notice that, if the diameter of the pulley wheel is doubled without altering the load upon the axle, this force will be halved.

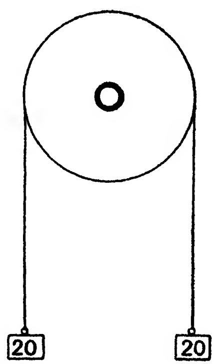


Fig. 68.

EXAMPLES IV.

1. A string is fastened at one end to a block of wood resting on a table, and passed along the surface of the table over a pulley at the edge. It is found that when a weight of 1 lb. is attached to the free end of this string, the block does not move. Neglecting the friction of the pulley, what is the force of friction exerted by the surface of the table on the block, and what is its direction? What is the force of friction exerted by the block on the table, and what is its direction? If the weight of the block is 8 lbs., what is the total normal reaction of the table on the block?

If the coefficient of sliding friction for the contact surfaces of the block and table is 0·35, what weight must be placed on the end of the string to keep the block moving steadily when given a start?

2. The following results were obtained in an experiment (carried out as described in Art. 47) to determine the sliding friction between a wood block and a polished steel plane for different loads:

Total load in lbs. wt.	0·87	1·37	1·87	2·37	2·87	3·37	3·87
Sliding friction in lb. wt.	0·17	0·28	0·37	0·48	0·59	0·67	0·78

Plot on squared paper the values of the force of sliding friction in terms of the corresponding values of the normal reaction of the plane on the block.

Take the following scales: Normal reaction $1''=1$ lb. wt., Friction $1''=0{\cdot}2$ lb. wt.

What conclusion do you draw from the line thus obtained?

With the aid of your graph, determine the coefficient of friction for the rubbing surfaces.

3. To slide an empty box, weighing 40 lbs., along a floor with steady motion is found to require a horizontal force of 13·5 lbs. wt. What will be the force required if a load of 75 lbs. is now placed in the box?

What is the coefficient of friction for the rubbing surfaces?

4. A thin slide, resting on a horizontal surface, has a contact area of 12 square inches and the load upon it is 36 lbs. wt. A horizontal force of 12 lbs. wt. is required to keep this slide moving with steady motion. Neglecting the weight of the slide, what is the normal reaction on each square inch of its contact surface; what is the force of friction on each square inch?

If the contact area of the slide is now reduced to 9 square inches, what is now the normal reaction on each square inch; what is now the friction on each square inch of its contact surface? Hence, what is the total friction in the second case?

5. The hook of a spring-balance is connected to the end of a long plank resting wholly on a level table, and the ring of the spring-balance is pulled horizontally with a force sufficient to keep the plank moving steadily.

Will the reading of the spring-balance alter as an increasing length of the plank projects over the edge of the table?

6. Two similar blocks rest on a smooth uniform horizontal surface and are connected together. The horizontal force required to keep the two blocks sliding steadily is 4·5 lbs. wt. What will this force be if one block is now placed on top of the other? Give reasons for your answer.

7. It is found that a horizontal force of 7 lbs. wt. can keep a load, whose weight is 26 lbs., in steady motion along a horizontal surface. What is the coefficient of friction for the rubbing surfaces?

8. A block of stone, which weighs 5 cwt., is pulled along a horizontal wooden surface. If the coefficient of friction for the rubbing surfaces of the stone and wood is 0·4, calculate the horizontal force required to keep the block moving with steady motion, when once it has been started. Find also how much work must be done to slide the block a distance of 20 feet.

9. The force of friction on a body, when sliding over another, is 3 lbs. wt. If the coefficient of friction for the rubbing surfaces is 0·25, with what total normal force is one body being pressed against the other?

10. How do you account for the following statements:

(i) If a horse on a slippery surface fails to move a cart on account of his shoes slipping, he is sometimes enabled to do so if a man gets on his back.

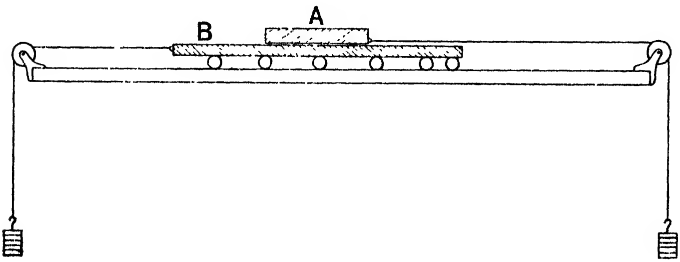
(ii) When work is transmitted from one pulley to another by means of a belt, it is found that a tight belt is less liable to slip than a loose one.

Why are the wheels of a locomotive coupled together?

11. What horizontal force must be applied to a 28 lb. weight to keep it moving steadily along a level table if the coefficient of friction is 0·4?

Compare the amount of work which has to be done to move the weight a distance of 10 feet along the table with that which has to be done to lift it through a vertical distance of 10 feet.

12. (Fig. *a*.) A flat polished slide of cast iron A is placed on a polished cast iron slab B, which is separated from a smooth hard surface by a number of steel balls. Horizontal strings connected to A and B pass over pulleys as shewn. The weight of the slide A is 3 lbs. and the coefficient of sliding friction for the contact surfaces of A and B is 0·2. If rolling friction and the friction of the pulleys are negligible, find what weights must be attached to the ends of the strings to maintain steady sliding between A and B.



(*a*)

13. A horizontal pull of 48 lbs. wt. is required to slide a trunk along the floor. The coefficient of friction for the rubbing surfaces is 0·25, and the trunk, when empty, weighs 60 lbs. What weight of goods does the trunk contain?

14. When a load of 14 lbs. is placed on a thin wooden slide supported on a horizontal surface, the force of sliding friction is found to be 3·5 lbs. wt.

The area of the rubbing surface of the slide is now diminished by one-third, and the load is increased to 31·5 lbs.

Assuming the weight of the slide in each case to be negligible, what is now the force of sliding friction?

15. The saddle of a lathe weighs 4 cwt. If the coefficient of friction for the sliding surfaces of the saddle and its bed is 0·1, what is the horizontal force required to slide the saddle?

16. The following results were obtained when experimenting (by the method described in Art. 47) to determine the coefficients of starting and sliding frictions for various polished surfaces in contact with a polished steel plane.

Steel plane. Contact area of slide, about 3 sq. inches. Surfaces dry.

Slide	Load placed on slide in lbs. wt.	Starting Friction	Sliding Friction
Steel slide weighing 11 ozs.	1	·28	·27
	2	·30	·26
	3	·31	·28
Wrought iron slide weighing 11·5 ozs.	1	·33	·29
	2	·33	·29
	3	·37	·29
Cast iron slide weighing 10 ozs.	1	·31	·21
	2	·36	·20
	3	·38	·20
Lignum Vitae slide weighing 2 ozs.	1	·41	·21
	2	·35	·22
	3	·39	·22

Make out a table giving the total normal reactions on the slide (weight of slide + load), and the coefficients of starting and sliding frictions in each case. Calculate the mean values of these coefficients for each pair of surfaces. State any conclusions you draw from your results.

17. A drawer of a chest of drawers weighs 14 lbs. and contains articles having a total weight of 35 lbs. If the coefficient of sliding friction for the runners and their bearings is 0·2, calculate the total horizontal force required to pull out the drawer when once it has been set in motion.

18. With what horizontal force must a block, weighing 10 lbs., be pressed against the face of a wall to just prevent it from slipping down? The block is only supported by the friction between the block and the wall, the coefficient of starting friction for the contact surfaces being 0·4.

19. To push up a window, whose area is 8 square feet, when no wind is blowing, requires a force of 4 lbs. wt. If a force of 7 lbs. wt. is required when a wind is producing a pressure on the window of 0·75 lb. wt. per square foot, what is the coefficient of friction for the rubbing surfaces of the window and its guides?

20. A fishing-boat, which weighs 3 tons, has, before being launched, to be dragged across 100 yards of level sandy beach. In order to make the task easier, flat boards are placed transversely underneath the keel. If the coefficient of friction for the iron of the keel and the wood is 0·32, find what

horizontal force will keep the boat in motion, and the work which has to be done to get her to the sea. Supposing there were not enough men to exert the necessary force, what methods could you suggest for rendering the task still easier?

21. A boy of 8 stone pulls a heavy box along a stone floor. If the coefficient of limiting friction for the soles of his boots and the floor is 0·4, what is the greatest horizontal pull he can exert?

22. A horse, weighing 15 cwt., is engaged in pulling a cab along a level wood surface. If the coefficient of starting friction for its shoes and the wooden surface is 0·25, what is the greatest horizontal steady pull the horse is capable of exerting?

If, when the surface is wet, the horse cannot exert a greater steady pull than $1\frac{1}{2}$ cwt. without slipping, what is now the coefficient of friction?

23. Water exerts a total force of 3 tons wt. on one side of a vertical sluice-gate. If the weight of the gate is 2 cwt. and the coefficient of starting friction for the sliding surfaces of the gate and its guides is 0·22, calculate the vertical force required to start the gate moving upwards.

24. The working face of a slide valve of a steam-engine measures $8'' \times 14\frac{1}{2}''$, and the steam pressure on the back of the valve is 140 lbs. wt. per sq. inch. If the coefficient of friction is 0·08, calculate the force required to overcome the friction.

25. A small metal planing machine, the table of which weighs 140 lbs., makes 7 backward and 7 forward strokes, each of 4 feet, in a minute, and the coefficient of friction for the rubbing surfaces is 0·07.

How much work is done against friction per minute?

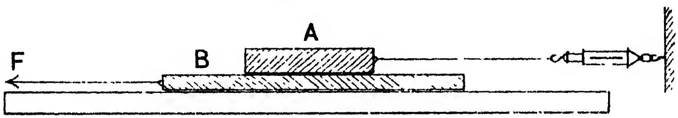
26. A man weighs 12 stone: what is the greatest weight he can pull along a floor by a horizontal rope if the coefficient of friction between the weight and the floor is 0·3, and that between his boot soles and the floor 0·5?

27. A horse-shoe magnet is fixed with the surfaces of its ends in a vertical plane, and a smooth block of iron, weighing 2 lbs., is placed symmetrically against these ends. If the coefficient of friction for the contact surfaces is 0·18, calculate what must be the least force of attraction of the magnet to just keep the iron block from slipping.

28. The load on the driving wheels of a locomotive is 25 tons and the coefficient of starting friction between the wheels and the rails is 0·2. Find the greatest horizontal force the locomotive can exert on the rails before the wheels begin to slip.

29. The weight on the driving wheels of a locomotive is 30 tons and the coefficient of friction between the wheels and the rails is 0·19. Find the weight of the heaviest train it can draw on the level if the tractive resistance is 12 lbs. wt. per ton.

30. (Fig. *b*.) A block *A*, weighing 5 lbs., rests on the horizontal top of a slab *B*, weighing 12 lbs., and is connected by a horizontal string to a fixed spring-balance. The slab *B* rests on the level surface of a table. What force *F* will be required to keep *B* sliding, when once started, and what will be the reading of the spring-balance connected to *A* ?



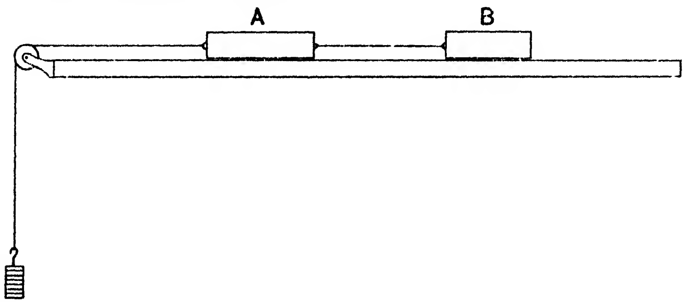
(*b*)

The coefficient of friction for the contact surfaces of *A* and *B* is 0·25, and that for the surfaces of *B* and the table 0·33.

31. Suppose that you hold two rectangular blocks of wood, one in each hand, and press them horizontally against the sides of a brick, which is thereby prevented from falling solely by the friction between its surfaces and those of the blocks. If the weight of the brick is 6 lbs. and the coefficient of starting friction for the contact surfaces is 0·5, find the least *horizontal* force with which you must press with each hand to prevent the brick from falling.

If each wooden block weighs 1 lb., what *vertical* force must each hand exert?

32. (Fig. *c*.) The blocks *A* and *B* resting on a table are connected in tandem and a string from *A* passes over a pulley.



(*c*)

The weights of A and B are 10 and 8 lbs. respectively and the coefficients of friction for their rubbing surfaces and that of the table are 0·2 and 0·3 respectively. Neglecting the friction of the pulley, calculate what weight must be attached to the end of the string to maintain steady motion when the blocks have once been started.

What is the tension of the string between A and B?

If B is placed upon A, what weight must now be placed on the end of the string to maintain steady motion?

33. A cart weighs $\frac{3}{4}$ ton. If the resistance offered to its motion on a level road is 44 lbs. wt. per ton, find the work done when the cart is drawn 1000 yards.

How do you account for the resistance to the cart's motion?

34. A horse pulls a cart, which weighs 12 cwt. and is loaded with 1 ton, along a level road. If the resistance is 40 lbs. wt. per ton, what force does the horse exert on the cart, and how much work is done in pulling the cart a distance of 300 yards?

35. To draw out one note-book from a pile, it is pulled horizontally with the right hand while the one above it is prevented from moving by the left hand.

If you have a pile of such books, each weighing 1 lb., and the coefficient of friction for all the surfaces of their covers is 0·3, calculate the horizontal force required to draw out (i) the 4th book from the top, (ii) the 9th book from the top.

36. A row of 10 note-books, each weighing 1 lb., is held horizontally between the two hands (in the same manner as a concertina), the books between the extreme ones being prevented from falling solely by the friction between their covers. The coefficient of starting friction for all the covers is 0·3. If the horizontal pressure of the hands is gradually reduced, where will sliding first occur and why?

What is the least horizontal force which each hand must exert to prevent the books from falling? What vertical force is each hand exerting?

37. In going downhill, the brake of a carriage is kept on one wheel for a distance of 200 yards. If the normal force of the brake block on the rim of the wheel is 120 lbs. wt. and the coefficient of friction for the rubbing surfaces is 0·36, what is the force of friction on the wheel? Calculate also the total work done against the friction of the brake and the amount of heat produced.

38. A flywheel is mounted on a spindle, the ends of which rest in horizontal bearings. The total thrust on the bearings is 80 lbs. wt., and

the diameter of the spindle is 1″. If the coefficient of friction for the rubbing surfaces at the bearings is 0·05, find the work done against friction when the flywheel makes 2000 revolutions.

39. A lathe spindle, 2″ in diameter, runs at 90 revs. per min. The total load on the bearings is $3\frac{1}{2}$ cwt., and the coefficient of friction for the surfaces at the bearings is 0·04.

How many foot-pounds of work are wasted in friction per minute?

40. A shaft has a diameter of 4″ and makes 200 revs. per min. The total load on the bearings is 25 cwt., and the coefficient of friction is ·025. Find the heat generated at the bearings per minute.

41. The brakes on all the wheels of a moving railway carriage are put on so hard that the wheels skid on the lines. If the coefficient of friction for the surfaces of the wheels and the lines is 0·18, and the weight of the carriage is 20 tons, calculate the heat generated when the carriage skids a distance of 20 yards.

42. The following is a record of an experiment on the friction of a simple pulley.

The pulley was suspended from a fixed support, and a cord was passed over the sheave. A weight P was attached to one end of the cord and the other end was then loaded with a weight Q, such that, when given a start, Q moved down with steady motion.

Weight of pulley sheave, 5 lbs.

P lbs. wt.	5	10	15	20	25	30	35
Q lbs. wt.	5·45	10·74	16·05	21·36	26·65	31·94	37·25

Tabulate the values of the total reaction of the bearings, namely ($P+Q+$weight of sheave), and the values of the force which is effective in overcoming the friction, namely $(Q-P)$. Plot on squared paper these values of the effect of friction in terms of the total reaction.

What conclusion do you draw from the result?

If the diameter of the sheave is 5″ and the diameter of the bearing is 0·75″, calculate the coefficient of friction for the rubbing surfaces at the bearings.

43. A weight of 100 lbs., attached to a rope passing over a pulley, is raised by pulling vertically downwards on the free end of the rope. The pulley wheel has a diameter of 4″, and the diameter of its axle is $\frac{3}{4}$″.

If the coefficient of friction for the rubbing surfaces at the axle is 0·2, calculate the effort required. (You may take the total load on the axle as 200 lbs. wt.)

44. When is the lubrication of a bearing said to be perfect?

How do the laws of friction for perfectly lubricated bearings differ from those for bearings whose lubrication is imperfect?

45. A two-wheeled cart has wheels of 5 feet diameter, and the diameter of the axles is $2\frac{1}{2}$ inches. The total load on the axles is 1·2 tons and the coefficient of friction for the rubbing surfaces is 0·08. Calculate the work done against axle friction when the cart travels $\frac{1}{4}$ mile.

46. To pull a garden roller up a slope requires a force of 50 lbs. wt. The axle has a diameter of $1\frac{1}{2}''$ and the coefficient of friction is 0·12. Calculate the work done against axle friction during 10 revolutions of the roller. (The weight of the handle and its attachment to the axle is negligible.)

47. A block of wood rests on an inclined plane. It is found that a force of 25 lbs. wt. is necessary to keep it sliding up the plane, and a force of 7 lbs. wt. is necessary to keep it sliding down the plane, the force in each case acting parallel to the plane.

What is the force of sliding friction of the plane on the block?

If the coefficient of friction is 0·3, what is the normal reaction of the plane on the block?

48. The following data were obtained in an experiment with a wheel and drum (see Fig. 45 on p. 41).

Combined weight of wheel, drum and axle = 44 lbs.

Diameter of wheel = $3'$.

Diameter of drum = $9''$.

Diameter of axle = $2''$.

Load W on rope fastened to drum = 160 lbs.

Effort applied vertically downwards to end of cord on wheel to maintain steady upward motion of the load = 42 lbs. wt.

Calculate the coefficient of friction for the rubbing surfaces at the bearings.

49. What is meant by the term *friction-effect* when applied to a machine?

How do you account for the fact that this friction-effect is expressed by an equation of the form

$$\text{Friction-effect} = a + c \,.\, W,$$

where a and c are constants belonging to the machine?

CHAPTER V

MOMENTS

56. Tendency of a force to turn a body about a fixed axis. Moment. In this chapter we propose to examine the tendency of a force to turn a body about a fixed axis.

Let us consider a door which is free to turn about the axis of the hinges. How do we try to open a door which has become jammed? In addition to pushing or pulling as hard as we can, thus employing as great a force as we can command, we instinctively apply that force as far from the line of the hinges as possible, for by doing so, we know that we are using our muscular effort to the best advantage. The turning effect of the force we use upon the door depends in part on the magnitude of the force and in part on the distance from the axis of turning to the point at which we push. To produce the greatest effect with a given force we apply it to the body at the greatest possible distance from the axis about which the body can turn: it is easier to turn a nut with a long spanner than with a short one; the longer the lever used to turn a screw-jack the greater the load which can be lifted by a given effort.

But is it only a question of the magnitude of the force and the

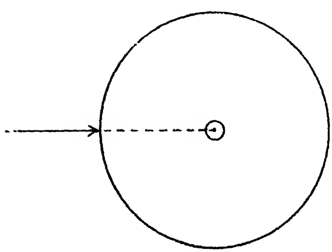


Fig 69.

distance from the axis of turning to the point of application of the force?

Let us consider a wheel which is free to turn about a fixed axle. We may try to turn the wheel by pushing on the rim, and if we keep to the rim the point of application of the force we use is always at the same distance from the axis of turning. Keeping the magnitude of the force constant as well as the distance of the point of application from the axis of turning, we can change the direction of the force. We know intuitively, if not from experience, that however hard we may push in the direction of the centre of the wheel, the force we use has no turning effect upon the wheel. (Fig. 69.) We know that the force we use will be most effective if we direct it tangentially. It seems therefore that to measure the turning effect of a force we must take account of the magnitude, of the direction, and of the point of application of the force. We shall in future use the term MOMENT of a force, meaning by 'moment' the turning effect or turning value or tendency to turn, the 'importance' of the force in turning. We must now find by experiment how to measure moment of a force.

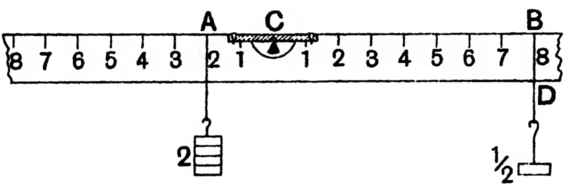


Fig. 70.

Exp. 1. Take a wooden rod of uniform rectangular cross-section, fitted, as shewn in Fig. 70, with a notched strip so that it can be balanced on a peg, like the beam of a weighing machine. If the balance is imperfect, correct it by cutting away a little from the arm which goes down.

When balanced in this way the forces which act on the rod are its weight, and the reaction of the peg acting upwards. The line of the reaction of the peg passes through C, and has therefore

no moment on the rod about C; neither has the weight of the rod itself any moment about C.

Suspend a 2-lb. weight from one side of the rod by a loop of cotton from some point such as A. There is now acting on the rod at A a force of 2 lbs. wt. having a moment upon it about C in a direction opposite to that of the hands of a clock; this is called a *counter-clockwise* direction. From the other side suspend similarly a weight of $\frac{1}{2}$ lb. from some such point as B. Then there is acting on the rod at B a force of $\frac{1}{2}$ lb. wt. having a *clockwise* moment on the rod about C. Adjust the distances CA and CB until the rod once more balances in a horizontal position. When this is done, the moment of the force of 2 lbs. wt. on the rod is just equal and opposite to that of the force of $\frac{1}{2}$ lb. wt.

The moments of these forces are equal. The directions of the forces are the same. The forces are of different magnitudes, and are applied at different distances from the axis of turning. In these circumstances we find that whatever length we make the arm CB, the other arm CA is such that

$$CA \times 2 = CB \times \tfrac{1}{2}.$$

If we repeat this experiment several times, using different weights and altering their positions, we find in each case, when the rod is balanced, that on multiplying the number of units in each weight by the number of units in the length of its arm we get the same product.

The conclusion is that the moment of a force about a turning axis is properly measured by the product of the magnitude of the force and the distance of the point of application from the turning axis.

By driving a fine pin through the lower edge of the rod at D, vertically below B, we can hang the weight of $\frac{1}{2}$ lb. from D instead of B. We find that the rod balances equally well, whether the weight is suspended from B or from D. DC is greater than BC (it may be much greater if the rod is a deep one), therefore since the moment of the force applied at D is the

same as the moment of the force applied at B, it cannot be correct to measure moment by the product of the force and the distance of the point of application from the axis of turning. The perpendicular distance from the axis C on to the line of the force, whether applied at D or B, is the same. It is therefore possible that the moment of a force is measured by the product of the magnitude of the force and the perpendicular distance from the axis on to the line of action of the force. We will test this possibility in another experiment.

Exp. 2. Take the same rod as was used in the last experiment. Suspend a 2-lb. weight from each arm. Fasten the cotton which supports the right-hand weight tightly round the rod at some point B, and pass it over a pulley as shewn in Fig. 71. Move the point of support of the other weight until the rod balances in a horizontal position.

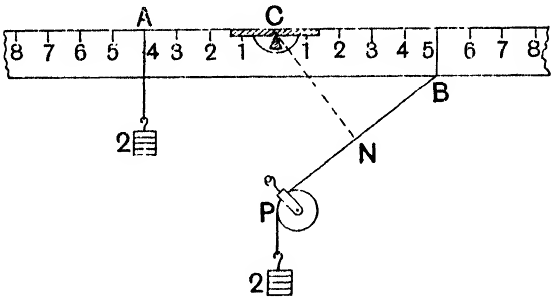


Fig. 71.

In this case the forces which tend to turn the rod about the pivot are a force of 2 lbs. wt. acting vertically downwards at A, and an equal force of 2 lbs. wt. acting at B along the string BP.

Since the rod is at rest the moments of these two forces are equal.

Measure the perpendicular distances CA and CN of the lines of action of these two forces from the edge C, using a foot rule

and a set square to measure CN. We find that these arms are equal. Hence

$$CA \times 2 = CN \times 2.$$

Move the pulley so as to alter the angle of inclination of the right-hand string. We now have to alter A to balance the rod horizontally, but on measuring, we again find the perpendicular distances on the lines of action of the forces from C to be equal.

In each of these experiments, when the rod is at rest in a horizontal position, the moments of the two forces on the rod in opposite directions must be equal. And we notice also that if we multiply the number of units in each force by the number of units of length in the perpendicular drawn to its line of action from the point about which the rod can turn, the products which we obtain are equal. Hence we are led to a rule for measuring the moment of a force on a body about its point of support: **Multiply the number of units in the force by the number of units of length in the perpendicular drawn from the point to the line of action of the force.**

This product measures the **Moment of the Force** about the point.

Notice that when a body turns, it does not do so about a *point* but about a *line* or *axis*. However, we are dealing only with forces acting on a pivoted body in a plane at right angles to the axis about which the body can turn, and in all diagrams this plane is represented by the plane of the paper and the axis perpendicular to this plane is represented, by a point. Hence, although it is customary to define the moment of each force with respect to this point, it is clear that the expression 'Moment of a force about a point' is a measure of the turning effect of the force about an *axis* through this point, perpendicular to the plane in which the forces are acting.

57. Units of Measurement of Moment. If the force is measured in lbs. wt. and the perpendicular or arm in inches, the unit in terms of which moments are measured is the moment of

a force of 1 lb. wt. having an arm of 1 inch. This is called a pound-inch, and is written lb. in.

Moments may be measured in other convenient units such as a pound-foot or a ton-foot, etc. These units are expressed in this form to distinguish them from the foot-pound, foot-ton, etc., which are units we have already employed for the measurement of Work.

Suppose a force of 4 lbs. wt. to be applied to a body at A in the direction shewn by the arrow (Fig. 72). Firstly, let the body be supposed to turn about an axis which is perpendicular to the paper and passes through P.

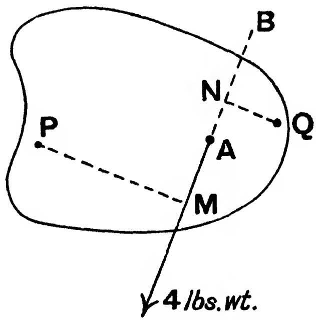


Fig. 72.

Draw PM perpendicular to AB, the line of action of the force, and measure its length. Suppose it to be 10 inches. Then the moment of force about P is

$$4 \times 10 \text{ lb. ins.}$$

Since the force tends to turn the body about P in the direction of the hands of a clock, the moment is said to be clockwise.

Secondly, suppose the body to turn about an axis through the point Q. Measure the perpendicular QN; suppose it to be 4 inches. Then the moment of the force is

$$4 \times 4 \text{ lb. ins.}$$

This force tends to turn the body in a direction opposite to that of the hands of a clock, and the moment is said to be counter-clockwise.

Lastly, suppose moments to be taken about an axis through a point such as N on the line of action of the force. Since N is on this line, no perpendicular can be drawn from it to the line. The moment is zero. This means that, as is obvious, the force does not tend to turn the body about N. The expression 'to

take moments' is commonly used to mean measuring moments according to the rule we formulated in Art. 56.

58. Condition of equilibrium of a pivoted body acted on by several forces. To test further the validity of our rule for measuring moments, we will take an example of a body under the action of several forces.

Exp. 3. Pivot the rod as before at the middle of its length and suspend from it loads of 4 lbs. and 2 lbs. at the points A and C respectively (Fig. 73). Pass a third string carrying a load of

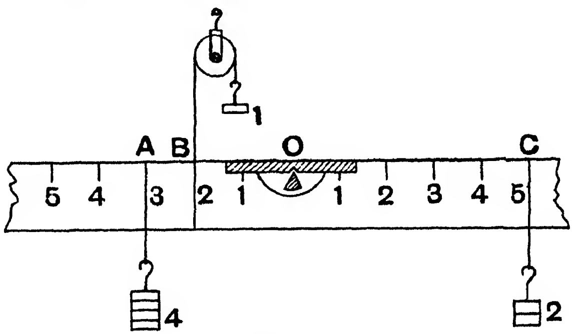


Fig. 73.

1 lb. over a pulley and fasten it to the rod at B. Arrange the position of the pulley so that the string at B is vertical. Let the points A, B and C be chosen so that the rod will remain at rest horizontally.

The rod is now in equilibrium under the action of five forces; a force of 4 lbs. wt. acting downwards at A, a force of 1 lb. wt. acting upwards at B, and a force of 2 lbs. wt. acting downwards at C. In addition we have the weight of the rod and the reaction of the support, neither of which has any moment about the pivot.

To find the moments of the forces tending to turn the rod about O, measure in inches the arms OA, OB, and OC.

Then the moment of the force of 4 lbs. wt. is

$$4 \times OA \text{ lb. ins.}$$

and is counter-clockwise.

The moment of the force of 1 lb. wt. is

$$1 \times OB \text{ lb. ins.}$$

and is clockwise.

The moment of the force of 2 lbs. wt. is

$$2 \times OC \text{ lb. ins.}$$

and is clockwise.

Our results will shew, with a reasonable degree of accuracy, that when the rod is in equilibrium,

$$4 \times OA = 1 \times OB + 2 \times OC \text{ lb. ins.}$$

If instead of three weights we employ four or more, and calculate the moments of all the forces tending to turn the rod about the pivot, we find a similar condition to hold good, provided the rod is balanced.

This condition may be stated shortly thus: "the sum of the clockwise moments is equal to the sum of the counter-clockwise moments."

59. Principle of Moments. (Forces parallel.) Although so far we have only considered a body balanced about a fixed pivot, we can extend the above principle to the case of any body which is kept at rest by forces in one plane, observing that we are at liberty to consider *any* axis at right angles to this plane as a fixed axis about which each of the forces tends to turn the body.

To make this clear let us pivot the rod at its centre and load it with 3 lbs. on one side and 4 lbs. on the other. If the load of 3 lbs. is suspended at A, at a distance of 6 inches from the centre, the load of 4 lbs. must be placed at B, distant $4\frac{1}{2}$ inches from the centre, to produce a balance, for then the moment of each force is equal to 18 lb. ins. Let us now replace the fixed

pivot by a stirrup connected to a spring-balance by a vertical string (Fig. 74*a*). We find that all the strings remain vertical

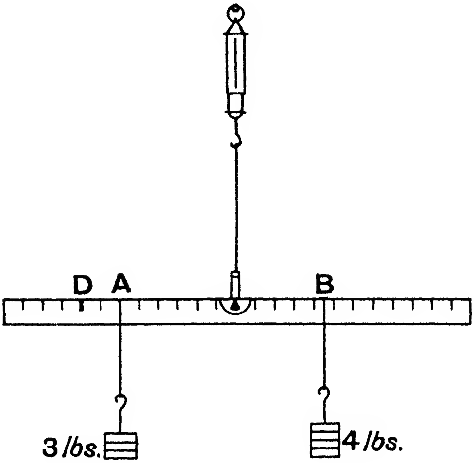


Fig. 74 *a*.

and the rod is still in equilibrium for the pull of the string now replaces the reaction of the pivot. This force, which we will call P lbs. wt., will be registered by the spring-balance. The rod is now in equilibrium under the action of the three forces shown in Fig. 74 *b*. (Since probably the weight of the rod is less than

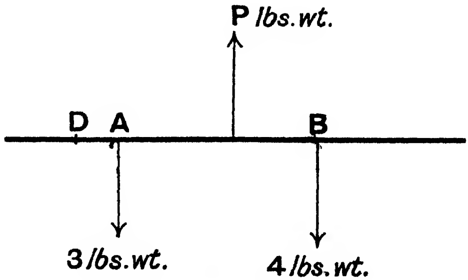


Fig. 74 *b*.

2 ozs., its effect will be barely appreciable, so that for simplicity we will neglect it.)

Now suppose we bore a hole through the rod at A and stick a bradawl through it into the board behind. Since the bradawl exerts no force on the rod the equilibrium will be undisturbed. Moreover the load of 3 lbs. can now be removed without disturbing the rod, since it is clear that the bradawl will in this case exert a force of 3 lbs. wt. downwards. Now, taking moments about the fixed axis of the pivot at A, we have

$$P \times 6 = 4 \times 10\tfrac{1}{2} \quad \text{or} \quad P = 7 \text{ lbs. wt.}$$

This result we find to correspond closely with the reading of the spring-balance. Observe that this force is equal to the sum of the downward pulls, a result we should expect from the fact that the forces are parallel.

Similarly, by taking moments about a pivot at B, we obtain the same result, for in this case we have

$$P \times 4\tfrac{1}{2} = 3 \times 10\tfrac{1}{2}, \text{ hence } P = 7 \text{ lbs. wt.}$$

Now let us take moments of all the forces about any point of the rod. For instance, taking moments about D, a point 2 inches outside A, we have

Moment of force of 4 lbs. wt. $= 4 \times 12\tfrac{1}{2} = 50$ lb. ins. clockwise.

Moment of force of 3 lbs. wt. $= 3 \times 2 = 6$ lb. ins. clockwise.

Moment of force of 7 lbs. wt. $= 7 \times 8 = 56$ lb. ins. counter-clockwise.

The sum of the clockwise moments is $50 + 6 = 56$ lb. ins., which is equal to the counter-clockwise moment. This relation we also find to hold good if we take moments about any axis at right angles to the paper, whether the axis passes through the body or not.

We can now state that: **When a body is in equilibrium under the action of forces in one plane, and the moments of all the forces are taken about any axis at right angles to this plane, the sum of the clockwise moments is equal to the sum of the counter-clockwise moments.**

Although we have confined our attention to a body in equilibrium under the action of three parallel forces, we can apply the same reasoning to shew that this principle holds good for all cases of equilibrium of forces in one plane. This can be verified experimentally by such a method as that described in the following article.

60. Principle of Moments. (Forces inclined.)

Exp. 4. Take a disc of cardboard and bore four holes through it at various points not far from its edge. Rest this disc on a few steel balls placed on a small sheet of glass on a horizontal table (Fig. 75 *a*). Arrange that the disc is kept at rest by any

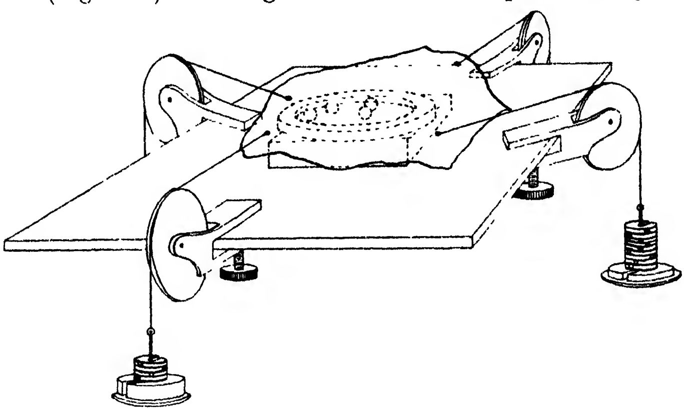


Fig. 75 *a*.

four horizontal forces. Each of these forces is applied by putting a hook, to which a thread is attached, into one of the holes and passing the thread over a pulley, a weight being fastened to the other end. Adjust the weights and positions of the pulleys, which can be clamped to the edge of the table, so that the disc is kept at rest in about the middle of the table.

The forces which act on the disc are the pulls along the four strings, which are severally measured by the weights which are attached to them.

Now choose any point O which is not in the line of action of any of the forces and stick a pin through the cardboard at this point so as to form a fixed pivot. This pin exerts no force on the disc.

Although each of the forces, acting alone, would turn the body about the pin, their total turning effect is nil. Calculate the moments of all the forces about this axis through O. To measure the moments, it is first necessary to draw with a ruler the lines of action of the forces on the disc, being careful not to disturb its position. Mark along each line the magnitude of the force which acts along it and remove the disc. With a set square draw perpendiculars from O to each line and measure them. By multiplying the number of lbs. wt. in each force by the number of inches in its perpendicular distance from O we get its moment about the axis selected.

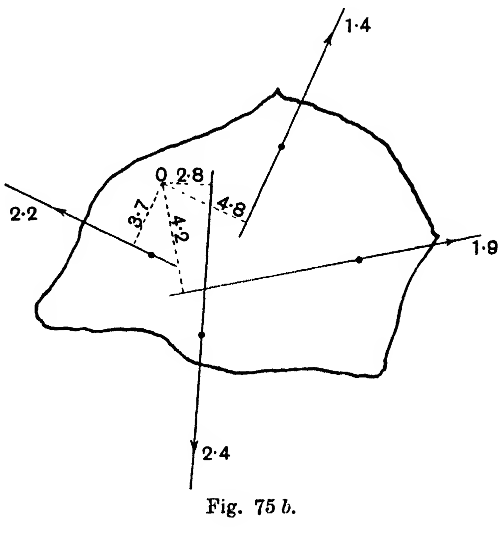


Fig. 75 *b*.

In a certain experiment the results shewn in Fig. 75 *b* were obtained. The forces were 2·4, 2·2, 1·4 and 1·9 lbs. wt. respectively and the corresponding perpendicular distances (or arms) were found to measure 2·8, 3·7, 4·8 and 4·2 inches respectively.

The clockwise moments about O are

$$2{\cdot}4 \times 2{\cdot}8 + 2{\cdot}2 \times 3{\cdot}7 = \mathbf{14{\cdot}86\ lb.\ ins.}$$

The counter-clockwise moments are

$$1{\cdot}4 \times 4{\cdot}8 + 1{\cdot}9 \times 4{\cdot}2 = \mathbf{14{\cdot}7\ lb.\ ins.}$$

Similarly it will be found that, whatever axis we select, the sum of the clockwise moments equals the sum of the counter-clockwise moments about this axis.

It should be noticed that if we agree to call the moments tending to turn a body in one direction *positive*, and those tending to turn it in the opposite direction *negative*, we can express the *Principle of Moments* as follows: *When a body is in equilibrium under the action of forces in one plane, the algebraical sum of the moments of all the forces about any axis at right angles to this plane is zero.*

61. Exercises in Taking Moments.

Ex. 1. (Fig. 76.) A uniform bar, 6 feet long, is pivoted 1 foot from one end and is loaded at this end with a weight of 80 lbs. The weight of the bar is 8 lbs. Find what vertical force (F) must be applied at the other end to balance the bar horizontally and find also the force (R) exerted by the pivot.

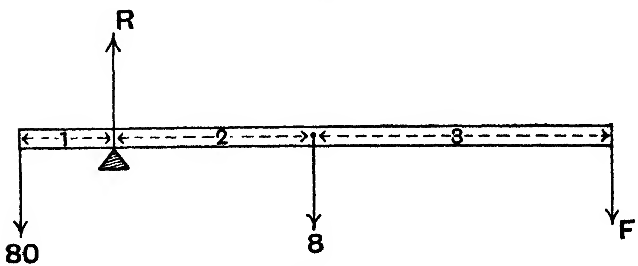


Fig. 76.

We must first specify **all** the forces acting on the bar. We shall assume that the weight of a uniform bar acts through its middle point; an assumption we shall justify later.

The forces acting on the bar are shewn in the figure. R is the upward reaction of the pivot called into play to keep the bar in equilibrium.

Taking moments about the axis of the pivot, we have

Moment of R lbs. wt. $= R \times 0 = 0$.

Moment of F lbs. wt. $= F \times 5$ lb. ft. clockwise.

Moment of the weight of the rod $= 8 \times 2$ lb. ft. clockwise.

Moment of 80 lbs. wt. $= 80 \times 1$ lb. ft. counter-clockwise.

Since the sum of the clockwise moments is equal to the counter-clockwise moment, we have

$$F \times 5 + 8 \times 2 = 80 \times 1 \text{ lb. ft.}$$

Hence $$F = \frac{80 - 16}{5} = \mathbf{12{\cdot}8\ lbs.\ wt.}$$

We can now determine the force R by taking moments about any other point. If we decide to take moments about the right-hand end of the bar, and equate the clockwise and counter-clockwise moments, we obtain the equation

$$R \times 5 = 8 \times 3 + 80 \times 6 \text{ lb. ft.};$$

hence $$R = \mathbf{100{\cdot}8\ lbs.\ wt.}$$

Notice that, since F has already been determined, R can be found more quickly by adding together the downward forces, *i.e.*

$$80 + 8 + 12{\cdot}8 = \mathbf{100{\cdot}8\ lbs.\ wt.}$$

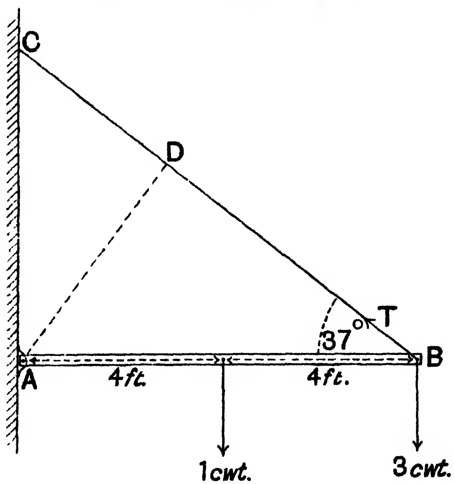


Fig. 77.

Ex. 2. (Fig. 77.) A uniform bar AB, 8 ft. long and weighing 1 cwt., is hinged to a vertical wall at A and supported in a horizontal position by

a tie BC inclined at 37° to the rod. Find the tension in the tie when a load of 3 cwt. is suspended from B.

Call this tension T cwt. Now the rod is in equilibrium under the action of four forces, namely, the three forces shewn in the diagram and also the reaction of the hinge. We know that the latter force acts through A, but we do not know in what direction. However, if we choose A as the point about which to take moments, the moment of the reaction will be zero, since a force has no moment about a point on its line of action. Therefore we select A as the point about which to take moments, and thereby obtain the equation

$$T \times AD = 1 \times 4 + 3 \times 8 \text{ cwt. ft.},$$

where AD is the perpendicular drawn from A to the line of action of the force T. If a diagram is drawn to scale, AD can be found by measurement; otherwise we can calculate it thus,

$$\frac{AD}{AB} = \sin 37° \quad \text{or} \quad AD = AB \sin 37° = 8 \times \cdot 6 = 4 \cdot 8 \text{ feet.}$$

Thus $$T = \frac{28}{4 \cdot 8} = \mathbf{5 \cdot 8 \text{ cwt.}}$$

Note particularly that we cannot use the principle of moments to find the reaction of the hinge in this case as we did in Ex. 1, since, the forces not being parallel, we do not at present know how to determine the direction of this force.

62. Note on Taking Moments.—Important Rule.

In applying the principle of moments to the forces keeping a body at rest we are at liberty to take moments about *any* axis at right angles to the plane of the forces. However, if we judiciously select this axis through a point in the line of action of one of the forces, the moment of this force being zero, we may obtain a simple relation connecting the remaining forces. Therefore, as in the examples above, we shall generally simplify a problem by taking moments about an axis through a point in the line of action of one unknown force.

It will be well to emphasize here the importance of the following rule to which we have already drawn attention and which should always be rigidly adhered to when attacking any problem dealing with forces: **Before applying any principle to the solution of a problem state definitely the body whose equilibrium you are going to consider, and specify all**

the forces acting on this body. A diagram of the body should be drawn shewing the known directions of the forces acting upon it.

63. Reactions of the Supports of a Loaded Horizontal Beam. As a further example, we will use the principle of moments to calculate the reactions of the supports of a loaded beam and will verify our results by experiment.

Exp. 5. Rest a uniform graduated rod on two compression spring-balances which carry small wedges so as to give definite points of support. Arrange the rod so that the points of supports A and B are equidistant from the ends (Fig. 78 *a*). Suspend a load of 4 lbs. at some point D. The weight of the rod is supposed to be 1 lb. and may be taken as acting at the middle point C.

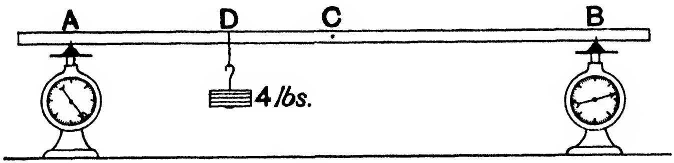


Fig. 78 *a*.

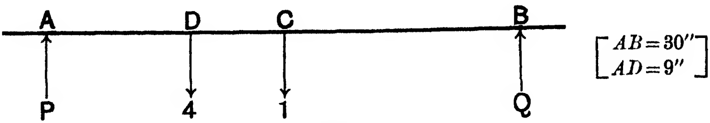


Fig. 78 *b*.

The forces acting on the rod are shewn in Fig. 78 *b*, in which P and Q are the upward reactions of the supports at A and B respectively. Suppose, in a certain case, that AB is 30 inches and AD 9 inches. Since we have here two unknown forces P and Q, we take moments about a point in the line of action of one of them. Thus, taking moments about A, the moment of P is zero, and by the principle of moments we have

$$Q \times AB = 1 \times AC + 4 \times AD \text{ lb. ins.}$$

or $\quad Q \times 30 = 1 \times 15 + 4 \times 9$, hence $Q = \frac{51}{30} =$ **1·7 lbs. wt.**

Similarly, taking moments about B, the moment of Q is zero, and we get the equation

$$P \times AB = 1 \times BC + 4 \times DB,$$

or $\qquad P \times 30 = 1 \times 15 + 4 \times 21$, hence $P = \frac{99}{30} =$ **3·3 lbs. wt.**

Compare these results with the readings of the spring-balances. Notice that $P + Q = 1{\cdot}7 + 3{\cdot}3 = 5$ lbs. wt., which is equal to the sum of the downward forces. Moreover, we find that wherever we suspend the weight, the readings of the two balances add up to 5 lbs. wt.

As a further exercise let us calculate where the load of 4 lbs. must be placed to produce readings of 3 and 2 lbs. wt. on the spring-balances at A and B respectively.

Let the required position be x inches from A.

Taking moments about A, we have

$$4x + 1 \times 15 = 2 \times 30 \text{ lb. ins., hence } x = \frac{60 - 15}{4} = \mathbf{11\tfrac{1}{4}\ inches.}$$

Verify this result by reading the balances when the load is placed $11\frac{1}{4}$ inches from A.

Ex. A uniform beam AB spans an opening of 16 feet and rests on supports at its ends. It carries loads of 6 cwt. and 8 cwt. at distances of 4 feet and 13 feet respectively from the end A. If the weight of the beam is 4 cwt., determine the reactions at its ends. Let these reactions be P and Q cwt.

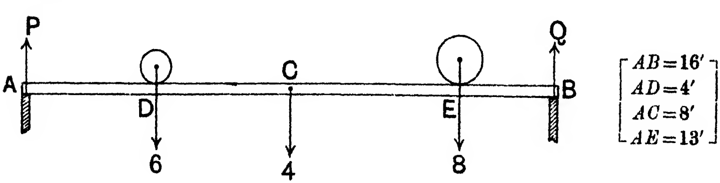


Fig. 79.

Draw a sketch of the beam and shew all the forces acting upon it. (Fig. 79.)

Here we have two unknown forces. However, by taking moments about

an axis through A, the moment of P is zero, and by the principle of moments we obtain the equation

$$16Q = 6 \times 4 + 4 \times 8 + 8 \times 13 = 160 \text{ cwt. ft.},$$

hence $$Q = \frac{160}{16} = \mathbf{10\ cwt.}$$

Since the sum of the upward forces must equal the sum of the downward forces,

$$P + Q = (6 + 4 + 8) \text{ cwt.} = 18 \text{ cwt.}$$

so that $$P = 18 - 10 = \mathbf{8\ cwt.}$$

Check this result by taking moments about an axis through B.

64. Simple Machines.—Levers. A lever is a machine, which in one form or another has been employed from the earliest times. It consists of a rigid bar, either straight or bent, which is pivoted at some point of its length. This pivot about which it turns is called the fulcrum. For instance, a crow-bar when used for raising a body is an example of the simplest type of

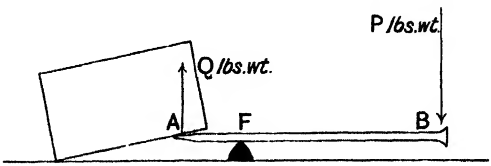


Fig. 80 *a*.

lever. This is a bar of iron having one end flattened. The flat end is inserted under the body, and beneath the bar is placed a block of wood, or stone, to serve as a support about which it may turn (Fig. 80 *a*). When a downward force is applied to the free end of the bar an upward force is communicated to the body. Suppose a vertical force of P lbs. wt. to be applied to the crow-bar at the point B and that, in consequence, a vertical force of Q lbs. wt. is transmitted to the body at A, the bar being taken as horizontal.

Since we are going to consider the equilibrium of the bar we must specify all the forces acting on it. These forces are shewn in Fig. 80 *b*; we have here left out of consideration the weight of the bar, the effect of which is assumed to be negligible. Notice that Q is equal and opposite to the force which the bar exerts on the body, and R is the reaction of the fulcrum which is brought

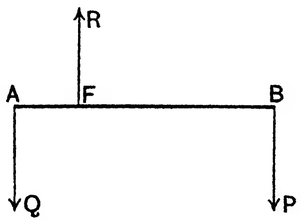


Fig. 80 *b*.

into play by P and Q, and balances them. This reaction, however, has no moment about F, so that, on taking moments about this point we have as the condition of equilibrium

$$P \times FB = Q \times FA \quad \text{or} \quad \frac{Q}{P} = \frac{FB}{FA}.$$

For example, if FB is 30 inches and FA is 5 inches, and we wish to overcome a resistance Q of 600 lbs. wt., then

$$P \times 30 \text{ lb. ins.} = 600 \times 5 \text{ lb. ins.}$$

or

$$P = 100 \text{ lbs. wt.},$$

and the mechanical advantage is $\dfrac{Q}{P} = \dfrac{600 \text{ lbs. wt.}}{100 \text{ lbs. wt.}} = 6.$

By placing the fulcrum nearer to the end A we can increase the ratio of the arms, namely $\dfrac{FB}{FA}$ (sometimes called the 'leverage'), and thus obtain a larger mechanical advantage.

As another example of the use of levers, we will take one

such as that used to work the plunger of a hydraulic press (Fig. 81 *a*).

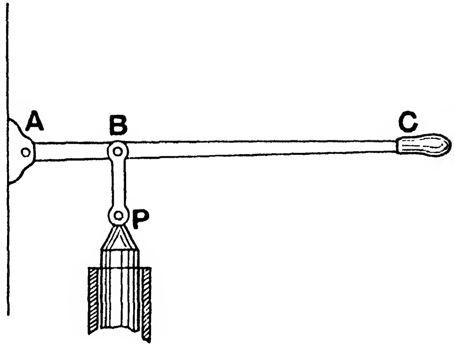


Fig. 81 *a*.

AC is the lever which can turn about a fixed pivot at A. At B, a loosely jointed rod connects the lever to the top of the plunger P.

If a force of 50 lbs. wt. is applied to the lever at C in a vertically downward direction, what force will act on the plunger if $AB = 6$ ins., $AC = 24$ ins., and BP is vertical? Consider the equilibrium of the lever. The forces acting upon it (neglecting, as before, its weight) are shewn in Fig. 81 *b*, namely, 50 lbs. wt.

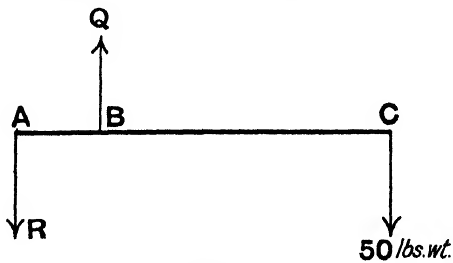


Fig. 81 *b*.

vertically downwards at C; the reaction of the connecting-rod, Q lbs. wt., which, for the position shewn in the diagram, acts vertically upwards at B; and the reaction R of the pivot A

which has no moment about this point. If, then, we take moments about A, we have for the condition of equilibrium,

$$Q \times 6 \text{ lb. ins.} = 50 \times 24 \text{ lb. ins.} \quad \text{or} \quad Q = \mathbf{200\ lbs.\ wt.},$$

and this is equal and opposite to the thrust produced on the plunger.

The ratio $\frac{\text{load}}{\text{effort}}$ or mechanical advantage is given by

$$\frac{200 \text{ lbs. wt.}}{50 \text{ lbs. wt.}} = 4.$$

Do not forget, however, that we are here neglecting both the weight of the lever and the friction at the pivot and joints.

We often use double levers of various types, such as scissors, pliers, nut-crackers, shears, etc. In Fig. 82 is shown one type of a

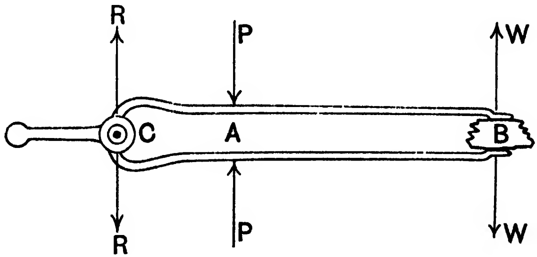


Fig. 82.

double lever, namely, a pair of tongs, which may be regarded as a combination of two equal levers pivoted at their common fulcrum C. Considering, for simplicity, the two levers in the same horizontal plane, and shewing only the horizontal forces acting upon them, we see that each lever is acted upon by the force P, exerted by the grasp of the hand; the force W with which the lump of coal reacts on the end; and the reaction of the pivot R. Considering the equilibrium of either lever, and taking moments about the pivot C, we have $P \times CA = W \times CB$ or $\frac{W}{P} = \frac{CA}{CB}$. You may notice that this type of lever differs from a pair of pliers

and nut-crackers in that the mechanical advantage is less than 1, the use of such tongs being, not to exert large forces, but simply to avoid handling the coal.

65. Continuous lever.—Simple winch. When using a simple lever for lifting a weight, we can only raise this weight through a short distance. However, by a simple modification which we have called a 'Wheel and Drum,' we can secure continuous action. As an example of this type of continuous lever, let us consider a simple winch,

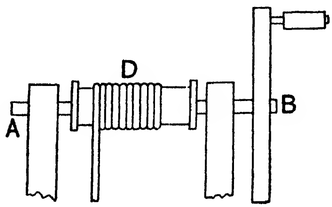


Fig. 83*a*.

such as is used for raising water from a well (Fig. 83*a*). To one end of a spindle AB, which is mounted horizontally in two bearings, is keyed a wheel, or simple iron crank, to which a handle is fastened. On AB is mounted a drum D. One end of the rope is made fast to the drum and the other end carries the bucket.

Let the radius of the drum be r inches; for more accurate calculations this distance should be measured from the centre of the drum to the middle of the rope. Let the effective radius of the wheel, that is, the distance of its centre from the centre of the handle, be R inches.

Suppose that a constant force of P lbs. wt., applied to the handle in a direction always at right angles to the crank, is required to lift a bucket of water weighing W lbs.

Since the weight of the wheel and drum and the reactions of the bearings upon it all act through the central axis, P and W are the only forces which tend to turn it about this axis (Fig. 83*b*). For equilibrium, the moments of these forces about this axis must be equal, that is,

$$P \times R = W \times r \quad \text{or} \quad \frac{W}{P} = \frac{R}{r}.$$

For example, if the effective radius of the handle is four times that of the drum, we should be able to balance a load of 40 lbs. by an effort of 10 lbs. wt.

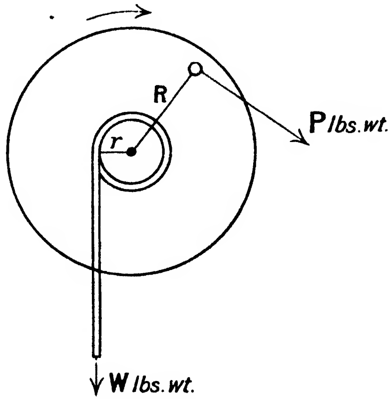


Fig. 83 *b*.

In practice, the effort required just to balance a given weight is found to be somewhat smaller, and the effort to raise the weight is found to be somewhat larger, than that calculated from the above equation; why is this?

Examples of various types of this machine are frequently met with in practice; for instance, a capstan or windlass: two toothed wheels on the same spindle such as you can observe in any 'train of wheels': two belt pulleys on the same shafting: etc.

66. Alternative solution, using the Principle of Work. The above relation between the load and effort in the case of the simple winch can be verified by the same experiment as that employed in Art. 25 to verify the principle of work; or it can be deduced from that principle as follows. If we assume, as we did above, that the friction at the bearings of the winch is negligible, then the principle of work states that the work done

by the effort or applied force is equal to the work done on the load during any given movement of the machine.

Let us consider this movement to be one revolution of the axle.

Then the work done by P

$$= P \times \text{distance moved in direction of } P$$

$$= P \times 2\pi R \text{ inch lbs.}$$

Similarly, the work done on the load $W = W \times 2\pi r$ inch lbs.

Neglecting frictional resistances, we have

$$P \times 2\pi R = W \times 2\pi r$$

or

$$P \times R = W \times r,$$

that is, the moment of the force tending to turn the body clockwise is equal to the moment of the force tending to turn it counter-clockwise.

It will be found that the majority of problems in Statics can be solved by alternative methods. Ability to recognise the easiest method of solution can only be cultivated by frequent practice in solving such problems by the application of different principles.

67. Compound Levers. Fig. 84*a* represents a type of compound lever such as is used to operate railway points. It consists of a combination of a straight lever turning about a fixed pivot C, and a bent (or bell-crank) lever whose fixed pivot is F.

The connecting-rod DB is jointed to the levers at B and D; the bent lever is also jointed at E to another rod EG. What force P must be applied horizontally at A so that the rod EG may overcome a resistance of W?

We shall here leave out of consideration the effects of friction at the pivots and joints and also the effects produced by the weights of parts of the machine itself.

The equilibrium of each part of the mechanism must be considered separately.

If T is the tension produced in DB, it is clear that this rod is

in equilibrium under the two equal and opposite forces shewn in Fig. 84*b*, these forces being equal and opposite to the forces which this rod exerts on the levers. Figs. 84*c* and 84*d* represent the two levers and the forces which tend to turn them about their respective pivots. Each lever is also acted upon by the reaction of its pivot. In the case of the bent lever this reaction is not shewn since we do not yet know its direction, but we know that it has no moment about the pivot F.

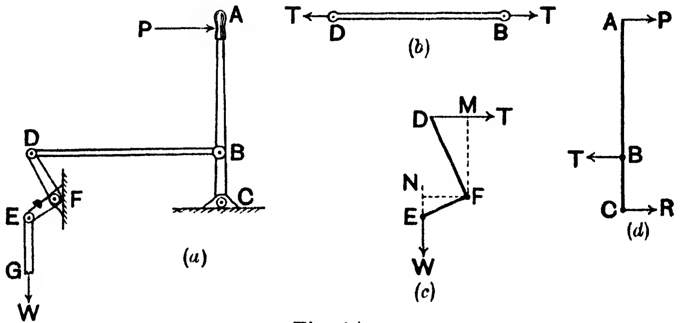


Fig. 84.

Let us first consider the equilibrium of the bent lever. Draw perpendiculars FN and FM from F on to the lines of action of the forces W and T. Then, taking moments about F, we have

$$T \times FM = W \times FN \quad \text{or} \quad T = W \times \frac{FN}{FM}.$$

Secondly, considering the equilibrium of the straight lever, and taking moments about C, we have

$$P \times AC = T \times BC \quad \text{or} \quad P = T \times \frac{BC}{AC}.$$

Substituting in this equation the above value of T, we have

$$P = W \times \frac{FN}{FM} \times \frac{BC}{AC}.$$

Note that the mechanical advantage $\frac{W}{P} = \frac{FM}{FN} \times \frac{AC}{BC}$, which is the ratio of the arms, or leverage of the first lever multiplied by the leverage of the second. Hence, to obtain a great leverage we often find that a compound lever is employed in place of a single lever of inconvenient length.

68. Crab Winch. As an example of a compound continuous lever or compound 'wheel and drum,' we will consider a crab winch with single gear, the essential parts of which are shewn in Fig. 85*a*. Such a machine you have probably seen in use for hauling boats up a beach or for hoisting bags of coal. The rope is wound round a barrel, the spindle of which also carries a large toothed wheel. Secured to a second parallel spindle is another smaller toothed wheel, or pinion, in gear with the first, and also a crank-handle. To raise the weight W, this handle is rotated in a clockwise direction.

Leaving out of consideration frictional resistances, what force P must be applied to the handle at right angles to the crank to balance a load W?

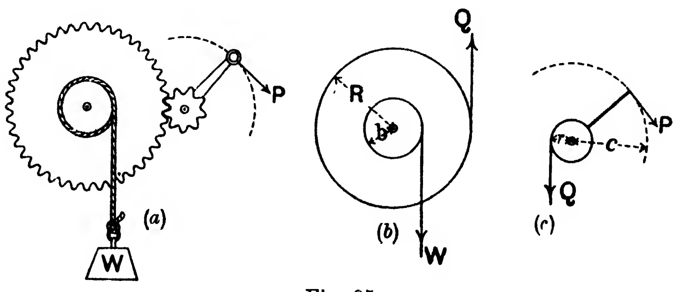


Fig. 85.

Here again, we must consider the equilibrium of each pivoted body separately. In Figs. 85*b* and 85*c*, the forces tending to turn the two spindles are shewn, the toothed wheels being represented by their effective or 'pitch' circles. The action of the

teeth of the pinion on the wheel and the equal and opposite reaction on the pinion are represented by the forces marked Q. Remember that each spindle is also acted upon by the reactions of the bearings which, however, produce no turning effect.

Considering the first spindle and taking moments about its central axis, we have

$$Q \times R = W \times b \quad \text{or} \quad Q = W \times \frac{b}{R},$$

where R and b are the effective radii of the toothed wheel and barrel respectively.

Similarly, the condition of equilibrium of the second spindle is given by

$$P \times c = Q \times r \quad \text{or} \quad P = Q \times \frac{r}{c},$$

where c and r are the effective radii of the crank circle and pinion respectively.

Substituting the above value of Q in this equation, we have

$$P = W \times \frac{b}{R} \times \frac{r}{c} \quad \text{or} \quad \frac{W}{P} = \frac{R}{r} \times \frac{c}{b}.$$

Now $\frac{R}{r}$, being the ratio of the effective radii of the two gear wheels, is therefore equal to the ratio of the circumferences of their pitch circles, which is equal to the ratio of the numbers of teeth in the two wheels, for the teeth are all of the same size. To make this clear, let N and n be the numbers of teeth in the large and small wheels respectively, and let p be the distance from the centre of one tooth to the centre of the next, measured along the circumference.

Then $$\frac{R}{r} = \frac{2\pi R}{2\pi r} = \frac{Np}{np} = \frac{N}{n}.$$

Hence, we can write the above equation

$$\frac{W}{P} = \frac{N}{n} \times \frac{c}{b}.$$

The importance of this lies in the fact that it is often easier to

count the number of teeth in a wheel than to measure its effective radius.

Let us work out the following example by this method and also by applying the principle of work.

Ex. Neglecting friction, find what weight can be hoisted with a simple-geared crab winch by applying a force of 20 lbs. wt. to the handle, given that

Radius of barrel = 3″, Length of crank = 8″,

and the numbers of teeth in the wheel and pinion are 45 and 10 respectively.

(1) *Principle of Moments.* Working with these numerical values on the lines indicated above, we get

$$\frac{W}{20} = \frac{45}{10} \times \frac{8}{3}.$$

Hence $$W = \frac{20 \times 45 \times 8}{10 \times 3} = \mathbf{240\ lbs.\ wt.}$$

(2) *Principle of Work.* Since we are supposing that no work is done against friction we can state that

Work done by effort

= Work done on load during any given movement of the machine.

Let this movement be one revolution of the crank. Then the work done by the effort of 20 lbs. wt. $= 20 \times 2\pi \times 8$ inch lbs.

Now, during one revolution of the pinion the wheel turns through an angle subtended by 10 teeth or $\frac{10}{45}$ of a revolution, and since the barrel turns through the same fraction of a revolution, it follows that the rope is wound up a distance equal to $\frac{10}{45}$ of the circumference of the barrel or $\frac{10}{45} \times 2\pi \times 3$ inches.

Therefore the work done on the load

$$= W \times \tfrac{10}{45} \times 2\pi \times 3 \text{ inch lbs.}$$

Equating, we have

$$W \times \tfrac{10}{45} \times 2\pi \times 3 = 20 \times 2\pi \times 8$$

or $$W = \frac{20 \times 2\pi \times 8 \times 45}{10 \times 2\pi \times 3} = \mathbf{240\ lbs.\ wt.}$$

69. Inclined Plane. An inclined plane facilitates the transfer of loads from one level to another. For instance, a garden roller may be pulled up a slope by a force much smaller than the weight of the roller; the less the slope, the smaller the effort required. The question naturally arises 'How does the relation of the weight of the body to the effort required to pull it up a slope depend on the inclination of the slope?' The equilibrium of a body on an inclined plane we have already partially dealt with in Art. 25, where we illustrated by experiment the principle of work. Here, however, we will shew that we can answer the above question by applying either of the two principles which we have so far established; and since the answer is more simply arrived at by applying the principle of work we will apply this in the first place.

(1) *Principle of Work.* A uniform cylinder or roller of weight W is steadily pulled up a plane from A to B by a force P applied to the axis a in a direction parallel to the plane (Fig. 86).

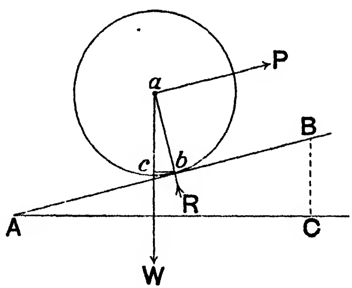


Fig. 86.

The work done by P is measured by $P \times AB$, and since during this movement the load is lifted a vertical distance BC, the work done on the load is measured by $W \times BC$. If no appreciable

amount of work is done against friction, these quantities must be equal, that is

$$P \times AB = W \times BC \quad \text{or} \quad P = W \times \frac{BC}{AB} = W \sin BAC.$$

(2) *Principle of Moments.* Since the cylinder is uniform, we will assume that its weight acts through the centre of its axis. The forces keeping the body in equilibrium are shewn in the figure, which represents a central section of the cylinder. Note that, since the reaction R of the plane acts at b the point of contact, we shall not require for our present purpose to know its direction, for we shall select this point to take moments about. However, it is not difficult to understand that in this case it must act at right angles to the plane.

Now, P and W are the only forces tending to turn the roller about its line of contact with the plane, and hence, for equilibrium, the moments of these forces about this line must be equal. Thus, if bc is the perpendicular drawn from b to the line of action of W, we have

$$P \times ab = W \times bc \quad \text{or} \quad P = W \times \frac{bc}{ab}.$$

Seeing that the triangles abc and ABC are equiangular (why?) and therefore similar, we have $\frac{bc}{ab} = \frac{BC}{AB}$, hence

$$P = W \times \frac{BC}{AB} = W \sin BAC.$$

This result can be easily verified experimentally by attaching a spring-balance to the axle of a metal roller on an inclined plane, as described in Art. 25.

$\frac{BC}{AB}$ or $\sin BAC$ is called the *slope* of the plane. For instance, when we speak of a road having a slope of 1 in 20, we mean the road rises 1 foot for every 20 feet measured along the road; that is, $\frac{1}{20}$ is the sine of the angle at which the road is inclined to the horizontal. The force necessary to pull a cart up this incline

would be equal to $\frac{1}{20}$th of the weight of the cart. Bear in mind, however, that we are here neglecting the rolling friction of the wheels and the friction at their bearings.

The above important property of the inclined plane is made use of in other machines, such as the wedge and the screw, which we shall refer to later. These machines are especially remarkable for the important part that friction plays in their behaviour.

70. Effect of Friction. We have treated the foregoing simple machines from a rather unpractical standpoint, having, for simplicity, left out of consideration in our calculations the influence of friction. However, in predicting the behaviour of any machine, or in designing a machine for any given purpose, due consideration must obviously be given to the frictional forces brought into action between the various parts of the machine. It is for this reason that we emphasized early in the book the importance of carrying out practical tests with actual machines under working conditions, in order to determine the effect of friction upon the behaviour and efficiency of the machine in each case.

We discussed in Chapter IV the factors which govern the amount of friction brought into play in machines. The conclusions there reached apply as well to the machines described in this chapter.

For instance, in the case of the simple winch, the friction is principally that called into play between the rubbing surfaces at the bearings of the spindle. If the total reaction of the bearings on the spindle be denoted by R, and the coefficient of friction by μ, we know that when the spindle turns, there is acting upon it a force of friction given by μR. This force acts tangentially to the circumference of the spindle and therefore has a moment about the central axis given by $\mu R \cdot \frac{d}{2}$, where d is the diameter of the spindle at the bearings. Since this moment opposes the rotation of the spindle, it is clear that the effort required, as calculated for the frictionless machine, must be

increased till its moment about the central axis is greater by this amount.

Again, the force required to slide, instead of to roll, a body of weight W up a plane inclined at an angle $\alpha°$ to the horizontal will be greater than that deduced in the preceding article, namely $W \sin \alpha$, by the force of friction called into play at the sliding surfaces. This force, again, we know is given by μR, where R is the normal reaction of the plane on the body, and μ the coefficient of friction. We see, then, that in all cases, before we can calculate the effect of friction, we must know, in addition to other factors, the coefficient of friction, a reliable value for which can be obtained only by experimenting with the actual surfaces concerned. It follows, therefore, that the simplest method of determining the effect of friction is to subject the machine itself to direct experiment.

71. Resultant of Parallel Forces. We have already shewn in Art. 59 that two parallel forces acting in the same sense can be balanced by a single force whose magnitude is equal to their sum. For example, let two parallel forces of 4 and 6 lbs. wt. be balanced by a force of 10 lbs. wt. (Fig. 87). If

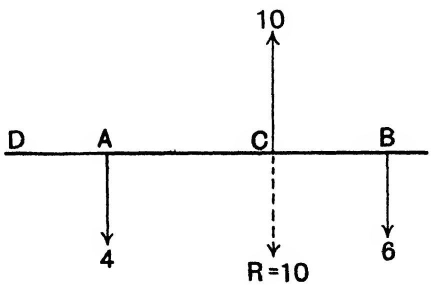


Fig. 87.

these forces of 4 and 6 lbs. wt. are replaced by a single force of 10 lbs. wt. acting along the same line as, but in the opposite direction to, their 'balancing-force,' equilibrium will still be main-

tained. This force, whose direction is shewn by the dotted line, is the *resultant* of the forces of 4 and 6 lbs. wt., since it has the same effect as these two forces acting together. The latter are called the *components* of the resultant.

Further, since by the principle of moments the sum of the clockwise moments about a point D of the 4 and 6 lbs. wt. is equal to the *counter-clockwise* moment of their balancing-force taken about the same point, it follows that the sum of the clockwise moments of these two forces is also equal to the *clockwise* moment of their resultant about the same point. This is simply expressing quantitatively the fact that the turning effect of the resultant is equal to the combined turning effects of its components about any point. It is clear that this must also be true for the resultant of any number of parallel forces.

So we see that **the resultant of any number of parallel forces acting in one plane, and in the same sense, is equal to their sum, and its moment about any point is equal to the sum of their moments about the same point.**

If this point is so chosen that the moments of some of the components are clockwise and the moments of others counter-clockwise, the above statement will still apply provided we call the moments in one direction positive and those in the opposite direction negative (see end of Art. 60).

Referring to the diagram above, it should be noted that the position of the point C, through which the balancing-force and the resultant of the forces of 4 and 6 lbs. wt. act, is expressed by any of the following relations, for, by taking moments about C, we have

$$4 \times AC = 6 \times BC \quad \text{or} \quad \frac{AC}{BC} = \frac{6}{4};$$

taking moments about A, we have

$$10 \times AC = 6 \times AB \quad \text{or} \quad \frac{AC}{AB} = \frac{6}{10};$$

taking moments about D, we have

$$10 \times DC = 4 \times DA + 6 \times DB \quad \text{or} \quad DC = \frac{4 \times DA + 6 \times DB}{10}.$$

Observe also that we can look upon any one of the three forces as the balancing-force of the other two and therefore as equal and opposite to their resultant. For example, the force of 6 lbs. wt. balances the forces of 4 and 10 lbs. wt. The resultant of these last two forces is therefore a force of 6 lbs. wt. acting *upwards* at B. Here, we see that *the resultant of parallel forces of opposite sense is equal to their difference* and its line of action can be found by taking moments, as shewn by the above equations.

Example. Four parallel forces of 7, 6, 4 and 9 lbs. wt. respectively are applied to a rigid body as shewn in Fig. 88. What is the magnitude of their resultant R and what is the perpendicular distance of its line of action from the point A?

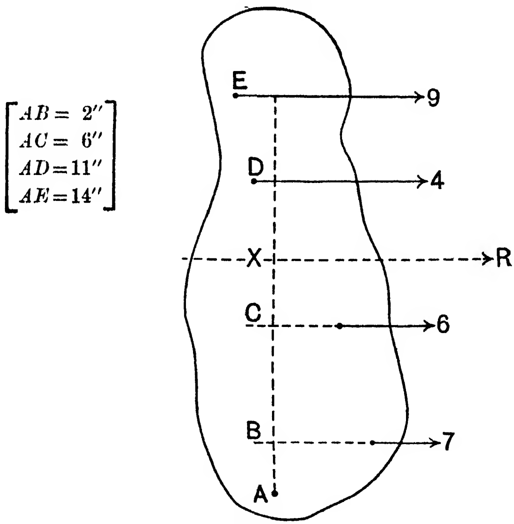


Fig. 88.

Draw the dotted line $ABCDE$ at right angles to the lines of action of the forces. Let X be the point where the line of action of the resultant cuts this line.

The magnitude of the resultant $R = 7 + 6 + 4 + 9 =$ **26 lbs. wt.**

Also, taking moments about A, we know that the moment of the resultant is equal to the sum of the moments of its components, that is,

$$26 \times AX = 7 \times AB + 6 \times AC + 4 \times AD + 9 \times AE.$$

Given that $AB = 2''$, $AC = 6''$, $AD = 11''$ and $AE = 14''$, this equation becomes

$$26 \times AX = 7 \times 2 + 6 \times 6 + 4 \times 11 + 9 \times 14,$$

and hence $$AX = \frac{220}{26} = \mathbf{8{\cdot}5''}.$$

That is, the resultant is a force of 26 lbs. wt. and the perpendicular distance of its line of action from A is 8·5 inches.

72. Couples. If two equal parallel forces act on a body in opposite sense and not in the same straight line, they are said to form a *couple*. The effect of a couple is to turn a body round without moving it as a whole in any direction. The following practical illustrations will make this clear.

Exp. 1. Take a circular wooden disc having a groove round its edge, and support it by a few steel balls on a horizontal table (Fig. 89 a). Fasten two threads to the groove, and coil them in

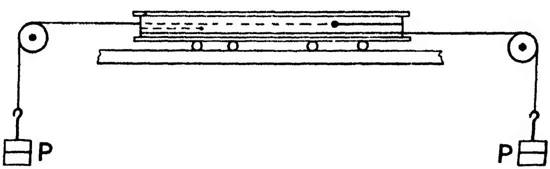


Fig. 89 *a*.

the same direction round the disc, and then lead the free ends away from the opposite extremities of a diameter. *Holding the disc at rest, pass these threads over pulleys, so arranged that the threads are parallel, and attach equal weights to their ends.* The threads exert on the disc two equal forces of opposite sense, each

equal to P (Fig. 89*b*). On releasing the disc it rotates about its centre but does not move bodily.

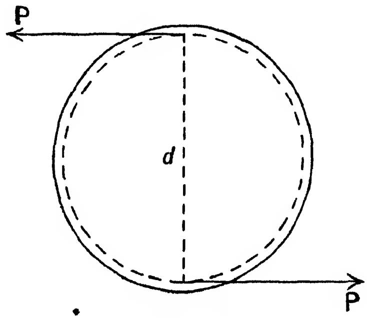


Fig. 89 *b*.

Exp. 2. Put a magnetised needle on a cork floating in the centre of a large dish of water. If the needle is released when it is not pointing N. and S., it turns round and comes to rest in the meridian, but does not move bodily in either direction. The reason of this is that the forces on the poles of the magnet caused by the earth's magnetism are equal, parallel, and opposite, and hence form a couple.

The following interesting properties of a couple can be deduced from the principle of moments and verified by experiments with the apparatus illustrated above; they are left as useful exercises for the more advanced student.

(1) A couple cannot be balanced by a single force.

(2) The turning effect of a couple is the same about any axis at right angles to its plane, for its moment is always equal to the product of one of the forces and the perpendicular distance between their two lines of action, *i.e.* in Fig. 89*b* above, the moment of the couple $= P \,.\, d$.

(3) Any two couples having equal moments but opposite turning tendencies will balance.

Examples V.

Note. *In answering these questions remember the rule given in Art. 62. Begin by making a sketch of the body you are going to consider, fill in all known quantities, and, as far as you can, shew the forces acting on the body.*

The weight of a uniform bar may be taken as acting on the bar at its centre.

1. A force of 40 lbs. wt. is applied to the pedal of a bicycle at right angles to the crank which is 6 inches long. What is the moment of the force about the axis of the spindle?

2. If a turning moment of 80 lb. ins. is necessary to unscrew a nut, what force must be applied at the end of a spanner 8 inches long?

3. To raise a weight by means of a screw-jack a force of 16 lbs. wt. is required at the end of a bar projecting 9 inches from the centre. What force would be required if the bar were 15 inches long?

4. A force of 9 lbs. wt. is applied to one end of a rod which is 10 inches long. Find the moment of this force about the other end of the rod when the force acts (i) at right angles to the rod, (ii) at an angle of 50° to the rod.

5. A uniform bar is balanced horizontally about a fixed pivot at its centre. If a load of 12 lbs. is suspended from the bar at a distance of 9 inches from the pivot, where must a load of 8 lbs. be suspended to keep the bar in equilibrium?

6. A uniform plank, 14 feet long, supported at its centre on a barrel, is used as a see-saw. A man at 5 feet from the centre balances a boy of 9 stone at the far end. What is the weight of the man?

If the plank weighs 1 cwt., what is the force on the barrel?

7. A man pulls on the handle of an oar with a force of 80 lbs. wt. If one-third of the oar is inboard, calculate the force exerted by the oar on the rowlock.

8. If a wheel can turn freely about a fixed pivot at its centre, and a force of 35 lbs. wt. is applied at right angles to a spoke at a distance of 9 inches from the centre, calculate the least force which must be applied to the circumference to keep the wheel from turning, the radius of the wheel being 16 inches.

9. A stiff uniform pole has one end embedded in a wall from which it sticks out horizontally a distance of 12 feet. Given that the pole would break if a weight of 50 lbs. were hung at the end, how far out along the pole may a man weighing 10 stone venture?

10. A pulley, driven by a belt, has a diameter of 15 inches. If the tension in the tight side of the belt is 120 lbs. wt., and the tension in the slack side 48 lbs. wt., what is the resultant turning moment about the centre of the pulley caused by the belt?

11. To stop a wheel which is rotating about its centre, a block of wood is pressed on the rim with a force of 30 lbs. wt. The diameter of the wheel is 3 feet and the coefficient of friction for the surfaces of the wood and rim is 0·3. Calculate the moment of the frictional force tending to stop the wheel.

12. The pressure of the water on the rudder of a yacht when sailing produces a total force of 160 lbs. wt. at right angles to the rudder at a point 9 inches from the axis about which it turns. Find the force which must be applied to the end of the tiller to balance this if the length of the tiller is $3\frac{1}{2}$ feet.

13. If the pressure of the wind on a front door when closed is equivalent to a force of 40 lbs. wt. at its centre, what is the force on the latch?

14. Given a 10-gram weight, describe how you would use it to find the weight of a metre rule.

15. A uniform girder lies on the floor. To raise one end requires a vertical force of 96 lbs. wt. What is the weight of the girder?

16. State the **Principle of Moments**. Explain, by taking a practical example, why, in applying this principle, we usually take moments about an axis through a point in the line of action of one of the forces.

17. A uniform rod AB, of weight 16 lbs. and length 12 ft., is hinged at A and supported in a horizontal position by a vertical rope at B. Find the tension in the rope when a load of 30 lbs. is suspended from the rod at a point 9 feet from A. Show that the result will be the same if the rod is not horizontal, provided that the rope remains vertical.

18. A uniform light rod, pivoted at its centre, carries on the left-hand side loads of 4 and 7 lbs. at distances of 3 and 4 feet respectively from the centre. Where must a load of 8 lbs. be placed to maintain the bar in equilibrium and what will be the force on the pivot?

19. A uniform light rod is supported on a fixed pin through a hole at its centre, and a weight of 4 lbs. is suspended from the rod at a distance of 9 inches from the centre. The rod is balanced in a horizontal position by applying to it a vertical force of 6 lbs. wt. Shew that this force may be applied at either of two points, and find the reaction of the pin in each case.

20. A uniform rod 2 feet long rests upon a support at one end and is kept in a horizontal position by a vertical string connecting the other end to a spring-balance. The balance reads $\frac{1}{2}$ lb. wt. A load of 6 lbs. is now suspended from the rod at a distance of 9 inches from the support. What is the reading of the spring-balance, and what is the reaction of the support?

21. The flap of a table is hinged along one edge and supported by a leg at a point distant 15 inches from the line of the hinges. The force on the leg due to the flap alone is 4 lbs. wt. Calculate the force on the leg when the following weights are placed on the flap together:

3 lbs. at 20″, 8 lbs. at 9″ and 14 lbs. at 6″ from the line of the hinges.

22. A crowbar ABC is 4 feet long and is placed on a fulcrum at B, 6 inches from C. Neglecting the weight of the bar, find what force applied at A will balance a load of 4 cwt. at C.

23. A man who weighs 13 stone, wishing to raise a rock, leans with his whole weight on one end of a horizontal crowbar 5 feet long, which is supported on a prop at a distance of 5 inches from the end in contact with the rock. What force is exerted on the rock, and what is the force on the prop?

24. What force applied at the end of a lever 30 inches long will raise a load of 80 lbs. attached to the other end, the fulcrum being 4 inches from this load? The weight of the lever is 6 lbs. and acts at its middle point.

25. Design a lever for a common pump such that, by applying a force of 9 lbs. wt. to the end of the handle, a force of 60 lbs. wt. is communicated to the pump rod.

26. A uniform straight lever, 5 feet long and weighing 12 lbs., has its fulcrum at one end and carries loads of 6 and 14 lbs. at distances of 2 and $3\frac{1}{2}$ feet from the fulcrum; it is kept horizontal by a vertical force F at the other end. Find the magnitude of F; find also the reaction of the fulcrum.

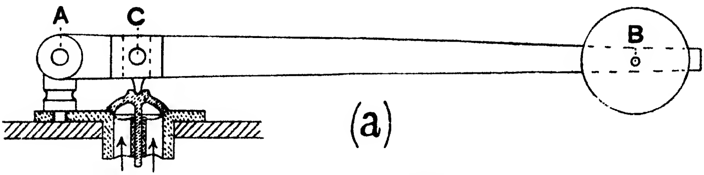
27. A bar 15 inches long carries at its ends loads of 18 and 48 lbs. Neglecting the weight of the bar, find about what point it will balance.

28. Given a weight of 7 lbs., a light stiff rod, and a foot rule, explain how you would proceed to determine roughly the weight of a basket of apples.

29. Describe an experiment to verify the **Principle of Moments** in the case of a rigid body in equilibrium under the action of parallel forces.

30. In Fig. (a) is shewn the application of the lever to the safety valve of a steam boiler. AB is the lever which can turn about the fixed pivot at A and is pressed on the top of the valve at C by means of a weight at B. If

the diameter of the valve is 3 inches, AC 4 inches, AB 30 inches, and the weight 36 lbs., find the force at C necessary to raise the valve and hence the

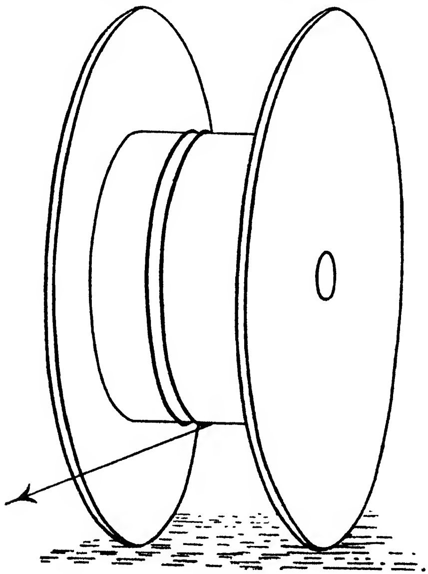


(a)

pressure at which the steam will blow off. Neglect the weights of the valve and lever.

31. The diameter of the safety valve of a steam boiler is 2·5 inches, the centre of the valve being 3 inches from the fulcrum. The valve is to be arranged so that it opens as soon as the steam pressure reaches 100 lbs. wt. per sq. inch. The end of the lever being 24 inches from the fulcrum, find what weight must be suspended from it in order that this may be effected. Neglect the weight of the lever.

32. A reel or spool (Fig. *b*) has a diameter of 9 inches, the drum having

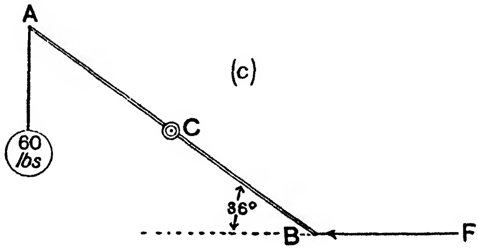


(*b*)

a diameter of 5 inches. A cord is fastened to the drum and wrapped round the middle of it. If the spool rests on the ground and a force is gradually applied to the cord horizontally, in which direction will the spool roll, and why?

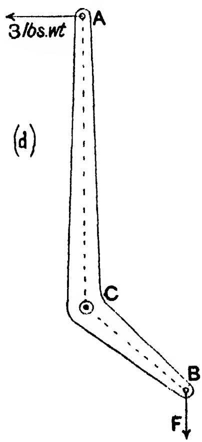
What will happen if the cord is pulled vertically upwards?

33. A uniform bar *AB*, pivoted at its centre *C*, carries a load of 60 lbs. at *A* (Fig. *c*). The bar is kept inclined at an angle of 36° to the horizontal by a horizontal force (*F*) applied to *B*. Find *F*.



34. Supposing that you know your own weight and that you wish to find approximately the weight of a stiff plank, how could you do this?

35. A uniform bar *AB* weighing 45 lbs. is hinged at *A* and supported in a horizontal position by a rope attached to the bar at *B*. If this rope is inclined at 40° to the bar, what will be the tension in it? By how much will this tension be increased if a weight of 25 lbs. is suspended from *B*?



36. A steam-engine has a crank 9 inches long. If the connecting-rod is under a compressive stress of 400 lbs. wt., calculate the turning moment on the shaft when the angle between the crank and the connecting-rod is (i) 135°, (ii) 35°.

37. A bent lever *ACB* (Fig. *d*) has a fixed pivot at *C* and is balanced by the pulls of the wires at *A* and *B*. The pull on *A* is 3 lbs. wt. horizontally. Find the vertical pull on *B*, given that *AC* is vertical and 9 inches long, *CB* is 4 inches, and the angle *ACB* is 130°.

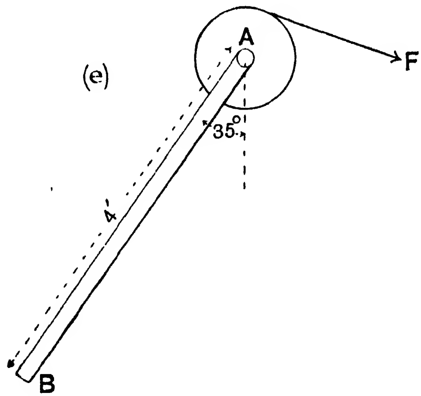
38. The arms of a bent lever *ACB* are perpendicular to one another and the lever is pivoted at

C. Find what horizontal force at B will balance a force of 150 lbs. wt. applied at A at 90° to AC. The arm AC is 5 inches long, the arm BC is 28 inches long and inclined at 40° to the vertical. Neglect the weight of the lever.

39. The weight of the lid of a chest is 26 lbs. and acts at its centre. The lid opens on hinges along one edge, and is held open at an angle of 60° to the vertical by a force applied at the other edge. Find the magnitude of this force when it is applied (i) vertically, (ii) horizontally.

40. A uniform bar AB, 4 feet long and weighing 20 lbs., is rigidly attached to the pulley A, which is pivoted at the centre (Fig. e). A cord wrapped round the pulley is pulled with a force F as shewn. If the diameter of the pulley is 1 foot, calculate the force F required to keep the bar inclined at 35° to the vertical.

(e)



41. In a 'wheel and drum' it is found that a force of 21 lbs. wt. applied to the circumference of the wheel balances a load of 1 cwt. Neglecting friction, find the radius of the drum if the radius of the wheel is 18 inches.

42. In a simple winch (Fig. 83, Art. 65) the distance of the handle from the centre of the axle is 10 inches, and the diameter of the drum is 6 inches. Neglecting friction, calculate what load on the rope can be raised by applying a force of 24 lbs. wt. to the handle.

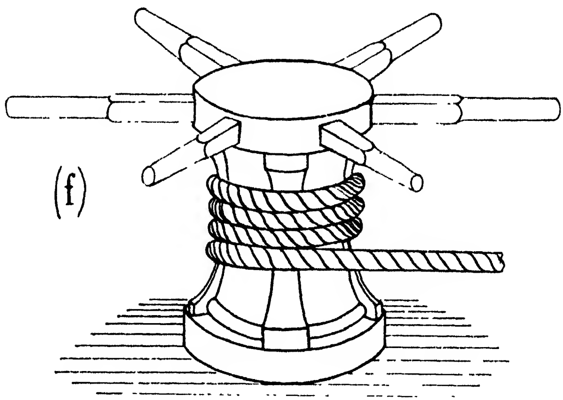
43. Design a simple winch to raise 1 cwt. by applying a force of 40 lbs. wt., assuming that 80 % of this force is being usefully employed.

44. To raise a weight of 210 lbs. on a windlass drum 14 inches in diameter, what moment must be applied?

If this moment is applied by means of a handle 2 feet from the centre, what force must be exerted on the handle?

45. Six men work a capstan (Fig. *f*), using handspikes projecting 5 feet from the centre. The barrel on which the rope is coiled is $2\frac{1}{4}$ feet in diameter. What force must each man exert in order to raise a weight of 1 ton, neglecting friction?

(f)

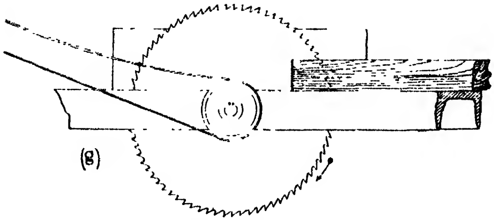


46. The cranks of a bicycle are each 7 inches long, and the chain-wheel on the same spindle has an effective diameter of 9 inches. When the cranks are horizontal a cyclist exerts on one pedal a vertical downward thrust of 60 lbs. wt. and on the other a vertical downward thrust of 4 lbs. wt. Assuming the bearings are frictionless, calculate the tension in the upper side of the chain, if the tension in the lower side is negligible.

47. A capstan is worked by four men; each man exerts a horizontal force of 45 lbs. wt. at a distance of $4\frac{1}{2}$ feet from the centre. The drum is 1 foot and the rope 1 inch in diameter. Neglecting friction, find the pull in the rope which balances the forces on the capstan bars.

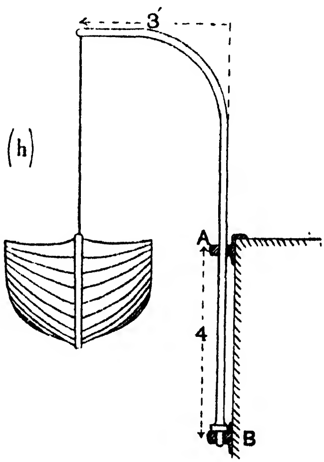
48. To keep a wheel, of 3 feet diameter, rotating at uniform speed about a fixed axle, $1\frac{1}{2}$ inches in diameter, a force of $\frac{1}{2}$ lb. wt. has to be applied to the rim tangentially. What is the force of friction of the axle on the wheel?

49. A circular saw (Fig. *g*) is driven by a belt from the flywheel of an engine. The belt-pulley keyed to the shaft of the circular saw has a diameter of 4 inches. What is the difference in tension in the two sides of the belt if the timber exerts a force of 18 lbs. wt. on the teeth at a distance of 7 inches from the centre?



(g)

50. A davit (Fig. *h*) is supported by a foot step at *B* and a collar *A*, 4 feet apart. The load on the end of the davit is 15 cwt. Calculate the horizontal force on the collar *A*, if the line of action of the weight is 3 feet from *AB*.



(h)

51. A bar, 30 inches long, is suspended at its ends from two spring-balances by vertical strings. Each balance reads 10 ozs. What is the weight of the bar? What will be the readings of the balances when a load of 5 lbs. is suspended from the bar at a distance of 12 inches from one end?

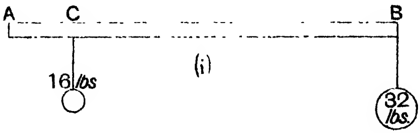
52. Taking the results obtained in Ex. 51, select a point not in the line of action of any of the forces, and by taking moments of all the forces about this point, shew that the principle of moments holds good.

53. A weight of 45 lbs. is hung on a light horizontal rod of length 5 feet, 2 feet from one end; the rod being supported at its ends by two men. What share of the load will each man bear?

54. Two men carry a load of 180 lbs. slung centrally on a 6 foot pole, whose weight may be neglected. The ends of the pole are on the men's shoulders; what share of the load does each man bear? Does a difference in the heights of the men affect their shares? If the load shifts 9 inches towards the shorter man, what share will each man bear?

55. A horizontal uniform beam of 20 feet span carries loads of 10 cwt. and 9 cwt. at distances of 5 feet and 12 feet from the left-hand end. If the weight of the beam is 1 cwt., find the reactions of the supports at its ends.

56. A uniform bar AB is 6 feet long and weighs 8 lbs. (Fig. *i*). From C and B loads of 16 and 32 lbs. are suspended; $AC=1$ foot. Calculate the distance from A of the point about which the rod will balance.



(i)

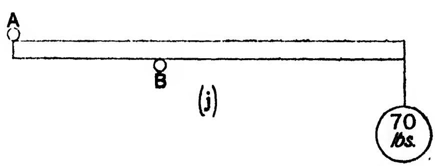
57. A uniform beam 14 feet long and weighing 1 cwt. carries loads of 5 cwt. at 2 feet, 7 cwt. at 5 feet, and 6 cwt. at 10 feet from the left-hand end.

(*a*) If the beam is kept in equilibrium by a single chain, what is the tension in this chain and where must it be fastened to the beam?

(*b*) If the beam is supported by two vertical chains fastened to its ends, what will be the tensions in these chains?

58. The horizontal roadway of a bridge is 30 feet long and its weight of 7 tons may be taken as acting at its middle point; it rests on similar supports at its ends. What are the reactions of the supports when a cart weighing 2 tons is one-third of the way across the bridge?

59. A uniform bar weighing 20 lbs. carries a load of 70 lbs. at one end. The bar is kept in a horizontal position by two fixed pegs A and B, as shewn in Fig. (*j*). The bar is 8 feet long, and the horizontal distance between the pegs is 3 feet. Calculate the reactions of the pegs.



(j)

60. Weights of 7, 5, 6, and 4 lbs. are hung on a light horizontal bar at distances of 2, 3, 5, and 8 feet from the left-hand end. The bar is supported by a single rope. If the weight of the bar is negligible, what is the tension in the rope, and at what distance from the left-hand end must it be fastened?

61. A bicycle weighs 50 lbs. and its rider 10 stone. Calculate the reactions of the ground on the two tyres if their points of contact with the ground are $3\frac{1}{2}$ feet apart, while the points through which the weights of the bicycle and the rider act are distant horizontally 10 and 6 inches respectively from the centre of the hind wheel.

62. A uniform beam is 20 feet long and weighs 9 lbs. per foot length. It is supported in a horizontal position by two props distant 5 and 6 feet from the two ends respectively. Find the force on each prop.

63. A uniform beam weighing $1\frac{1}{2}$ tons, and 22 feet long, is supported at 2 feet from its left end, and at 6 feet from its right end. It is loaded with 8 tons at 9 feet from its left end. Find the forces exerted by the supports.

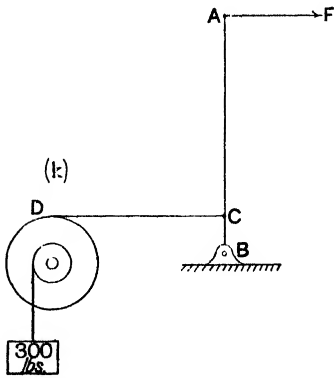
64. A uniform steel shaft, AB, is 12 feet long, and weighs 90 lbs. It is carried by two bearings, one at the end of A, and the other 3 feet from the end of B. The shaft carries a pulley weighing 36 lbs. at its middle point, and a second pulley, weighing 30 lbs., at B. Find the weight supported by each bearing.

65. A uniform bar, 6 feet long and weighing 10 lbs., has a movable weight of 6 lbs. placed 1 foot from the centre of the bar. The bar is supported by vertical cords at the ends. Find the tension in each of the cords. If the greatest tension that either of the cords can stand is 10 lbs. wt., find the greatest distance from the centre of the bar at which the 6-lb. weight can be placed.

66. A uniform beam, 12 feet long and weighing 100 lbs., rests horizontally upon a support at each end. From what point of the beam must a 200-lb. weight be suspended in order that the reaction at one support may be double that of the other?

What will be the value of these reactions?

67. A uniform log of teak, 20 feet by $1\frac{1}{2}$ feet by 1 foot, weighing 48 lbs. per cubic foot is stowed on two horizontal supports, A and B, 11 feet apart. If 5 feet of its length projects over the end A, find the reaction of each support.



68. (Fig. *k*.) AB is a lever pivoted at B. A wire rope from the wheel D is fastened at C. AC is $2\frac{1}{2}$ feet and CB 6 inches. The wheel D has a radius of 7 inches, and the

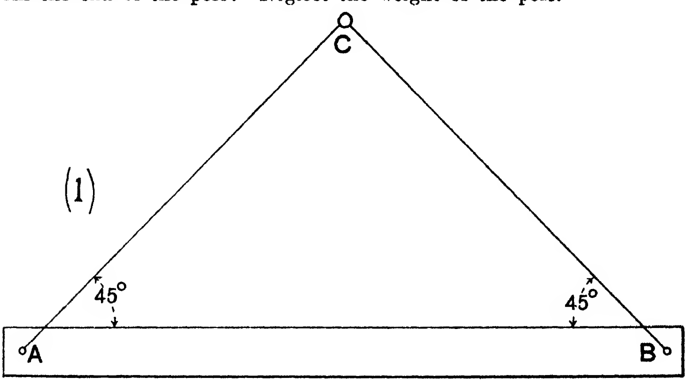
drum has a radius of 3 inches. A weight of 300 lbs. hangs from a rope coiled on the drum. Calculate what force must be applied horizontally at A to keep the system in equilibrium. Neglect friction.

69. A sack of coal weighing 1 cwt. is suspended from a fixed support by a rope 20 feet long. It is now pushed a *horizontal* distance of 5 feet. Calculate what horizontal force is required to keep it in this position. Calculate the horizontal force required under the same conditions if the length of the rope is 40 feet.

70. The foot of a post which is 18 feet high is freely hinged to a floor. Fastened to the top is a rope inclined at 32° to the vertical. Find what horizontal force applied to the post 11 feet from the floor will keep it upright when the rope is pulled with a force of 98 lbs. wt.

71. A pendulum consists of a heavy weight of 10 lbs. at the end of a light stiff rod which is 10 feet long. A string is attached to the rod at a point 3 feet from the 10-lb. weight and is pulled in a direction always at right angles to the rod. Find the tension in the string when the 10-lb. weight is 6 inches from the vertical line through the point of support.

72. A pole 24 feet long is used as a derrick; its top is supported by a rope 12 feet long fastened to a point 24 feet vertically above the foot of the pole. If the rope can bear a pull of 25 cwt., what load can be suspended from the end of the pole? Neglect the weight of the pole.



73. A uniform bar AB (Fig. *l*), weighing 2 cwt., is supported at its ends by equal ropes AC, CB, fastened to the same point C. Each of these ropes makes an angle of 45° with the bar. Find, by moments, the tensions in the ropes.

74. In helping a cart up a hill, is there any reason why a man should exert a force on the spokes of the wheel rather than on the body of the cart?

75. Describe an experiment to verify the **Principle of Moments** in the case of a rigid body in equilibrium under the action of forces whose directions are not parallel.

76. Find the least horizontal force which must be applied to the centre of a wheel of 3 feet diameter, to drag it over an obstacle 2 inches high. The weight of the wheel is 60 lbs. and acts at its centre.

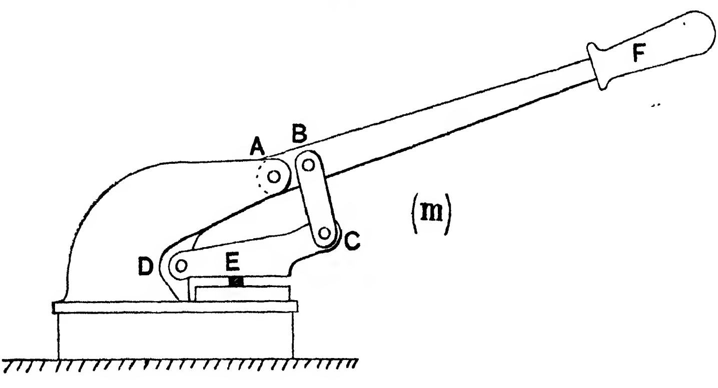
77. A platform projects 6 feet horizontally from a vertical wall to which it is hinged, and the outer edge is supported by two chains secured to the wall, the inclination of the chains being 45° to the platform. The weight of the platform is 2 cwt. and acts at its centre. If two weights of 4 and $2\frac{1}{2}$ cwt. are placed on the platform at 3 and 5 feet respectively from the wall, find the tensions in the chains, assuming these to be equal.

78. Two vehicles have the same weight but one has larger wheels than the other. Which vehicle is the easier to pull over a rough road, and why?

79. What is meant by the resultant of parallel forces? Two parallel forces of 7 and 5 lbs. wt., having the same sense, act upon a rigid body, the perpendicular distance between their lines of action being 18 inches. Calculate the magnitude of their resultant and the perpendicular distance of its line of action from that of the force of 7 lbs. wt.

80. Take an example of four like parallel forces acting on a rigid body and explain how to find the magnitude and direction of their resultant.

81. In the shears shewn in Fig. (*m*) we have an example of a com-

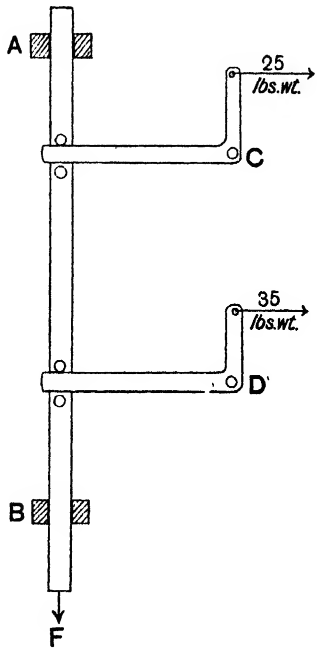


(m)

pound lever. One lever AF, 20 inches long, has a fulcrum at A and is connected at B, 2 inches distant from A, to a short link BC; the other end of this link is jointed to a second lever CD which is 8 inches long and has a fulcrum at D; it forms one edge of the cutting shears, the other edge being fixed to the framework. What force can be exerted on a piece of iron at E, 3 inches from D, by applying a force of 8 lbs. wt. at F?

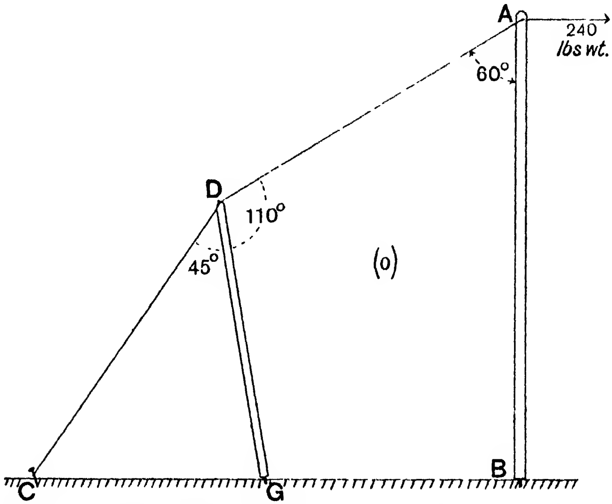
82. (Fig. *n*.) Two equal right-angled levers pivoted at C and D are operated by a vertical rod AB weighing 8 lbs. Neglecting friction and the weights of the levers, calculate the force F required to pull down the rod in

(n)



the position shewn, when the shorter arms of the levers are being pulled horizontally by forces of 35 and 25 lbs. wt. The effective length of the horizontal arm of each lever is 9 inches, and of the vertical arm 4 inches.

83. The vertical post AB is stayed as shewn in Fig. (o). Find the tensions in the parts of the stay wire AD and DC when a horizontal force of 240 lbs. wt. is applied at A.



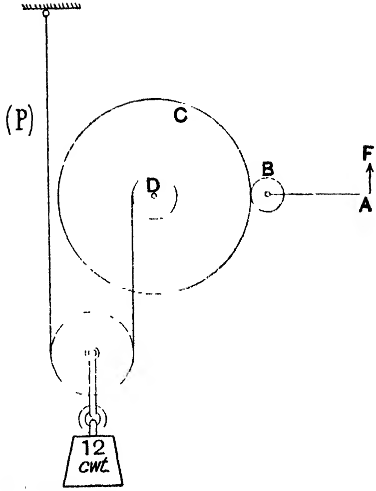
Apply the principle of moments to the equilibrium of AB, and then of DG, assuming that both posts can turn freely about their points of contact with the ground. Neglect the weight of the post DG.

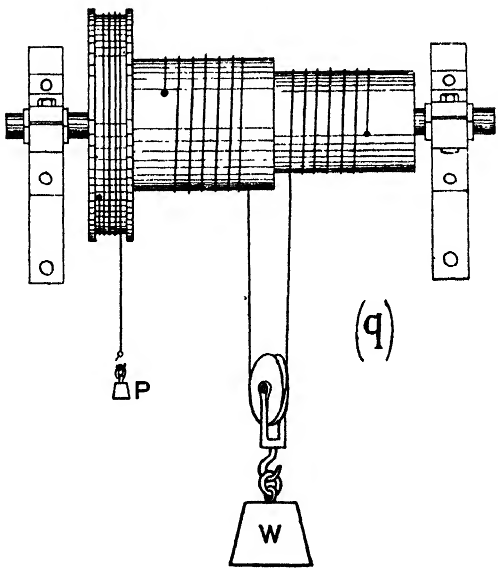
84. Design a crab winch (see Art. 68) to raise half a ton of coal by applying a force of 40 lbs. wt., assuming that 70 °/₀ of this force is usefully employed.

85. (Fig. p.) The weight of 12 cwt. is raised by applying a force F at right angles to the crank AB of a winch. The toothed wheel B, which is secured to the crank spindle, has a pitch circle of radius $2\frac{1}{2}$ inches and gears with the pinion C, the pitch circle of which has a radius of 15 inches. The drum D, on which the rope is wound, is secured to the spindle of C, and has a radius of $3\frac{1}{2}$ inches. Length of crank AB is 16 inches.

Use the principle of moments to calculate F; neglect friction and also the weight of the movable pulley. Check your answer by applying the principle of work.

86. Fig. (q) represents a model of a machine known as a 'Wheel and Differential Axle' or 'Chinese Windlass.' This differs from the simple


(p)
C
F
B
D
A
12
cwt.


(q)
P
W

'wheel and drum' in that the drum consists of two parts of different diameters. One end of a cord is fixed to the smaller drum, coiled round it several times and, after passing round the movable pulley, is coiled round the larger axle in the opposite direction.

If the diameter of the wheel is 2 feet and the diameters of the drums 6 and 4 inches, calculate, by moments, what load W can be balanced by an applied force P equal to 8 lbs. wt. Neglect friction and the weight of the movable pulley. Check your result by applying the principle of work.

87. Neglecting frictional resistance, calculate the force which must be applied to the centre of a roller to pull it up a slope of 1 in 7, the weight of the roller being 2 cwt. The force is applied parallel to the slope.

88. Referring to Fig. (j) of Ex. 59, calculate the horizontal force which must be applied to the bar to start it moving, if the coefficient of friction for the contact surfaces at the pegs is 0·3.

89. A body is acted upon by a *couple*, consisting of two equal and opposite parallel forces, each of P lbs. wt., the perpendicular distance between the lines of action of the forces being a inches. Prove that the turning moment of this couple is the same about every axis at right angles to the plane of the forces and is given by $P \times a$ lb. ins.

CHAPTER VI

CENTRE OF GRAVITY

73. Centre of Gravity.

Exp. 1. Take an ordinary metre rule and balance it across your finger. Observe that this can be done only when the metre rule is in one particular position, with its middle point above the point of contact with your finger. If it be moved, however little, to one side or the other, the balance is destroyed, and it falls over.

Repeat this experiment with, for example, a billiard-cue. The cue will balance when one particular point of it, in this instance not the middle point, is above the point of support. We call this point the centre of balance.

Exp. 2. Take two light tin, or thin glass tubes, each about half a metre long, and fitted with corks at each end; the tubes may be cylindrical or conical, or indeed any shape, but they must be exactly alike in shape and weight.

Weigh one of the tubes empty; then fill it with water, and by subtraction find the weight of the water in it.

Put in a small bag enough lead shot to make the bag of shot weigh as much as the water in the first tube. Tie the bag to the middle of a piece of string long enough that you may be able to draw the string through the second tube, and with the bag at any point inside the tube have the ends of the string coming out at each end of the tube.

Find the centre of balance of the tube full of water. A convenient way is to lay the tube of water flat on the table with its length at right angles to the edge of the table; then push the tube along so that a gradually increasing part of its length pro-

jects over the edge of the table. The point of balance of the tube will be just above the edge of the table when the tube begins to overbalance.

(We have tacitly assumed that the tube is symmetrical about a straight central axis, and if so the true centre of balance lies within the tube. With an L-shaped tube the centre of balance lies outside the tube, and experimental difficulties are rather greater, but the result is the same.)

Now by marking a point on the string attached to the bag of shot, draw the bag of shot into the second tube until it occupies in it a point similarly placed with the centre of balance of the first tube.

We find by experiment that the centre of balance of the tube with the shot is similarly placed with the centre of balance of the tube full of water; it is at the centre of the bag of shot. We may support first one and then the other tube as shewn in Figs. 90*a* or 90*b*, or indeed in any manner at all, and we find that the forces required to balance the distributed weight of the one are exactly the same as the forces required to balance the concentrated weight of the other. That is to say, the effect of the weight of the distributed water is the same as though the weight of the water were a single force acting vertically downward through the centre of balance.

Certainly the weight of the shot is not concentrated at a point, but it is much more nearly concentrated at a point than is the weight of the water, and it seems fair to presume that however great a concentration of weight we could have in the second tube the effect of the weight of this tube would still be the same as that of the tube of water.

So we are led to the conclusion that the resultant of the weight of a body is equivalent to a single force of the same magnitude acting through the centre of balance of the body.

We say, shortly, that the weight of a body acts through the centre of gravity, which is the centre of balance. In future we shall commonly refer to the point as the *Centre of Gravity*.

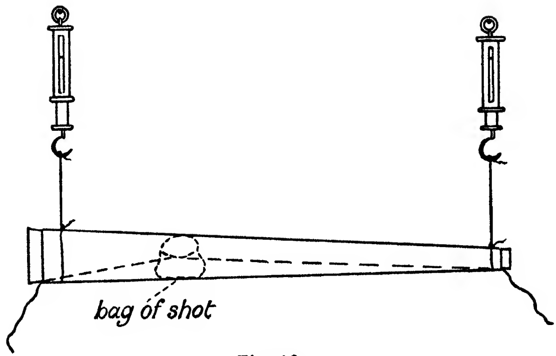


Fig. 90 *a*.

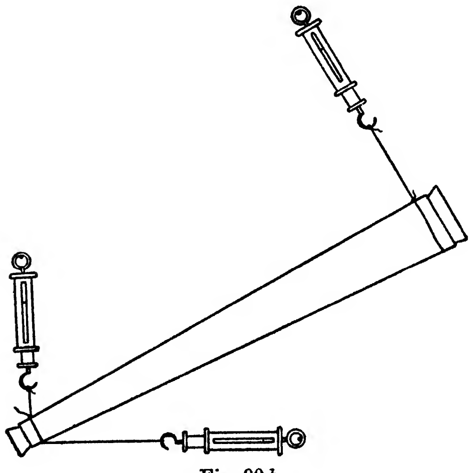


Fig. 90 *b*.

74. To determine experimentally the C.G. of a thin flat plate. Although it is difficult to find the Centre of Gravity of a solid whose shape is not regular, it is a comparatively simple matter to find that of a flat sheet of material, however

irregular its outline may be. The following experiment is an example of the method which is employed.

Exp. 3. Cut out of thin cardboard a figure of any irregular shape. Bore a hole through it near its edge. Through this pass a small wire hook *H*. Hang the cardboard up from a fixed support by a string attached to *H* (Fig. 91 *a*).

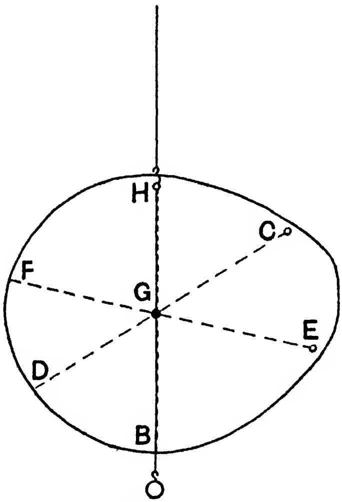


Fig. 91 *a*.

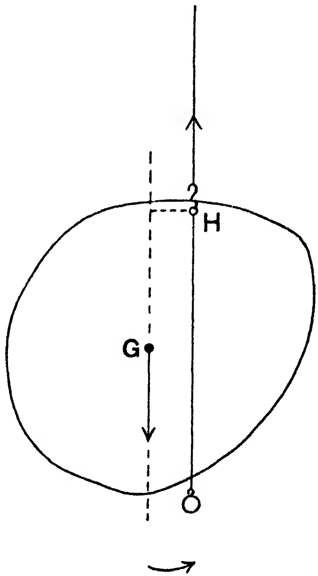


Fig. 91 *b*.

The forces which act on the cardboard disc are two in number, its weight, and the pull of the string, and for equilibrium these must act in opposite directions along the same straight line. Hence the Centre of Gravity, the point through which the weight acts, must be vertically below the point of support. Otherwise the weight would produce an unbalanced moment about *H* which would turn the disc (Fig. 91 *b*).

Now hang a light plumb-line from *H*, and by means of it

draw on the cardboard the vertical line through this point; the C. G. will be somewhere on this line.

Suspend the disc from some other point such as C, and draw on it CD, the vertical line through C, when it is in this position; the C. G. will be somewhere on this line. And since it also lies on AB, it must be at the point G, where these two lines intersect.

If G is the Centre of Gravity, then, since the line of action of the weight must always pass through it, a vertical line through any other point by which the disc may be suspended, such as E, should pass through G. Shew that this is so.

Further, if it is supported horizontally at G on a sharp point it will be found to balance. Note that G being on the surface is not actually the centre of gravity. This is really a point immediately over G and half-way between the two surfaces of the disc.

75. Uniform ring.

Exp. 4. Cut out a circular disc of cardboard and, as in the last experiment, find its C. G. This we find to be its centre as we should expect from the symmetry of the figure.

With the same centre describe on the disc a smaller circle, and cut it out carefully, so as to leave a flat ring (Fig. 92). Hang this up as before from the same points as were used in determining the C. G. of the circle, and from each suspend the plumb-line. There will be no need to trace fresh lines on the cardboard, for the plumb-line will lie along the parts of the original lines which have not been removed. Now these lines intersected at the centre of the original circle, which is also the centre of the ring. Hence the C. G. of the ring is its centre.

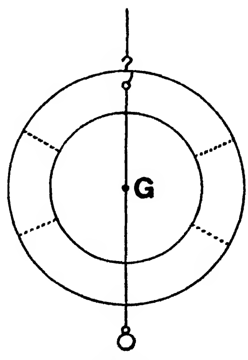


Fig. 92.

Observe that the C.G. may be a point which does not lie in the body. But however the body may be placed the line of action of the weight passes through this point.

76. Centre of Gravity: deduced from the Principle of Moments. The Centre of Gravity of a body was defined above as that point through which the resultant weight of a body acts. We are now going to examine this statement more closely.

Suppose two small particles A and B, whose respective weights are P lbs. and Q lbs., to be placed so that the line AB is horizontal (Fig. 92a). In the preceding Chapter (Art. 71) we

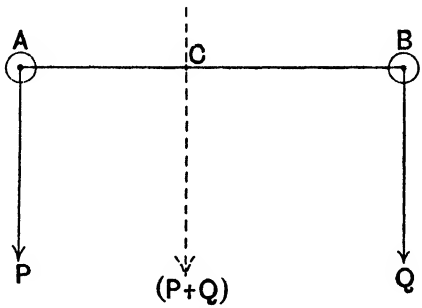


Fig. 92 *a*.

shewed that the resultant of these two weights was a force of $(P+Q)$ lbs. wt. acting vertically downwards through a point C in the line AB, such that the moments of the two weights taken about this point were equal and opposite, that is

$$Q \times BC = P \times CA,$$

whence $$\frac{BC}{CA} = \frac{P}{Q}.$$

Now suppose that, while their distance apart remains the same, A and B take up some other position, such as that shewn in Fig. 92*b*. As before, the resultant of the two weights is a force of $(P+Q)$ lbs. wt. acting vertically downwards. Let its line of action cut AB in the point D. And let EDF be a line drawn through D perpendicular to the lines of action of the weights. We know the position of D must be such that

$$Q \times FD = P \times DE,$$

that is $$\frac{FD}{DE} = \frac{P}{Q}.$$

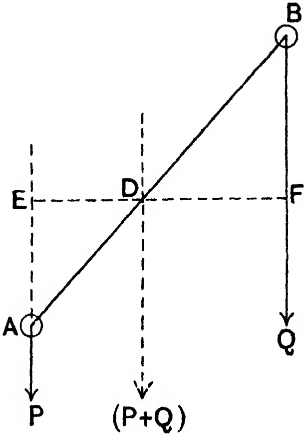


Fig. 92*b*.

Now the triangles AED, BDF are equiangular and therefore similar.

Hence $$\frac{FD}{DE} = \frac{BD}{DA},$$

or $$\frac{BD}{DA} = \frac{P}{Q}.$$

It follows that D is the same point as C for it cuts the line AB in the same ratio. Therefore whatever position the particles A and B may occupy, provided that their distance apart does not change, the resultant of their weights passes through a fixed point in the line AB.

The above argument can be extended to any number of particles, whose distances from one another do not alter, and it can be shewn that their resultant weight will always pass through a point whose position is fixed relatively to that of the particles.

Now any body may be considered to be made up of a large number of small particles. The weights of these particles form a system of parallel forces. Their resultant, the weight of the whole body, passes through a point whose position is fixed relatively to the body, *so long as the shape of the latter does not alter.*

In other words, the position of the C.G. of a rigid body relative to the body is independent of the position of the body.

77. Uniform straight rod. Let AB be a thin rod of uniform cross-section and density. Imagine it to be divided up into a very large number of equal parts, each of weight w. Take any pair of these whose distances from the middle point of the rod are the same (Fig. 93). Their resultant is a force of $2w$

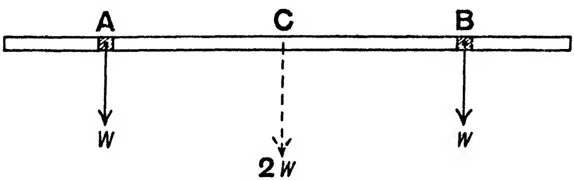


Fig. 93.

acting through C, the middle point of the rod. Similarly the resultants of all similar pairs are each equal to $2w$, and act through C. But the sum of the weights of all the particles is the weight of the whole rod. Hence C, the point through which this resultant acts, is the centre of gravity.

78. Rectangle and parallelogram. Strictly speaking, a plane figure cannot have a centre of gravity. When we talk of the C.G. of a rectangle we are thinking of the C.G. of a sheet of very thin uniform material, whose shape is rectangular.

When dealing with plane figures, this point is sometimes called the *centroid*, or *centre of area.*

Suppose we cut out a rectangle and a parallelogram from a

thin sheet of uniform material and imagine the whole areas of these figures divided up into a large number of strips parallel to one pair of sides (Fig. 94). The line joining the middle points of

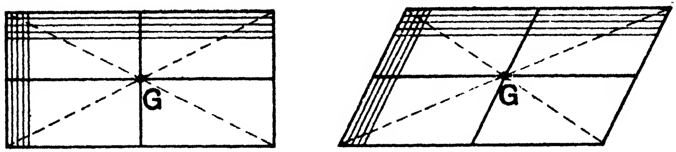


Fig. 94.

these sides bisects all these strips. Now we may consider each strip as a uniform rod whose C.G. is at its centre and therefore lies on this line. Consequently the C.G. of the whole figure lies on this line. Similarly if we imagine the figure divided up into thin strips parallel to the other pair of sides the C.G. must likewise lie on the line bisecting these sides. Hence the point of intersection of these lines is the C.G. Is this also the point of intersection of the diagonals of the rectangle or parallelogram?

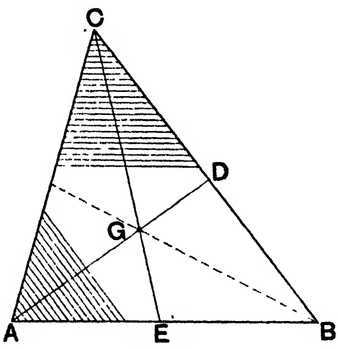


Fig. 95.

79. Triangle. By similar reasoning we can arrive at the position of the C.G. (or centroid) of a plane triangular surface. Suppose the whole area of the triangle *ABC* (Fig. 95)

to be divided up into very narrow strips by lines drawn parallel to the side AB. Take any one of these strips AB. This may be considered to be the equivalent of a thin rod, and therefore its C.G. is at E the middle point of AB. Similarly the C.G. of each of the other strips is its middle point. Now for geometrical reasons the median CE bisects all lines drawn parallel to AB, and therefore passes through the C.G.'s of all the strips. Therefore the C.G. of the whole triangle will lie somewhere on this line.

Again, by dividing the triangle into strips parallel to another of its sides, such as BC, we find that the C.G. of the triangle must lie also somewhere on the median AD. Therefore the C.G. is the point G, where these two medians intersect. The position of G in AD may be found by geometry; or experimentally as follows:

Exp. 5. Draw a triangle on a piece of cardboard. Bisect the sides of the triangle and join the middle points to the opposite angles. Cut out the triangle and find if it will balance on a pin placed at the point where these medians intersect. Find also in what ratio this point divides each of the medians.

80. Quadrilateral. The above construction for finding the C.G. of a triangle can be extended to locate that of a quadrilateral. Thus, to find the C.G. of the uniform sheet or lamina in

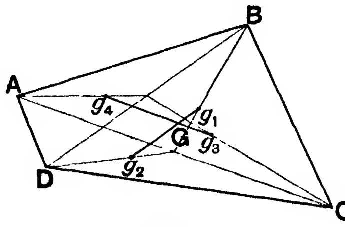


Fig. 96.

Fig. 96, divide it into two triangles by the line AC. Determine the C.G. of each triangle by finding the point of intersection of two of its medians. These points are denoted by g_1 and g_2.

The C.G. of the whole lamina must lie on the line joining g_1 and g_2. Now divide the figure into a second pair of triangles by the line BD and find their C.G.'s. These points are denoted by g_3 and g_4. The C.G. of the whole lamina must also lie on the line joining these points. Hence the point G, where these lines intersect, must be the C.G. of the whole figure.

Exp. 6. Draw an irregular quadrilateral on a sheet of cardboard and determine the position of its C.G. by the construction given above. Carefully cut out the figure and find by experiment if this point is correct.

81. Body composed of two simple parts. The process of dividing a body into simpler parts, as illustrated in the preceding article, will also enable us to calculate the position of the C.G. of a body when the weights and C.G.'s of these parts are known.

For example, suppose we have a solid metal cylinder, a certain length of which has been turned down to a smaller diameter

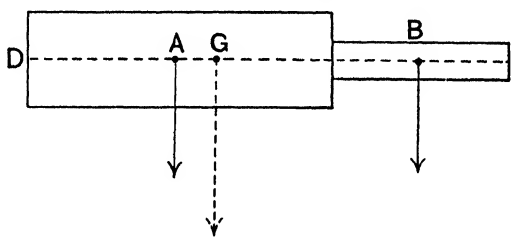


Fig. 97.

(Fig. 97). The body so formed consists of two cylinders, the C.G.'s of which are at A and B, the centres of their respective axes.

Example. If the diameters of the two cylinders are 3″ and 1″ respectively, and their corresponding lengths 10″ and 6″, how far from the end D is the C.G. of the whole body?

If w lbs. denotes the weight of a cubic inch of the material,

then the weights of the cylinders are $\pi\left(\frac{3}{2}\right)^2 10 \,.\, w$ and $\pi\left(\frac{1}{2}\right)^2 6 \,.\, w$ pounds.

We know $DA = 5''$ and $DB = 13''$. Let $DG = x$.

Taking moments about D, we have:

The moment about D of the weight of the whole body =

The sum of the moments about D of the weights of the two parts of the body.

That is,

$$\left[\pi\left(\tfrac{3}{2}\right)^2 10 + \pi\left(\tfrac{1}{2}\right)^2 6\right] w \,.\, x = \pi\left(\tfrac{3}{2}\right)^2 10 \,.\, w \,.\, 5 + \pi\left(\tfrac{1}{2}\right)^2 6 \,.\, w \,.\, 13 \text{ lb. ins.}$$

which gives $$x = \mathbf{5\tfrac{1}{2}}''.$$

This result is independent of w; that is, the position of the C.G. does not depend on the material the body is made of, providing that this is uniform. Moreover, to find the C.G. of such a body, it should be observed that, in addition to knowing the positions of the C.G.'s of its parts, it is only necessary to know the relative volumes of these parts.

82. Body composed of several parts whose C.G.'s lie in one straight line. If a body is composed of a number of parts whose weights and C.G.'s are known, we can determine the C.G. of the whole body. We will first illustrate this by extending the method used in the preceding article to the case of a body composed of a number of parts whose C.G.'s all lie in one straight line.

$$\begin{bmatrix} AB = 6'' \\ AC = 11'' \\ AD = 15'' \end{bmatrix}$$

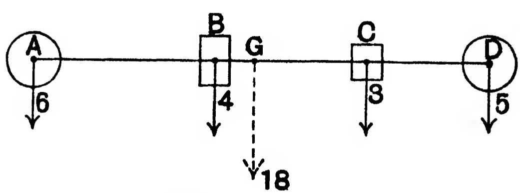


Fig. 98.

For example, suppose we have four lumps of metal rigidly connected by a light rod AD whose weight may be neglected (Fig. 98). If the weights of the separate parts are 6, 4, 3, and

5 lbs., and their corresponding C.G.'s are A, B, C, and D, then it follows that the weight of the whole body is 18 lbs., and its C.G. lies in the line AD. To fix this point we have therefore only to find the line of action of the resultant of the four weights. Observe that, since the C.G. is a point fixed relatively to the body, we are at liberty to consider the body in any position which may be convenient for our purpose.

Thus, for simplicity, we will choose to consider AB horizontal.

Let the C.G. of the whole body be denoted by G.

Now we know that the sum of the moments of the weights of the four parts is equal to the moment of their resultant about any point and therefore, by taking moments about A, we obtain the equation

$$18 \times AG = 6 \times 0 + 4 \times AB + 3 \times AC + 5 \times AD.$$

Or, supposing that $AB = 6''$, $AC = 11''$ and $AD = 15''$, then

$$18 \times AG = 4 \times 6 + 3 \times 11 + 5 \times 15,$$

hence $$AG = \frac{24 + 33 + 75}{18} = \frac{132}{18} = \mathbf{7\tfrac{1}{3}}''.$$

That is, the C.G. of the whole body is a point distant $7\frac{1}{3}$ inches from A.

Example. To illustrate the application of the foregoing method, let us determine the C.G. of the uniform plate whose dimensions are given in Fig. 99. Notice that we can divide the figure into three rectangles as shewn by the dotted lines, and that the C.G.'s of these rectangles all lie on the centre line, which must therefore contain the C.G. of the whole plate. This centre line is represented by CD in the second figure.

Let w lbs. be the weight of 1 sq. inch of the plate; then the weights in lbs. of the separate rectangles are $24w$, $12w$, and $32w$, and they act at the points whose distances from AB are shewn in the figure. Now the sum of the moments of these weights about this line is equal to the moment of their resultant. Perhaps you will realise this better if you imagine yourself holding the plate in a horizontal plane by grasping the edge AB.

This resultant, being the weight of the whole plate, is given by $24w + 12w + 32w = 68w$, and acts at a certain distance (which we will call y inches) from C. If we consider CD to be horizontal and take moments about the axis AB, we get

$$68w \,.\, y = 24w \,.\, 12 + 12w \,.\, 7 + 32w \,.\, 2 \text{ lb. ins.,}$$

hence $$y = \frac{436w}{68w} = \mathbf{6{\cdot}41''}.$$

This result is independent of w, and therefore does not depend on the material of which the plate is composed providing it is uniform. This we should expect since it is clear that the weight of the plate or any part of it is proportional to its area. When working out examples of this type, we can therefore take the areas to represent the weights.

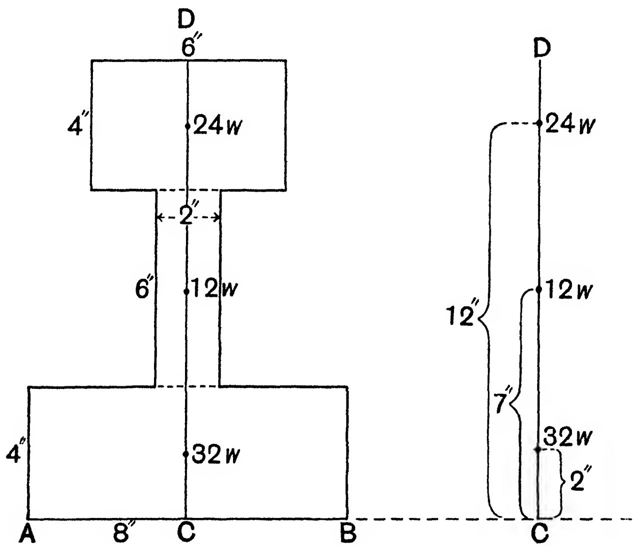


Fig. 99.

Exp. 7. Draw on cardboard a figure similar to that shewn in Fig. 99, and calculate the position of its C.G. Mark in this

point, and then carefully cut out the figure. Now test the position of the point you have calculated by observing if it lies in each case behind a plumb-line hung from a pin, this pin supporting the cardboard first at one, and then at another well chosen point. The points should be taken so that the lines drawn from them should intersect at as wide an angle as possible.

83. Body composed of parts whose C. G.'s lie in one plane. Let us take a thin uniform rectangular board and load it with flat circular weights of 1 lb. and 3 lbs., as shewn in Fig. 100*a*. The board measures $20'' \times 16''$, and its surface is

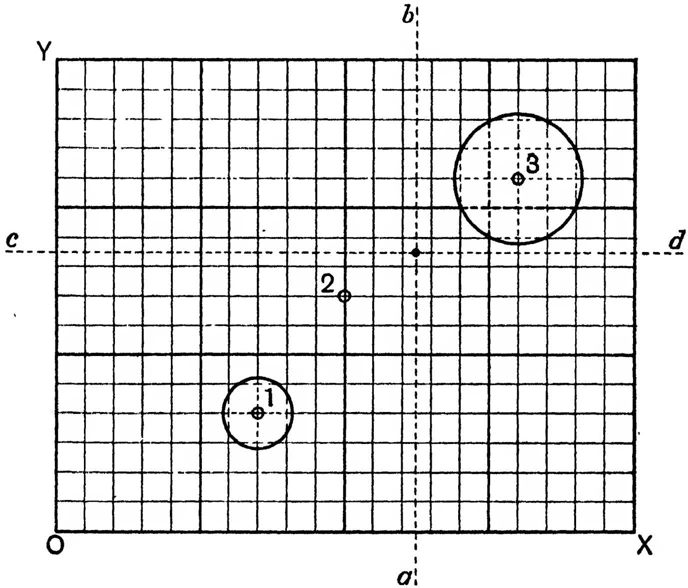


Fig. 100 *a*.

divided up into square inches by transverse lines, so that we can read off the distances of the centres of these weights from the edges of the board. The weight of the board itself is 2 lbs. and

is equivalent to an equal load at the centre. The whole body therefore consists of three parts whose C. G.'s lie nearly in one plane. The weights of these parts form a system of parallel forces whose resultant is the weight of the whole body, namely 6 lbs., acting at its C. G., the distance of which from the edge OY we will call x inches.

We may suppose the board (Fig. 100b) held horizontally grasped in the hands by the edge OY. The moment exerted about OY by the hands measures equally the sum of the moments

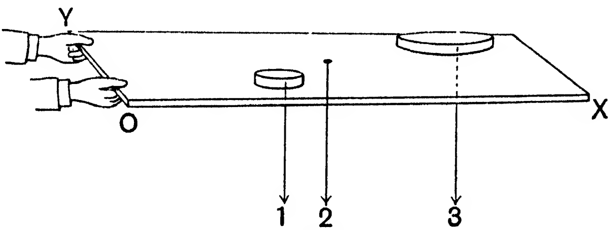


Fig. 100 b.

of the separate weights, *or* the moment of their resultant about this axis. The arms of the moments of the component forces can be read off on the surface of the board.

Equating these moments, we get

$$6x = 1 \times 7 + 2 \times 10 + 3 \times 16 \text{ lb. ins.},$$

or $$x = \mathbf{12\tfrac{1}{2}}''.$$

From this we know that the C. G. of the whole body lies somewhere on the line ab. This we can verify by observing that the board will balance on the edge of a rule placed under it in the direction of this line.

Similarly let us imagine the board held horizontally by the edge OX.

If we call the distance of the C. G. of the whole body from this edge y inches and take moments about this axis, we get

$$6y = 1 \times 4 + 2 \times 8 + 3 \times 12 \text{ lb. ins.}$$

or $$y = \mathbf{9\tfrac{1}{3}}''.$$

Hence we know that the C. G. lies on the line *cd*. We can verify this as before by shewing that the board will balance when supported along this line. It follows that the point of intersection of these lines *ab* and *cd* gives us the position of the C. G. of the whole body. We can further demonstrate this by shewing that the board will balance on the end of a finger placed below this point.

As an example of the practical application of the foregoing method of locating the C. G. of a composite body by calculating its distances from two convenient axes at right angles, we will shew how we proceed to determine the C. G. (or centroid) of the sail plan of a yacht (Fig. 101).

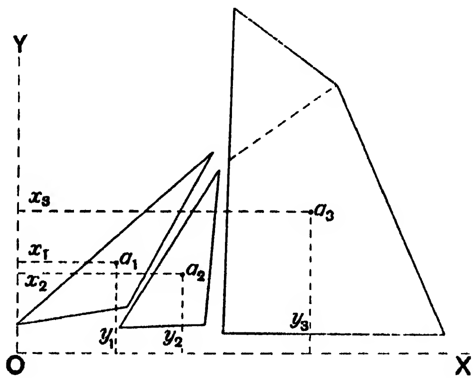


Fig. 101.

As already explained, we can consider the figures as thin uniform plates whose areas may be taken to represent their respective weights. The mainsail and topsail can be taken as one quadrilateral or as two triangles.

The areas of the various sails may be found geometrically, or, if the plan of the sails has been drawn on squared paper, by counting squares.

The centroid of each area may be found, after cutting out

equal areas in cardboard, by the experimental method of Art. 75, or by the method of Arts. 79 and 80.

Next, taking moments successively about OX and OY, the coordinates of the centroid of the whole area are found by the method used in the beginning of this article.

The importance of the position of the C. G. in the example we have taken is due to the fact that the resultant force of the wind on the sails of the yacht under certain conditions acts approximately through this point. Hence her ability to 'stand up' under the pressure of the wind on her canvas depends upon the height of this point above the axis about which she heels; and the horizontal distance of this point from the vertical axis about which she tends to turn governs the amount of 'helm' she carries.

84. C. G. of a Remainder. We sometimes require to ascertain the position of the C. G. of the portion of a body which remains after a part has been removed. This we can do if we know the weights of the whole body and the part removed, and also the original positions of their C. G.'s.

Let us illustrate this by the following example.

A laden vessel whose weight (including cargo) is 1200 tons has its C. G. at A, 11 feet above the keel K (Fig. 102). The cargo is 200 tons and its C. G. is at C, 5 feet above the keel. Where is the C. G. of the vessel after discharging the cargo?

Denoting this point by S, we see that we have acting here a weight of 1000 tons. If we now imagine the cargo to be replaced, it becomes clear that the resultant of the weight of 1000 tons at S and the weight of 200 tons at C is a weight of 1200 tons at A.

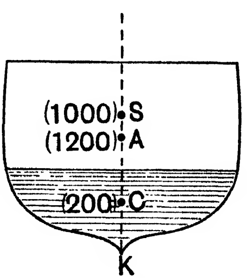


Fig. 102.

If we attempt to take moments about the keel, we are in

difficulties, for neither the weight of the ship nor the cargo has any moment about the keel. We know however that the C.G. of a body is independent of the position of the body, and we may therefore suppose the ship heeled over through an angle of 90°, so that the line $KCAS$, which was formerly vertical, becomes horizontal. We may now take moments about the keel, and we have

$$1000 \cdot KS + 200 \cdot KC = 1200 \cdot KA \text{ ton ft.,}$$

or

$$KS = \frac{1200 \cdot KA - 200 \cdot KC}{1000} = \frac{1200 \times 11 - 200 \times 5}{1000} = \mathbf{12 \cdot 2 \text{ feet.}}$$

Thus, by discharging the cargo the C.G. of the vessel has been raised 1·2 feet.

Exp. 8. Take a rectangular sheet of cardboard measuring about 15″ × 10″, and draw on it two circles some distance apart. Convenient radii for these circles are 2″ and $2\frac{1}{2}$″.

Taking two edges of the sheet as rectangular axes, calculate with reference to these axes the position of the C.G. of that part of the cardboard which will remain after the circular portions have been removed.

Now cut out the circles carefully with a sharp knife.

Find by experiment the C.G. of the remainder and see if it coincides with the point you have found by calculation.

85. Effect of displacing part of a body. It is sometimes necessary to find the change in the position of the C.G. of a body consequent upon a change in the position of the C.G. of a part of the body.

Suppose for example we have a body which weighs 12 lbs. and consists of two parts, a board $OYCX$ weighing 10 lbs. and a movable weight of 2 lbs. whose C.G. is at g; and that the C.G. of the whole body (the board and its movable load) is at G.

Find the change in the position of G consequent upon moving the 2 lb. weight from g to g', a distance of 10 inches.

First suppose that gg' is parallel to OX.

Both the C.G. of the board and of the movable weight have remained the same distances from the line OX, therefore, since no change has been made in the distance from OX of either part of the body, it follows that the C.G. of the whole body must also have remained the same distance from OX. So, if G' is the new position of the C.G. of the whole body, GG' is parallel to OX.

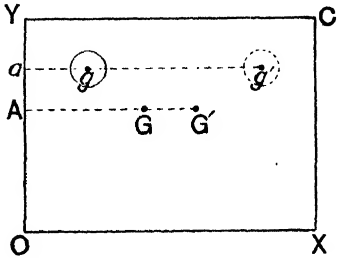


Fig. 103.

Both before and after the movement, the sum of the moments of the weights of the parts of the body about OY is equal to the moment of the weight of the whole body about OY. Therefore the change of moment of the weights of the parts about OY is equal to the change of moment of the weight of the whole about OY.

The moment of the weight of the board itself has not changed. Therefore the change of moment of the 2 lb. weight about OY = the change of moment of the weight of the whole body about OY; that is

$$2 \cdot ag' - 2 \cdot ag = 12 \cdot AG' - 12 \cdot AG,$$

or

$$2 \cdot gg' = 12 \cdot GG',$$

hence

$$GG' = 1{\cdot}67 \text{ inches.}$$

Suppose gg' had taken a direction oblique to the sides of the board, or suppose the board of an irregular shape; we could choose our axes of coordinates parallel and perpendicular to the direction of gg'; it is therefore clear that the change of position of the C.G. of the whole body is always parallel to the direction of the change of position of the part which is moved.

The method we have used may be extended to movements of a number of parts.

For example, let us find the displacement of the C.G. of a ship and its cargo, which together weigh 800 tons, when 20 tons of the

latter are transferred from one side of the deck to the other, a distance of 25 feet.

The change of moment of the part moved is 500 tons ft., therefore the change of moment of the whole is 500 tons ft., and the C.G. of the ship has moved $\frac{500}{800} = 0{\cdot}625$ ft. transversely, parallel to the deck.

EXAMPLES VI.

1. What is the C. G. of a body?

State how you would find by experiment the position of the C. G. of (*a*) a thin flat plate of irregular shape, (*b*) a long symmetrical body such as a connecting-rod of a steam-engine.

2. Explain what is meant by the C. G. (or *centroid*) of a plane figure. State the position of this point for each of the following plane areas: circle, circular ring, square, rectangle, parallelogram, regular hexagon.

3. State the position of the C. G. of each of the following bodies: golf ball, wheel, reel of cotton, top of an oval table, cask, three-blade propeller.

4. Sketch a uniform thin flat plate which is symmetrical about one line only. Explain why the C. G. of the plate must lie in this line.

5. Give an example of a uniform solid body which is symmetrical about one line only. Explain why the C. G. of the body must lie in this line.

6. A piece of uniform wire is bent to form the shapes V, W, Z, and S respectively. State the position of the C. G. in each case.

7. A pile of 10 equal note-books, each 1″ thick, stands on a table. At what height above the table is the C. G. of the pile? By how much will the C. G. be raised when 6 more note-books of the same kind are placed on top?

8. Uniform equal rods are fastened together to form the shapes H, Y, and T respectively. State the position of the C. G. in each case.

9. Give a construction to find the C. G. of a uniform triangular lamina.

An isosceles triangle has an altitude of 8″. What is the perpendicular distance of the C. G. of the area from the base?

10. State (giving reasons) the position of the C. G. of a uniform triangular prism.

11. Draw a triangle with sides of 2, 3, and 4 inches respectively. Taking this to be a uniform lamina, obtain the position of its C. G. Find, by measurement, the distance of the C. G. from each of the angular points.

Taking any side as base, shew that the perpendicular distance of the C.G. from this base is one-third of the altitude of the triangle. What is the reason of this?

12. Shew that the C.G.'s of triangular laminas on the same base, and of the same altitude, lie on a straight line. State the position of this line.

13. A uniform triangular prism rests with one side in contact with the ground. If the top edge is 33 inches above the ground, what is the height of the C.G.?

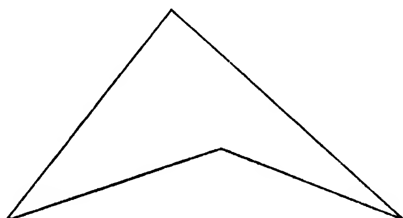
14. A uniform metal plate, in the shape of an equilateral triangle, lies flat on the floor. Shew that the force required to raise one corner is equal to one-third of the weight of the plate.

Shew also that this is true for any uniform triangular plate.

15. A piece of cardboard is in the shape of an isosceles triangle. The angle at the vertex is 28°, and the base measures 5″. Calculate the distance of the C.G. from the base.

16. Draw a plan to represent the mainsail of a yacht. Shew how to find the position of the centroid, by drawing.

17. Give a construction to find the C.G. of a uniform plate having such a shape as that shewn in Fig. (*a*).



(*a*)

18. Weights of 2 lbs. and 3 lbs. have their C.G.'s 10″ apart. Through what point does their resultant weight act?

19. A body consists of two portions. One portion weighs 8 lbs. and its C.G. is at A. The other portion weighs 12 lbs. and its C.G. is 15″ from A. Calculate how far the C.G. of the whole body is from A.

20. A uniform metre scale, which weighs 100 gms., is placed on a table. On one end of it is put a weight of 900 gms. How far can the unloaded end project beyond the end of the table without the scale toppling over?

21. A boat, 24′ long and weighing 1 ton, rests with its keel supported horizontally on two transverse timbers, of which one is 4′ from the bow and the other 6′ from the stern. If the C.G. of the boat is 11′ from the stern, calculate the vertical reactions of the supports in lbs. wt.

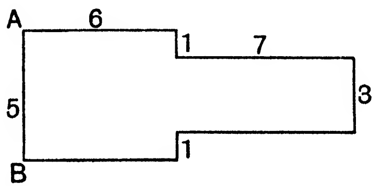
22. A uniform rod of wood, 4′ long and weighing 3 lbs., is joined end to end with a uniform rod of iron, weighing 10 lbs. If the C.G. of the whole lies at the joint, what is the length of the iron?

23. A boat, 14′ long, is slung up by two vertical ropes, one at each end. If the tension in the bow rope is $3\frac{1}{2}$ cwt., and that in the stern rope 4 cwt., calculate the distance of the C.G. from the stern.

24. Why is a heavy body carried more easily if supported at a point above, or below, its C.G.?

A man supports on his shoulder a light bamboo rod, from the ends of which hang weights of 20 and 25 lbs. respectively. If he is to carry his load as easily as possible, what point of the rod should rest on his shoulder?

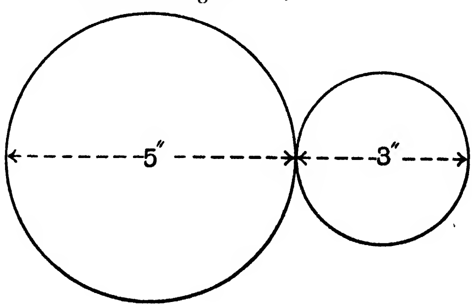
25. Fig. (*b*) represents a sheet of cardboard, the dimensions of which are given in inches. Upon what line does the C.G. lie? Calculate the distance of the C.G. from the end *AB*.



(*b*)

26. If Fig. (*b*) represents the section of a uniform solid cylinder, of which a length of 7″ has been turned down to a diameter of 3″, calculate the distance of the C.G. from the end *AB*.

27. Fig. (*c*) represents a uniform plate consisting of two contiguous circles having diameters of 5 and 3 inches respectively. Calculate the distance of the C.G. from the centre of the larger circle.



(*c*)

28. If Fig. (*c*) represents the section of two solid spheres, made of the same uniform material and having diameters of 5 and 3 inches respectively, calculate the distance of the C.G. from the centre of the larger sphere.

[Volume of a sphere $= \frac{4}{3}\pi r^3$.]

29. A plate of metal, 2′ square, is cut from a uniform sheet. A circular disc, of 4″ radius, is cut from the same sheet and fixed to the square, so that the centre of the disc is on a diagonal and 7″ from one corner. Find the distance of the C.G. of the whole from this corner.

30. Find the C.G. of a uniform cylindrical tin without a lid. The diameter of the bottom is 7″ and the height of the sides is 6″.

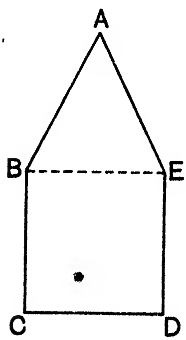
31. A uniform solid stone column consists of a cylinder, 8′ in length and 18″ in diameter, standing on a cubical base having an edge of 30″. Calculate the height of the C.G.

32. A ladder, 28′ long, lies on the ground. To raise one end off the ground a force of 65 lbs. wt. must be applied to it, and to raise the other end requires a force of 47 lbs. wt.

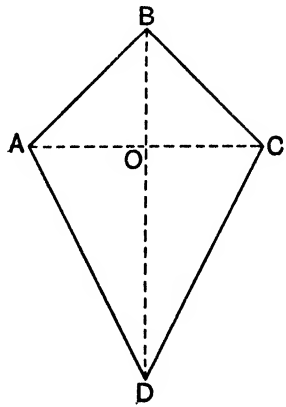
What is the weight of the ladder, and how far is its C.G. from its foot?

33. A billiard-cue is 56″ long, and weighs 1 lb. It is found to balance at its middle point when a weight of $\frac{1}{2}$ lb. is suspended from the tip. How far is the C.G. of the cue from the butt end?

34. Fig. (*d*) represents a uniform lamina which is divided by the diagonal BE into a square $BCED$ and an isosceles triangle ABE. The side of the square and the altitude of the triangle each measure 4″. On what line does the C.G. lie? Calculate the distance of the C.G. from A.



(*d*)



(*e*)

35. Fig. (*e*) represents a uniform quadrilateral lamina whose diagonals intersect at right angles at O. If $OA=OB=OC=6$ inches, and $OD=12$ inches, calculate the distance of the C. G. from A.

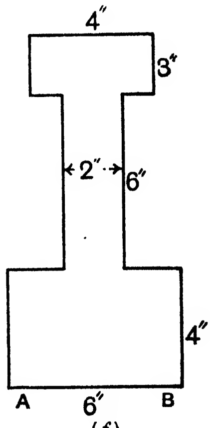
36. Four equal weights have their C. G.'s in one straight line at A, B, C, and D respectively. If $AB=3''$, $AC=7''$ and $AD=11''$, find the point through which the resultant weight acts.

37. Four weights of 2, 3, 4, and 5 lbs. have their C. G.'s in one line at A, B, C, and D respectively. If $AB=5''$, $AC=8''$ and $AD=12''$, calculate the distance from A of the C. G. of the four weights together.

38. A uniform rod, 24″ in length, weighs 2 lbs. Weights of 2, 4, and 6 lbs. are fastened to it so that the distances of their C. G.'s from one end are 3″, 9″, and 15″ respectively. Taking moments about one end of the rod, calculate the distance, from this end, of the point about which the loaded rod will balance.

39. Fig. (*f*) represents the section of a girder, the measurements being given in inches. Calculate the distance of the centroid from the edge AB.

40. Fig. (*f*) represents the section of a solid cylinder of uniform material which has been turned down over two portions of its length; the diameters and lengths of the three portions are given in inches. Calculate the distance of the C. G. from the end AB.



(*f*)

41. Two weights, 5 and 7 lbs., are placed on a rectangular table, so that the perpendicular distances of their C. G.'s from one side are $1\frac{1}{4}'$ and $2'$ respectively. Calculate the perpendicular distance, from this side, of the point through which the resultant weight of the two bodies acts.

Calculate also the perpendicular distance of this point from one end of the table, given that the C. G.'s of these weights are $2\frac{1}{2}'$ and $3\frac{1}{2}'$, respectively, from this end.

42. A uniform rectangular board, weighing 4 lbs., measures $24'' \times 16''$. A flat circular disc of metal, weighing 5 lbs., is attached to the board with its centre distant 5″ from one side, and 9″ from one end. Sketch a plan of the board, shewing the position of the disc.

Calculate the distances of the C. G. of the whole from the side and end mentioned.

43. A uniform drawing board, having a weight of 2 lbs. and a length of 24″, is held with its surface horizontal by grasping the edge at one end.

The board is loaded with flat weights of 1, 2, and 3 lbs. so that the perpendicular distances of their C. G.'s from this edge are 18″, 15″, and 6″ respectively. Calculate the sum of the moments of the weights of the loads and the board about this edge. Hence calculate the perpendicular distance of the C.G. of the whole from this edge.

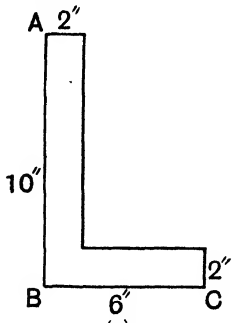
How would you verify your result by experiment?

44. A rectangular board, 24″ × 16″, rests flat on a table and is loaded with various weights, the weight of the whole loaded board being 12 lbs. It is found that the vertical force required to raise the edge at one end is 4 lbs. wt.; and to raise one side in a similar manner requires a force of 3 lbs. wt. Calculate the perpendicular distances of the C.G. of the whole from this end and from this side, respectively.

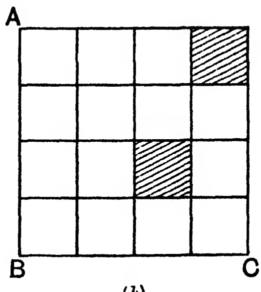
45. *ABCD* is a square whose side is 20″ long. Weights of 2 lbs., 3 lbs., 3 lbs., and 4 lbs. are placed so that their C.G.'s lie at the corners *A*, *B*, *C*, and *D* respectively. Calculate the distances of the C. G. of the four weights from the sides *AB* and *BC*.

46. A rectangle, *ABCD*, has weights of 5 lbs., 5 lbs., 3 lbs., and 3 lbs. placed at the four corners *A*, *B*, *C*, and *D* respectively. If $AB = 20''$, and $BC = 8''$, find the position of the C. G. of the weights with reference to the lines *AB* and *BC*.

47. Fig. (*g*) represents the section of a piece of angle iron. Find the distances of its C. G. from *AB*, and from *BC*.



(*g*)



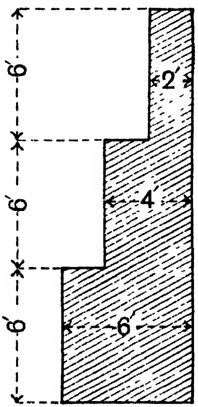
(*h*)

48. A straight piece of uniform wire, 20″ long, is bent at the centre to form two arms at right angles. Below what point in one arm must you put your finger so that the whole will balance with this arm horizontal?

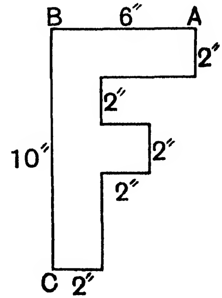
49. (Fig. *h*.) A uniform square plate, of side 12″, weighs 2 lbs. It is divided into 16 equal squares by lines drawn parallel to its sides. Two of these, denoted by the shaded portions, are covered with plates of sheet lead,

each of which weighs 1 lb. Calculate the position of the C. G. of the whole with reference to the sides AB and BC.

50. (Fig. *i*.) The section of a retaining wall has the dimensions shewn. Calculate the position of the centroid of this section with reference to the right-hand and bottom edges.



(*i*)

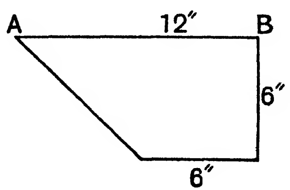


(*j*)

51. Fig. (*j*) represents a uniform plate. Calculate the distance of its C. G. from AB, and from BC.

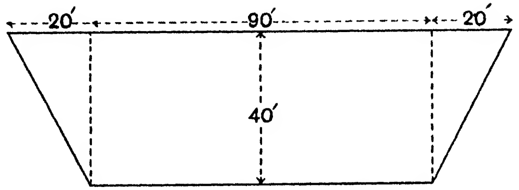
52. A right-angled triangle is made by fastening together three straight pieces cut from a uniform wire. The hypotenuse measures 10″, and the other two sides 6″ and 8″ respectively. How far is the C. G. of the triangle from each of these two sides?

53. Fig. (*k*) represents a uniform sheet of metal. From what point in AB must it be suspended so that it will come to rest with this edge horizontal?



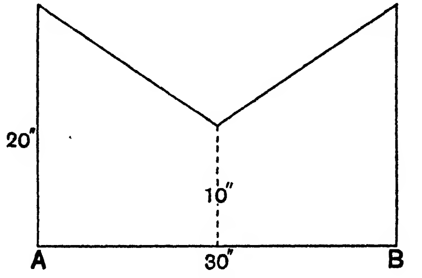
(*k*)

54. Fig. (*l*) represents the section of a reservoir. Calculate the height of the centroid of the section.



(*l*)

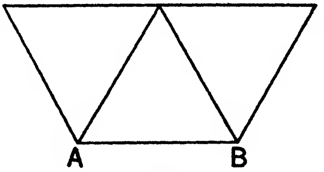
55. Fig. (*m*) represents a uniform lamina. Calculate the distance of its C.G. from *AB*.



(*m*)

56. Three equal uniform rods, each 15″ long, are fastened together to form an equilateral triangle. Find the position of the C.G.

57. (Fig. *n*.) Seven equal uniform rods, each 16″ long, are jointed together to form a framework consisting of three equilatoral triangles. Calculate the distance of the C.G. from *AB*.



(*n*)

58. Calculate the distance of the centre of the sail area of a cutter from the centre line of the mast, and also its distance from the deck, from the following data :

Sail	Area in sq. ft.	Distance of C.G. from centre line of mast in feet	Distance of C.G. from the deck
Mainsail	200	$6\frac{1}{2}$	$9\frac{1}{2}$
Topsail	48	3	18
Foresail	68	$4\frac{1}{2}$	6
Jib	54	10	7

59. A ship, when empty, weighs 2000 tons, and the height of her C.G. above the keel is 12 feet. How much will the C.G. be lowered by filling her ballast tanks with 600 tons of water, if the C.G. of this water is $2\frac{1}{2}$ feet above the keel?

60. The C.G. of a ship of 4000 tons is 15 ft. above the keel. 300 tons of coal are now stowed so that the C.G. of this coal is 20 ft. above the keel. How much is the C.G. of the ship raised?

61. A vessel, of 1000 tons, has its C.G. 12 ft. above the keel, and 60 ft. from the stern. Calculate the vertical and horizontal displacements of the C.G. when a load of 40 tons is placed on the deck, the C.G. of this load being 20 ft. above the keel and 90 ft. from the stern.

62. Given the weight of a body and the position of its C.G., and also the weight of a part of the body and the position of the C.G. of this part, shew how to find the position of the C.G. of the remainder when this part is removed.

A body weighs 12 lbs. and its C.G. is at A. A portion of the body weighs 5 lbs. and the C.G. of this portion is at B, distant 14″ from A. Where is the C.G. of the remainder when this portion is taken away?

63. A circular lamina, of diameter 14″, has a circular portion, of diameter 4″, removed from it, the centre of this portion being 4″ from the centre of the lamina. Where is the C.G. of the remainder?

64. A square of cardboard, of 8″ side, has a triangle cut off one corner, the base of this triangle being the line joining the middle points of adjacent sides. Calculate the distance of the C.G. of the remainder from the opposite corner of the square.

65. A laden ship weighs 4000 tons, and the C.G. is 14′ above the keel. The cargo weighs 800 tons, and its C.G. is 8′ above the keel. Where will the C.G. of the ship be after the cargo has been discharged?

66. A ship with ballast weighs 3000 tons, and the C. G. is 14′ above the keel. The ballast consists of 500 tons of water with its C. G. 2′ above the keel. How much will the C. G. of the ship be raised by emptying the ballast tanks?

67. Fig. (*h*) on p. 200 represents a uniform square plate, of side 12″, divided into 16 equal squares. If the shaded portions are now removed, calculate the position of the C. G. of the remainder with reference to the sides *AB* and *BC*.

68. An eccentric consists of a circular steel plate, 5″ in diameter, having through it a hole of 2″ diameter. If the circumference of the hole passes through the centre of the plate, calculate the distance of the C. G. of the eccentric from its centre.

69. *O* is the centre of a circular lamina, of 20-inch radius; *OB* and *OC* are two radii having an angle of 30° between them. Find the C. G. of the remainder when the sector *BOC* is removed.

70. *ABC* is a triangular lamina; the base *BC* is 10″, and the altitude of the triangle is 6″. The triangle is divided, by a line joining the mid-points of the sides, into a triangle and a trapezium. What is the altitude of the C. G. of the whole triangle, and also of that of the smaller triangle? Hence calculate the altitude of the C. G. of the trapezium.

71. A sailing boat of $1\frac{1}{2}$ tons is found to carry too much weather helm. To rectify this, a load of 3 cwt. is moved aft a distance of $3\frac{1}{2}$ feet in a horizontal plane. What is the change in the position of the C. G. of the boat?

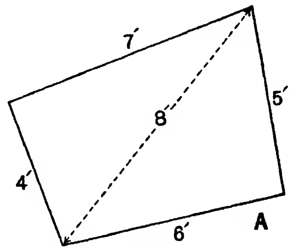
72. An excursion steamer, with passengers, weighs 500 tons. On nearing a pier the passengers, 400 in number, move to one side of the ship, each moving transversely an average distance of 12 feet. Find the transverse displacement of the C. G. of the ship.

Take the weight of 16 passengers to be 1 ton.

73. A 25-ton gun is transferred across the deck of a 15,000-ton battleship through a distance of 40 feet. How does this affect the position of the C. G. of the ship?

Miscellaneous Examples.

74. (Fig. *o*.) A uniform sheet of metal, having the dimensions given, lies flat on the floor. The corner A is now raised by applying here a vertical force. State, giving your reason, the edge about which the plate will turn. Find, partly by drawing and partly by calculation, the vertical force required to raise the corner A, if the weight of the sheet is 30 lbs.



(*o*)

75. An equilateral triangle is described on one of the sides of a square, the edge of which measures 6″. Calculate the distance of the c. g. of the whole figure from the vertex of the triangle.

76. A uniform metal rod ABC, weighing 3 lbs., is bent at B to form two arms at right angles, the arm AB having twice the length of the arm BC. If it is now laid flat on the table, calculate what vertical force will be required to raise the end A clear of the table. Calculate also the respective forces required to raise, in turn, the end C and the corner B.

If the bent rod is suspended from A, find the angle at which the arm AB will be inclined to the vertical.

77. In raising a ladder against a vertical wall, a man places the foot against the bottom of the wall and pushes on the under side. If the ladder weighs 1 cwt. and its c. g. is 12′ from the foot, with what force must the man push at right angles to the ladder at a point 7′ from the foot, when the ladder is inclined at 45° to the vertical?

78. A ship on entering a port weighs 5000 tons, and her c. g. is 18′ above the keel. When in port she takes on board 250 tons of coal, the c. g. of which is 12′ above the keel. What alteration does this produce in the height of her c. g.? Before leaving the port she discharges 400 tons of cargo, the c. g. of which was 14′ above the keel. Find the height of her c. g. above the keel on leaving the port.

79. An omnibus with passengers weighs 2 tons. Find the alteration produced in the height of the c. g. when 7 passengers, weighing 10 stone each, go from the inside to the top of the omnibus. The seats outside are 8′ above those inside.

80. A uniform wire is bent so as to form a triangle having sides of 4″, 5″ and 7″. Explain how you would find the c. g.

81. The mainsail of a yacht has an area of 200 sq. ft. and its centre of area is 7 ft. above the boom. The sail is now reefed by shortening it by 3 ft.

If the canvas taken in is assumed to be a rectangle 3 ft. high and 16 ft. broad, calculate the height of the centre of area of the reefed sail above the boom.

82. $ABCD$ is a square cut out of paper, having an edge of 12 cms. E and F are the middle points of the sides AD and DC. The triangle DEF is folded back along the line EF so that it lies flat on the remainder. Calculate the distance of the C. G. from B.

83. When a semicircular sheet of cardboard is suspended freely from one end of its straight edge, it is found that it comes to rest with this edge inclined at 23° to the vertical. Shew that this result verifies the fact that the C. G. of a semicircular lamina is situated at a distance from the mid-point of its straight boundary given by $\frac{4}{3\pi} \times R$, where R is the radius.

84. The altitude of a solid cone of homogeneous material is 18″, and the diameter of its base is 12″. When suspended freely from a point in the edge of the base, the plane of the base is found to be inclined at 53° 8′ to the horizontal. Shew that the distance of the C. G. of the cone from its base is equal to one-quarter of the height of the cone.

85. A pair of shear legs on the edge of a wharf are supported in a plane inclined at 60° to the horizontal, by a tie running back from the top at an angle of 30° to the horizontal. If each leg has a weight of 1 ton and has its C. G. at its centre, calculate the tension in the tie.

86. The displacement of a ship is 12,000 tons before heavy guns are mounted at two points A, B, which lie in a horizontal line 200 feet apart. The vertical line through the centre of gravity G intersects AB at D so that $GD=20$ feet, $AD=85$ feet and $DB=115$ feet. Find the vertical and horizontal displacements of the centre of gravity consequent on mounting a pair of guns at each of the points A, B, the weight of each pair being 160 tons.

87. A beam, 10 feet long, is supported at its ends in a horizontal position. The beam is 2 feet wide at one end, and gradually tapers to 1 foot at the other end, the depth being everywhere 1 foot. A concentrated load of 400 lbs. is placed on the beam at a distance of 4 feet from the large end, and the material of the beam weighs 50 lbs. per cubic foot. Find the reactions at the supports.

88. A uniform solid cone, 16″ high, is cut across half way up, the plane of the cut being parallel to the base. If the smaller cone so formed is removed, calculate the height of the C. G. of the remaining frustum.

The volume of a cone $=\frac{1}{3}\pi$ (radius of base)$^2 \times$ height, and the distance of its C. G. above the base is one-quarter of the height of the cone.

CHAPTER VII

CENTRE OF GRAVITY—STABILITY OF EQUILIBRIUM

86. Stable, Unstable and Neutral Equilibrium. In this chapter we propose to illustrate the practical importance of the position of the centre of gravity in certain cases.

This we shall first do in the case of a rigid body resting upon some support, our primary object being to shew how the position of the centre of gravity affects the behaviour of the body when it is slightly disturbed from its position of equilibrium. Consider a body pivoted about a horizontal axis, such as a disc of cardboard supported at some point O by a horizontal pin about which it can turn freely. (Fig. 104.)

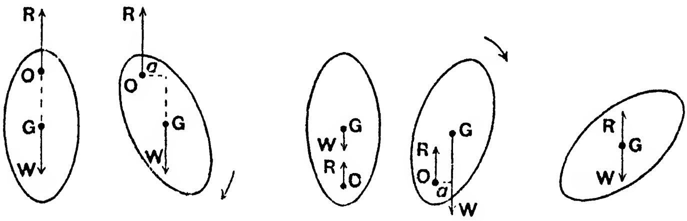


Fig. 104.

The centre of gravity of the disc (found by the method described in Art. 74) is denoted by G. When this point is vertically below O (i) the disc is in equilibrium, for its weight and the equal and opposite reaction of the pin act in the same line. If we disturb the disc we find that, after swinging about O, it always returns to its original position, for the weight of the disc when displaced has an unbalanced moment ($W.a$) about O which tends to turn it back again (ii). For this reason, the equilibrium is said to be **Stable.**

Note that the effect of any displacement is to raise the centre of gravity.

But the disc, with difficulty it is true, can be made to balance when G is vertically above O (iii); for, since the weight and the reaction of the pin again act against one another in the same line, the disc is also in equilibrium in this position. The equilibrium, however, is not stable, for the slightest disturbance gives the weight an unbalanced moment about O which tends to turn the disc farther from its original position (iv). Hence the equilibrium in this case is said to be **Unstable.**

Note that any displacement from this position has the effect of lowering the centre of gravity.

If we now pivot the disc by striking the pin through G, we find that it will rest in any position (v). In this case the weight and the equal and opposite reaction of the pin act at the same point and hence always balance one another. The disc therefore, when displaced, shews no tendency either to return to, or to move farther from, its original position. The equilibrium in this case is said to be **Neutral.** Here, any displacement of the disc does not alter the height of its centre of gravity.

To sum up: a body, when pivoted about a horizontal axis, is in stable, unstable, or neutral, equilibrium according as its centre of gravity is below, above, or on, this axis respectively.

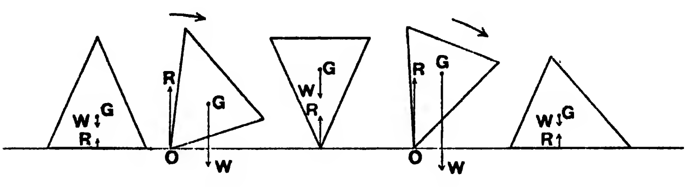


Fig. 105.

Another illustration of these three states of equilibrium is afforded by a uniform cone resting on a table (Fig. 105). When standing on its base (i) the centre of gravity G is above the

support, yet the cone is in **stable** equilibrium, for, when slightly tilted about any point O in the edge of the base and then released (ii), it turns back into its former position. One effect of tilting the cone is to transfer the normal reaction of the table to O, about which point the weight acting through G has a restoring moment; another effect is to raise the centre of gravity.

When placed on its apex with its axis vertical, the cone (iii) is clearly in **unstable** equilibrium, for the slightest displacement causes it to topple over since an overturning moment about the apex is brought into action (iv). The effect of tilting the cone in this case is to lower the centre of gravity.

Finally, if we place the cone on its side (v) and then displace it by rolling it through any angle, it shews no tendency either to return to, or to move farther from, its former position. Hence, for such displacements, this position is one of **neutral** equilibrium. This is explained by the fact that the total reaction of the table on the side of the cone always acts upwards through G.

If the cone is not quite true or the table not quite level, this will not give a convincing illustration of neutral equilibrium: a perfect example is afforded by a billiard ball on a billiard table.

At the same time it should be observed that if the cone, when resting on its side, is tilted slightly about either end of its line of contact with the table and then released, it falls back on to its side again, shewing that for such displacements the equilibrium is stable. We see then that a body in one position may be in different states of equilibrium according to the axis about which it may be supposed to be turned. The nature of the equilibrium may even depend upon the direction in which the displacement about a certain axis is supposed to be made. For instance, a uniform cylindrical ruler placed along the edge of a table is in a position of stable, unstable, and neutral, equilibrium according as we suppose it to be tilted *about one end*, rolled slightly *over* the edge of the table, and rolled *away from* this edge respectively.

Defs. *When a body is slightly disturbed from a position of equilibrium, the equilibrium is said to be* (1) **Stable,** *if the forces acting on the body tend to restore it to its original position;* (2) **Unstable,** *if the forces acting on the body tend to displace it farther from its original position;* (3) **Neutral,** *if the forces acting on the body neither tend to restore the body to, nor tend to displace it farther from, its original position.*

We will now consider a few cases of equilibrium of various kinds and note the conditions making for stability or instability.

87. Applications. In the foregoing illustrations we observed that when a body is in stable equilibrium, any slight displacement has the effect of raising its centre of gravity, and that the opposite effect is produced when the position is one of unstable equilibrium. This naturally suggests the statement: "a body tends to come to rest in such a position that its centre of gravity is as low as possible."

A plumb-line in equilibrium hangs vertically, for in this position the centre of gravity of the lead weight is as low as possible.

When a liquid is poured into a vessel, it comes to rest with its surface in a horizontal plane; if it did not, the centre of gravity of the liquid would not occupy the lowest possible position. The valuable property of a spirit-level also depends on this principle. This instrument is constructed of a glass tube slightly curved, with its convex surface uppermost. Except for a bubble of air, the tube is filled with spirit, and is then attached to a flat base. If accurately adjusted, the bubble will stand at the centre (the highest point) when the base rests on a horizontal surface, for it is only when it is in this position that the centre of gravity of the liquid is as low as possible.

Again, if you examine a self-shutting swing gate you will find that it is so constructed that the effect of opening the gate in either direction is to raise its centre of gravity. When left to itself, the gate comes to rest with its C.G. as low as possible, that is in the closed position.

A number of so-called 'balancing' toys depend for their success on this principle. Various bodies, such as models of men or animals, are made so that they will stand in stable equilibrium on a narrow edge or point, although at first sight their equilibrium would appear to be unstable.

This is achieved by fastening to them heavy weights low down, by means of a stiff wire. In this way the centre of gravity of the whole is carried below the point of support about which, when disturbed, the whole body will swing like a pendulum. This can be illustrated by the simple arrangement in Fig. 106; a nail is driven through a cork and into the side of this cork is thrust a stiff bent wire which carries at its other end a lead bullet. This brings the centre of gravity to some such point as G below the point of support and consequently the whole arrangement is in stable equilibrium.

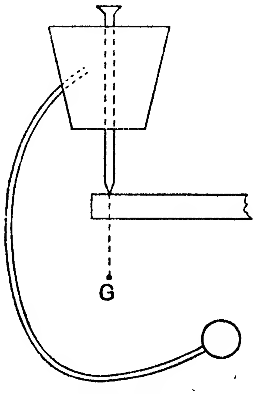
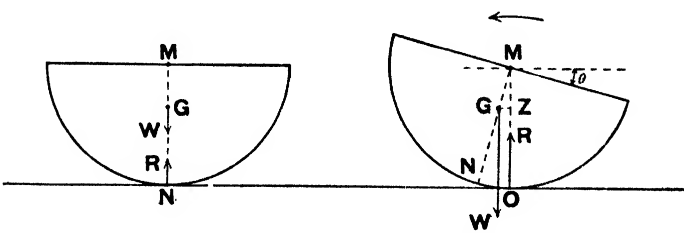


Fig. 106.

Other balancing toys are made by attaching some light model figure to a heavy hemispherical base. When disturbed, the figure rocks to and fro, but always regains its upright position which is, therefore, one of stable equilibrium.

Let us examine this case of a uniform solid hemisphere resting with its curved surface on a horizontal plane (Fig. 107a). Clearly



(*a*) Fig. 107. (*b*)

the centre of gravity G lies on the axis of symmetry MN and below M. If we tilt the hemisphere through any angle less than that subtended at G by a quadrant of the circle, and then release it (Fig. 107*b*), the forces acting upon it are (1) the weight W acting downwards through G and (2) the reaction R acting upwards through O the new point of contact. Since the section of the plane surface is a tangent to that of the curved surface of the hemisphere, it follows that OM is the vertical line along which R acts. Considering the moments of the forces about O (or M) we see that, while the moment of R is zero, the moment of W is such that it tends to restore the body to its former position which is, therefore, one of stable equilibrium.

The two forces W and R are equal and hence, when the hemisphere is tilted, these forces constitute a *couple* which in this case is called a 'righting couple' since it tends to right the body (Art. 72). The moment of this couple $= W \cdot GZ = W \cdot GM \sin\theta$, where θ is the angle OMN through which the body is tilted. We shall see later that an example closely resembling this is afforded by a ship when heeled through a small angle.

In the following example the position of unstable equilibrium is employed in practice.

Fig. 108 represents a steel skip or bucket in which coal is hoisted out of a vessel by means of a crane. The skip is pivoted about an axis which lies *below* the centre of gravity when it is full, and is locked in position by a clasp on the rim. When this clasp is unfastened, the loaded skip is in unstable equilibrium and hence a slight push suffices to tip it over. Moreover, it is generally arranged that the centre of gravity of the empty skip lies below the axis of support so that, when emptied, it rights itself.

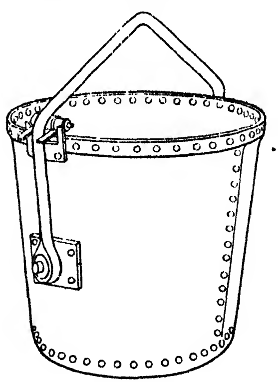


Fig. 108.

Another example is afforded by a loaded two-wheeled farm cart. If the centre of gravity of the

load lies almost vertically above the axle, no great force is required to tip up the body of the cart when the 'tip-stick' which locks it in front has been removed.

88. Condition for a body standing on a plane surface. Let us take two equal uniform rectangular blocks of wood. If we stand one block by itself on the table, the vertical line through its C.G. passes through the middle point of the base (Fig. 109 *a*), and the body is in stable equilibrium for the reason already explained in the case of the cone on p. 208. If we now strap the second block to the first with rubber bands as shewn in Fig. 109 *b*, the effect is to bring the C.G. of the whole to G, a point

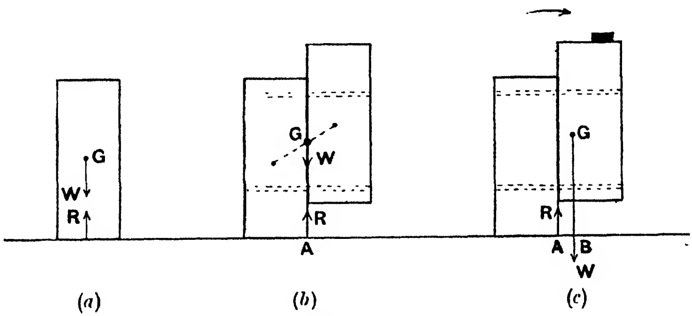
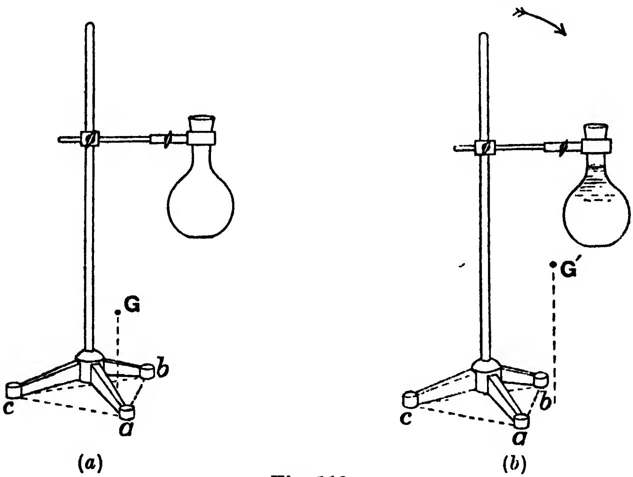


Fig. 109.

vertically above the edge A of the base, and we find that the whole body is just on the point of overturning to the right. But if a small weight is placed on the top of the second block (Fig. 109 *c*), the effect is to bring the C.G. of the whole farther to the right so that the vertical line through this point passes through B a point outside the base. The body now upsets, for the weight W of the whole has an unbalanced overturning moment $(W.AB)$ about the edge A.

We may state generally that *when a body is placed on a plane surface, it will stand or fall according as the vertical line through its* C.G. *passes within or without its base*.

In some cases the base of support is not quite so simple. For instance, Fig. 110 *a* represents a retort stand having a tripod base, the lines joining the three feet forming an equilateral triangle. An empty flask is shewn clamped to the stand and at some distance from it. If the vertical line through the C.G. of the whole arrangement falls within the triangle *abc* the equilibrium is stable, but if, on filling the flask with water, the C.G. is displaced so that the vertical line through it falls outside the line *ab*, the stand will overturn about this line (Fig. 110 *b*).



(*a*) (*b*)

Fig. 110.

It is clear then that we must take the term 'base' in the above statement to signify the area which would be enclosed by a thread drawn tightly round the lowest part of the body so as to enclose all the points of contact with the plane. Thus, the base of a railway truck is the rectangle whose angular points are the points of contact of the four wheels with the rails.

The celebrated leaning tower of Pisa is 182 feet in height, and the centre of its summit is $13\frac{1}{2}$ feet from the vertical line through the middle of its base. Yet, though appearing ready to fall over

at any moment, it remains perfectly secure since the vertical line through its C.G. falls well within its base.

Since a man is not a rigid body, the position of his C.G. depends upon his attitude. When he is on his feet his every movement is an illustration of the success which he instinctively achieves in keeping his C.G. above his base of support. (What is this base?)

For instance, if you carry a pail of water in one hand, you lean to the other side so as to keep the common C.G. of yourself and the pail over your base of support.

Stand with your back to the wall of a room with your heels against the skirting and try to pick up an object from the floor in front of you; or again, stand sideways to the wall with one foot against the skirting and try to raise the other foot; what is the cause of failure in each case?

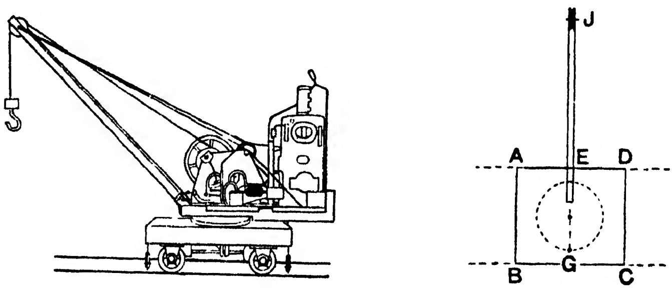
The art of tight-rope walking, or rope-dancing, consists in keeping the C.G. exactly over the rope. To accomplish this more easily the performer often carries a pole horizontally; as soon as he feels himself leaning to one side, he shifts the pole towards the opposite side. By so doing he shifts the C.G. the same way, and thus preserves his balance.

On the other hand, when you balance a body such as a billiard-cue on the end of your finger, you are continually restoring the equilibrium by shifting your finger so that the small base of support is brought vertically below the C.G. of the cue.

89. Application to a Travelling Crane. Fig. 111 *a* shews a travelling steam crane. The jib of the crane, the winch and the engine which works it, are all supported on a platform which can be turned round about a vertical axis passing through the centre of the trolly. A plan of the base of the trolly is shewn in Fig. 111 *b*, the points A, B, C, and D being the points of contact of the wheels with the rails. In this diagram a bird's-eye view of the jib in a position transverse to the rails is shewn by the line EJ.

The C.G. of the platform and everything upon it is brought

some distance from the central axis on the side opposite the jib by mounting the heavy boiler on this side; moreover, this distance will remain fixed when the platform is rotated, provided that the inclination of the jib remains unaltered. Further, since the C.G. of the trolly alone lies on the central axis, it follows that the C.G. of the whole crane will always be at a fixed distance from the axis; that is, as the platform is rotated, the C.G. of the whole crane always lies on a circle such as that indicated by the dotted circle in Fig. 111 (*b*).



(*a*) (*b*)

Fig. 111.

In order that the crane may not upset when raising a load, the vertical line through the C.G. of the crane and load together must fall within the base $ABCD$. Let us see what is the maximum load which can be raised when the jib is in the position shewn in Fig. 111 *b*. When the crane is on the point of upsetting the total reaction of the rail on the side BC is zero, and the forces acting on the crane are its whole weight acting downwards, through G, the weight of the load acting downwards through J, and the total reaction of the rail on the wheels A and D.

Taking moments about the line AD, we have

$$\text{Max. load} \times JE = \text{weight of crane} \times GE,$$

or

$$\text{Max. load} = \frac{GE}{JE} \times \text{weight of crane.}$$

Thus, if JE, the horizontal overhang of the jib, be three times the distance GE, the maximum load which could be raised would be one-third of the weight of the whole crane.

90. Stability of a body standing on a plane surface. Suppose that a cask filled with liquid stands on its end on level ground, and that we wish to upset it by applying a horizontal push P to the top (Fig. 112). We will assume that the friction called into play between the cask and the ground is sufficient to prevent any slipping taking place. When the cask is just on the point of tilting about O, the forces acting upon it are (1) its weight W acting downwards from G through the centre of the base (of radius r), (2) the horizontal force P acting horizontally at a height h, (3) the reaction of the ground acting in some direction through O.

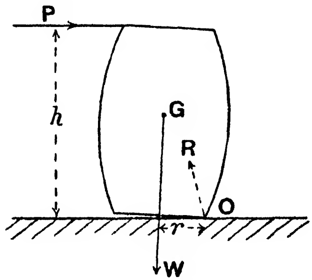


Fig. 112.

Taking moments about O, the moment of the reaction is zero, and we have

$$P \cdot h = W \cdot r \quad \text{or} \quad P = \frac{r}{h} \cdot W.$$

It is easy to see that, as the cask tips up, the arm of the moment of the force P *increases*, while that of the moment of W *decreases*; hence, the greatest force is that required to start the tilting. As the tilting proceeds, the force required to tilt it farther diminishes until it becomes zero when G is vertically above O.

Notice also that the steady force which must be applied at any given height, to tilt the body, does not depend on the height of the C.G. On the other hand, however, it is important to observe that, the higher the C.G., the smaller is the angle through which we have to tilt the body before its equilibrium becomes unstable.

This angle, which may be called the *angle of vanishing stability*, can be measured by standing the body on a rough board and finding to what inclination the board must be tilted so that the body will just upset. To demonstrate this we will stand a uniform cylinder on a board and tilt it until the cylinder is on the point of upsetting (Fig. 113). If the friction called into play is not sufficient to prevent the cylinder from slipping, a rubber band can be stretched round the board, and the edge of the base of the cylinder placed against it.

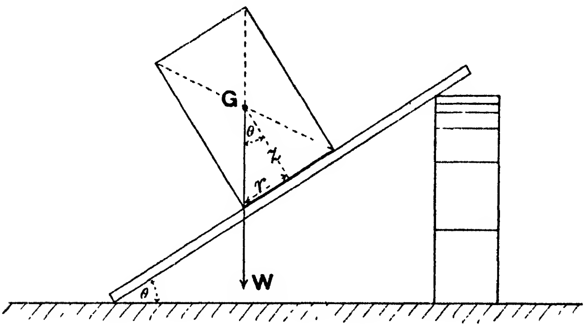


Fig. 113.

Let r be the radius of the cylinder and z the height of its C.G. when it stands on a horizontal surface, and let θ be the angle of vanishing stability, that is, the angle through which we have to tilt the board to bring G vertically above the edge of the base. It will be seen from the figure that

$$\tan\theta = \frac{r}{z}.$$

This shews that the angle of vanishing stability increases with the radius of the base and decreases with the height of the C.G.

What is the shape of a railway milk-churn, and why is it given this shape?

91. Experiments on Stability.

Exp. 1. *To determine indirectly the approximate weight of a cask of liquid.* You are provided with a 7-gallon cask of liquid standing on its end on the floor. Measure the height of the cask and the diameter of its end. Hook a spring-balance (reading to 30 lbs. wt.) to the top edge and pull it horizontally until the cask just begins to tilt. Read the spring-balance when the farther edge of the base is just clear of the floor. What forces are now acting on the cask?

By taking moments of these forces about the point of contact with the floor, calculate the weight of the cask.

If there is a weighing-machine in the laboratory, this result should be tested by direct weighing, the cask being rolled up a board on to the platform of the machine.

Exp. 2. *To find the* C.G. *of a uniform solid cone by determining its angle of vanishing stability when standing on its base.*

Stand a uniform solid cone on its base on a rough board. Measure the height of the cone and the diameter of its base. Now tilt the board until the cone is on the point of upsetting.

Measure the angle which the board makes with the table when the cone is just on the point of upsetting.

Hence calculate the distance of the C.G. from the centre of the base. What fraction is this distance of the height of the cone?

92. Tilting or Sliding? In Art. 90 we shewed how to find the horizontal force required to tilt a body standing on a horizontal surface, assuming that no sliding occurs. It will be found an instructive exercise to consider the equilibrium of such a body more fully. Suppose, for example, we place a bottle on a table and apply to it a horizontal force P at a height h above the table (Fig. 114). This force may be insufficient to move the bottle at all, but if it does so, it will cause the bottle either to slide or to tilt, or to do both simultaneously.

When there is no movement or when steady sliding occurs,

the four forces acting on the bottle are those shewn in the figure. The horizontal forces are P and the force of friction F exerted by the table on the base; F is equal to P but of opposite sense. The vertical forces are the weight W and the equal normal

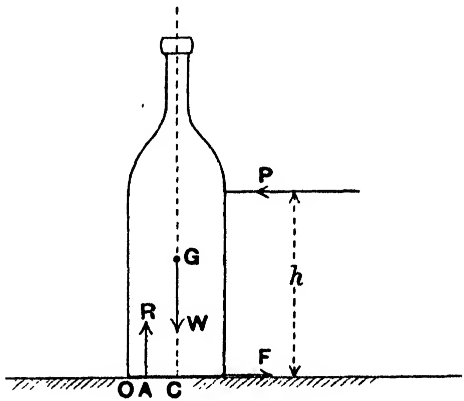


Fig. 114.

reaction R. Since the body is in equilibrium, this resultant normal reaction must act upwards through a point A, such that the moments of the forces about this point are balanced, that is, such that $P.h = W.AC$, the moments of R and F both being zero.

Note that the two vertical forces form a couple and likewise the two horizontal forces. For equilibrium, the moments of these two couples must be equal, and this is expressed by the above equation.

If steady sliding occurs, P is equal to the limiting force of friction, which is μR or μW, where μ is the coefficient of friction for the contact surfaces.

On the other hand, if no sliding occurs, then, as we increase P, AC will increase; that is, the line of action of R will be displaced farther and farther to the left until this force acts upwards through the edge O, and the bottle will then begin to tilt.

Denoting OC, the radius of the base, by r, and taking moments about O we have $P.h = W.r$ or $P = \frac{W.r}{h}$. (Art. 90.)

We see then that the bottle will first begin to slide or to tilt according as $\mu W <$ or $> \frac{Wr}{h}$, that is, according as $\mu <$ or $> \frac{r}{h}$.

If $\mu W = \frac{Wr}{h}$ or $\mu = \frac{r}{h}$, then if P is increased till movement occurs, the bottle will begin to slide and tilt at the same moment.

Note that the condition determining whether the body will slide or tilt does not depend on the weight of the body.

To demonstrate this for yourself by experiment, find at what height you must push an empty bottle with your finger so that it will begin to tilt and slide simultaneously, and then shew that the same result is obtained when the bottle is full of liquid and standing on the same horizontal surface.

Example. If the radius of the base of a bottle is $1\frac{1}{2}''$ and the coefficient of friction between this base and the table is 0·25, will the bottle begin first to slide or to tilt, when a gradually increasing horizontal force is applied to it at a height of $5''$ above the table?

The horizontal force required to make the body slide is equal to the limiting force of friction which is **0·25 W**, taking W as the weight of the bottle.

If the bottle tilts, the necessary force P is found by taking moments about the point of the edge of the base about which it turns, *i.e.* $5P = 1{\cdot}5\,W$ or $P =$ **0·3 W.**

Since this force is greater than that necessary to produce sliding, the bottle will not tilt, but will slide as soon as the horizontal force reaches the value $0{\cdot}25\,W$.

93. Sensitiveness of a Balance. Fig. 115 illustrates a form of laboratory balance used for accurate weighing. The main principle here made use of is that of a lever with equal

arms, for if two bodies suspended from the ends of these arms balance one another, their weights must be equal.

But to attain a high degree of accuracy in weighing it is essential that a balance should obey certain conditions, and in order that it may do so the chief adjustments necessary in its construction are as follows.

The metal beam is supported on a central steel knife edge resting on polished agate bearings. This is to ensure that the beam shall turn about a definite line and with as little friction

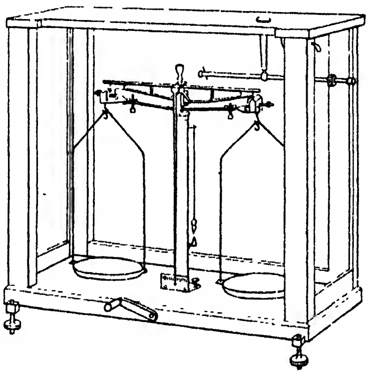


Fig. 115.

as possible. For the same reason the hooks which carry the scale pans also rest on knife edges. These knife edges must be fastened to the beam at exactly equal distances from the centre and the line joining them must pass through the central knife edge. When horizontal, the beam must be perfectly symmetrical about the vertical line through the central knife edge, and the C.G. of the beam must lie vertically below this edge. This ensures that the beam will come to rest in stable equilibrium in a horizontal position when the scale pans are empty or when they carry equal loads, and that, when disturbed, it will swing to and fro about this position. But, for accurate weighing, in addition to the beam being stable, it must at the

same time be ***sensitive***, that is, a very small inequality of the loads placed in the two pans must cause an appreciable deflection of the pointer attached to the beam. Let us see how the balance may be made to fulfil this condition.

In Fig. 116, which represents the beam only, O is the central knife edge about which the beam turns, and D and E are the

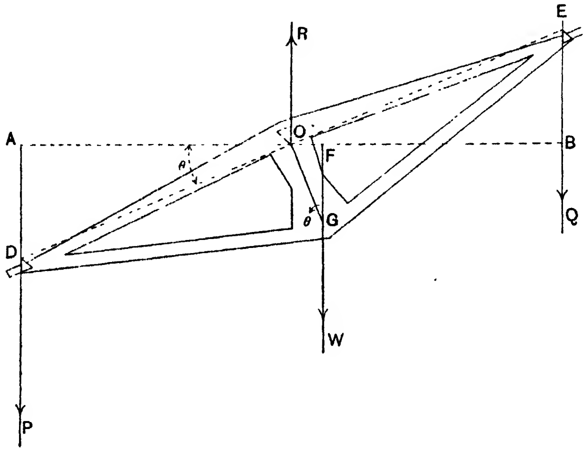


Fig. 116.

knife edges which carry the hooks from which the scale pans are hung. G is the centre of gravity of the beam shewn at an exaggerated distance below O for the sake of clearness. The beam is shewn in a position of equilibrium such as it takes up when the loads in the pans are slightly unequal. These loads (including the weights of the pans) are denoted by P and Q (P being slightly greater than Q).

In addition to these forces, the beam is acted upon by its weight W acting downwards through G, and the reaction of the bearing acting upwards through O.

Taking moments about O we have, for equilibrium,

$$P \cdot OA = Q \cdot OB + W \cdot OF.$$

But $OA = OB$ for the arms of the balance are equal and the triangles AOD, BOE are similar.

Hence the equation may be written $(P-Q)\,OA = W.\,OF$.

If θ be the angle which the beam makes with the horizontal, then the angles AOD and OGF each equal θ, and if l be the length of either arm of the balance, we can write

$$OA = l \cos\theta \quad \text{and} \quad OF = OG \sin\theta.$$

Substituting these values in the above equation, we have

$$(P-Q)\,l \cos\theta = W.\,OG \sin\theta$$

or
$$\frac{\tan\theta}{P-Q} = \frac{l}{W.\,OG}.$$

Up to about 8°, and we shall have no deflection greater than this, $\tan\theta$ is very nearly equal to θ, so we may write

$$\frac{\theta}{P-Q} = \frac{l}{W.\,OG}.$$

$\dfrac{\theta}{P-Q}$, the angular displacement per unit difference of weight, is a measure of the sensitivity of the balance, and it is clear that this is great in proportion as l is great and W and OG are small.

However, since it is found that the shorter the beam, the more quickly it swings, it is important that its length should not be too great; otherwise, it will prove too sluggish. Hence a balance is usually made sensitive by (i) constructing the beam like a girder so as to combine lightness with stiffness, (ii) decreasing the distance of the C.G. of the beam below the central knife edge. Some balances are provided with a small brass weight carried on a fine screw above the centre of the beam. By raising or lowering this weight, the height of the C.G., and hence the sensitiveness, of the balance can be adjusted as required.

94. Equilibrium of a Floating Body. We may look upon a body floating freely as under the action of two resultant forces: these are (1) the weight of the body, equivalent to a single force through the C.G. of the body, the resultant of the distributed

forces which make up the weight of the body, and (2) the buoyant force of the water; this again is the resultant of a distributed force, the pressure of the water on the bottom ends and sides of the floating body. We need not for the present consider how the pressures of the water exerted in different directions combine together to form one single force, but it is clear that they are together equivalent to a single force equal and opposite to the resultant weight of the body. If there is any doubt about it, the following experiment will be conclusive.

Set a tank, not quite full of water, upon a table balance of the kind illustrated in Fig. 5. Suspend a block of wood from another balance. The upper balance indicates the weight of the wooden block; the lower balance indicates the weight of the tank and its water. Now lower the block of wood gradually into the water, noting the readings of the two spring-balances during the process (Fig. 117). The upper balance indicates less and less weight, until the block is wholly supported by the water; the lower balance indicates more and more weight until the block is wholly supported by the water, and then the increase of weight indicated by the lower balance is equal to the decrease of weight shewn by the upper balance, that is to say the weight of the block of wood. The weight of the block, which was formerly balanced by the vertical pull of the string, is now balanced by the equal vertical thrust of the water. The reaction of the block is an equal and opposite downward thrust upon the water, and this thrust on the water is indicated by the difference between the first and last readings of the lower balance.

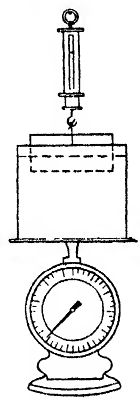


Fig. 117.

It is also evident that the resultant buoyant force of the water acts in the same vertical line as the weight of the block, for otherwise the block would be subject to an unbalanced couple which in fact it is not.

When a body floats in water, that part of the body which lies below the level of the water displaces an equal volume of water, and we may shew by an experiment that the weight of this volume of water displaced by the submerged part of the floating body is equal to the weight of the whole floating body. A very direct experiment on this question is the following:

Into a tank which has in one side a small spout, pour water until this just overflows from the spout. When the water is quite at rest, and has ceased overflowing, lower a large block of wood into the tank, and catch the water which in consequence overflows from the spout. It will be found that the weight of the block of wood is equal to the weight of water which is displaced by the block and overflows.

It may be interesting to consider the submerged part of the floating body and the equilibrium of the water which exactly fills this space when the floating body is taken out of the water. The shapes are the same, therefore the pressures exerted by the surrounding water are the same in the two cases; therefore since the same total pressure is required to support each, the floating body must weigh the same as the water which fills the space formerly occupied by the submerged part of the floating body.

95. Stability of a floating body. So far we have considered only a body floating freely, and we have two conclusions:

(1) that the buoyant force of the water acts vertically upwards through the C.G. of the floating body,

(2) that the weight of the floating body is equal to the weight of the water displaced.

In order that we may consider the stability of the floating body, it will be necessary to consider the effect of the buoyancy of the water when the floating body is forcibly displaced from its natural position of rest, when it is therefore not in equilibrium under the actions alone of its weight and the force of buoyancy of the water, and when therefore the buoyant force of the water is not vertically upwards through the C.G. of the floating body.

Consider for example a piece of wooden plank held half submerged, with its side making an angle of say 45 degrees with the horizontal. We know that a plank will not float freely in this position, and that in fact if it is let go it will turn and float on its face. We want to determine the direction of the buoyant force of the water when the plank is held obliquely in the water.

By an experiment similar to that illustrated in Fig. 117, we can shew that in this case also the buoyant force of the water is vertically upwards and equal to the weight of the volume of displaced water, but this does not tell us the line of action of the buoyant force.

Let us consider the equilibrium of the water which takes the place of the obliquely submerged plank when that is removed; this water, of course, floats in equilibrium, so we can say that the buoyant force of the outer water upon this water acts vertically upwards through the C.G. of this water which took the place of the submerged portion of the plank. So we conclude that upon any submerged body whatever, whether in equilibrium or not, the buoyant force of the water acts vertically upwards through the C.G. of the displaced water. This expression, 'the C.G. of the displaced water,' is in common use; a more accurate expression would be 'the centroid of the submerged volume.'

In the light of what we have just said, we will examine two positions of a floating plank: (1) when the plank is on its face, the ordinary position in which it does float in stable equilibrium, and (2) when the plank is on its edge, in which position it is in unstable equilibrium.

Fig. 118 shews the plank floating upon its face, but slightly displaced from its position of equilibrium. (For the sake of simplicity we have illustrated a plank whose density is half that of water, and which, therefore, floats half submerged. We have also shewn the plank displaced so that the diagonal plane DE is at the water surface.)

When the plank is let go it is under the action of two forces: (1) the weight of the plank vertically downwards through O, the

C.G. of the plank, and (2) the buoyant force of the water vertically upwards through the C.G. of the 'displaced water,' that is to say B, which lies in the median OA, OB being one-third of OA. Evidently these two forces form an unbalanced couple which will

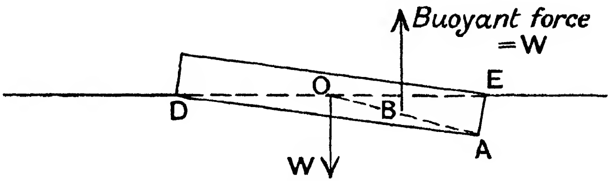


Fig. 118.

turn the plank back towards its original position of equilibrium; just what we should expect, knowing that the plank was in stable equilibrium on its face.

Fig. 119 shews the plank floating upon its edge, but slightly displaced from the vertical position. It is evident from the figure that the C.G. of the 'displaced water,' B, lies to the left of the vertical line through O, and that consequently the weight of the

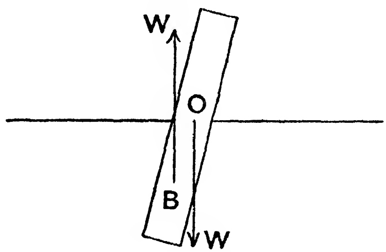


Fig. 119.

plank and the buoyant force of the water form together an unbalanced couple which will turn the plank farther from the vertical position, in which position we know that it was in fact unstable.

In future we shall call the centre of gravity of the 'displaced water' the **centre of buoyancy,** for the resultant buoyant force always acts in a vertical line through this point.

96. Stability of a ship. If a boat or ship, when floating upright in still water, is made to heel slightly and is then released, it rolls to and fro, but always comes to rest again in an upright position. It is therefore in stable equilibrium. To understand why this is so, and to ascertain upon what the stability depends, we must examine the forces acting on the ship in its inclined position. Fig. 120 represents a ship when heeled through an angle θ. LL represents the original, and $L'L'$ the new, water-line; G is the centre of gravity of the ship; B is the position in the ship of the centre of buoyancy when the ship is on an even keel. The centre of buoyancy in the inclined position is B'. The buoyant

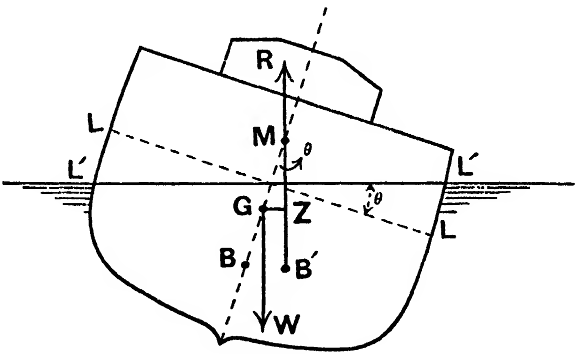


Fig. 120.

force is still equal to the weight of the ship but it now acts vertically upwards through B'. Let the vertical line through B' intersect the original vertical line through B in M. For stable equilibrium M must be above G, for only so can the weight W of the ship acting downwards through G, and the buoyant force R acting upwards through B', form a couple which tends to right the ship.

If GZ is drawn at right angles to $B'M$, the moment of this couple is given by $W . GZ$.

Now $GZ = GM \sin\theta$, for the angle GMZ is equal to θ, the angle of heel.

Hence, the moment of the righting couple is given by $W \,.\, GM \sin\theta$. Note that this moment, which is called the *moment of stability* for a heel of $\theta°$, is proportional to the distance GM.

Compare this with the example of the hemisphere resting on a table (Fig. 107 on p. 211). With the hemisphere we observe that the reaction of the table acts through M, the centre of the spherical surface, for all angles of heel up to 90°.

If the hemisphere were floating in water, the buoyancy would likewise act upwards through the same point M, but only as long as the angle of heel did not bring the edge of the hemisphere below the surface of the water. (Why?)

In the case of a ship it is found that the position of M is nearly constant for angles of heel up to about 15°. This important point is called the **Metacentre,** and the distance GM the **Metacentric Height.**

Let us now consider what happens when G is raised without altering the position of M. We can suppose this to be effected

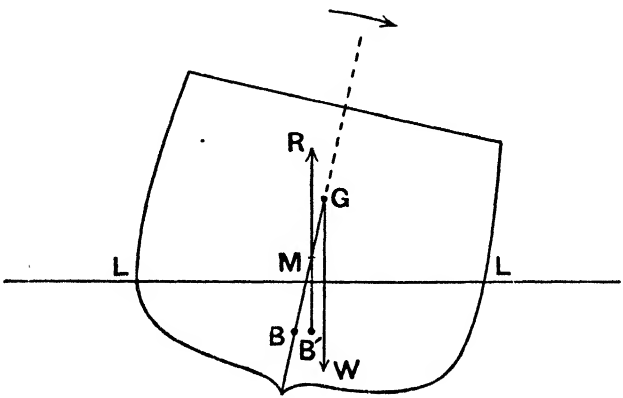


Fig. 121.

by bringing the cargo up on deck. As G is raised, the metacentric height GM becomes less and therefore the stability decreases also. If G is brought above M, then the ship becomes unstable, for, as shewn in Fig. 121, the couple now brought into play, on heeling, is such as to overturn the ship.

At the beginning of the preceding article we summed up the two conditions that must be satisfied so that a body shall float in equilibrium. In order that this equilibrium shall be stable, a third condition must be added, namely, *the centre of gravity must lie below the metacentre.*

97. Value of the Metacentric Height. Since the moment of the couple required to heel a ship through a small angle θ is given by $W.GM \sin\theta$, it is clear that to increase the 'stiffness' of a vessel we must increase the metacentric height GM. As already indicated, one way of doing this is to lower the centre of gravity. For instance, in the case of a sailing boat, G is brought low by putting lead on the keel, or by placing ballast in the bottom of the boat; in the case of a merchant ship, by storing the cargo and coal supply as low as possible. But GM can also be increased by raising the position of M.

We have said that in ships the metacentric height is nearly constant for small angles of heel. The magnitude of the couple which rights the ship when heeled over depends, for any given angle, upon the metacentric height. The greater the righting couple for a given angle, the 'stiffer' will the ship be, that is to say the shorter will be the periods of her rolls about the middle position. Stiffness does not necessarily mean safety however. The safety of a ship in bad weather conditions depends more upon M remaining above G for all angles of heel, even very large angles, than upon the magnitude of GM for small angles of heel.

98. Heeling of a sailing-vessel. Since an interesting application of the foregoing principles is afforded by the method used for calculating the capability of a vessel for carrying sail,

we will briefly consider the effect of a certain amount of sail in heeling a particular vessel having a given metacentric height. For this purpose, we consider the heeling effect of the wind pressure on the sail area assuming that the sails are braced fore and aft. The resultant force of the wind is assumed to act at the centre of gravity of the sail area which is denoted by E in Fig. 122. This point is called the *centre of effort* and its position is calculated as described in Art. 83.

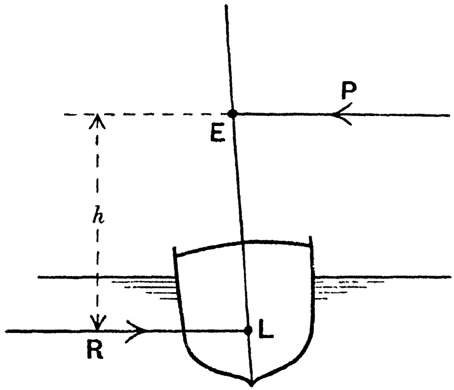


Fig. 122.

The horizontal force of the wind brings into play an equal reaction of opposite sense which is exerted by the water on the under-water body of the vessel. This reaction of the water on the lee side of the vessel is called the *lateral resistance*, and it acts through L, a point called the *centre of lateral resistance*. The depth of this point is usually taken as half the draught of the vessel.

The total force of the wind P and the lateral resistance of the water R thus form an inclining couple whose moment is given by $P.h$, where $h = LE$.

This moment is balanced by the moment of stability which we have shewn is given by $W.GM \sin\theta$ (Art. 96).

That is, $$W \cdot GM \sin\theta = P \cdot h,$$

or $$\sin\theta = \frac{P \cdot h}{W \cdot GM}.$$

For example, suppose the sail area of a ship of 1000 tons to be 5000 sq. ft., the height of the centre of effort above the centre of lateral resistance 30 ft., and the metacentric height 2 ft. What angle of heel may be produced when a fresh breeze is blowing?

If we reckon a fresh breeze to produce a force of 4 lbs. wt. on each sq. ft. of sail, we have

$$\text{Total force of wind} = 5000 \times 4 \text{ lbs. wt.} = \frac{20,000}{2240} \text{ tons wt.}$$

$$\text{Moment of inclining couple} = \frac{20,000 \times 30}{2240} \text{ tons ft.}$$

$$\text{Moment of righting couple} = 1000 \times 2 \sin\theta \text{ tons ft.}$$

Hence $$2000 \sin\theta = \frac{20,000 \times 30}{2240},$$

or $$\sin\theta = \frac{20,000 \times 30}{2000 \times 2240} = 0{\cdot}134.$$

That is, $\theta = 7^\circ\ 42'$.

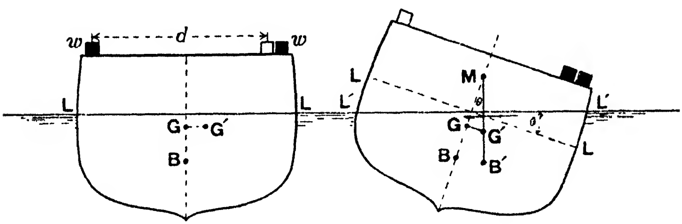
It is true that, as the ship heels, the effect of a horizontal wind pressure will not be so great as in the upright position, since the surface on which it blows is at an angle less than a right angle. The same applies to sails which are not braced fore and aft. In practice, therefore, the angle of heel will be somewhat less than that calculated. However, our reasoning is sufficiently correct to provide a useful working rule, and, moreover, the error introduced into our calculations will be on the side of safety.

99. Experimental determination of the Metacentric Height. The metacentric height of a ship is determined experimentally by carrying out what is known as the 'inclining experiment,' the main features of which are as follows. Equal loads of ballast are placed symmetrically one on each side of

the upper deck as shewn in Fig. 123 *a*. The ship being upright, the load on one side, of known weight w tons, is shifted across the deck through a measured distance of d feet. The effect of this on the centre of gravity of the ship is to shift it from G to G' parallel to the deck, and

$$GG' \times W = w \times d \qquad \text{(see Art. 85),}$$

where W tons is the total weight of the ship found by calculating its displacement. The ship heels over until the new centre of buoyancy B' is such that it lies vertically below the new centre of gravity G' (Fig. 123 *b*). The point M, where the vertical through B' cuts the original vertical through B, is the metacentre.



(*a*) Fig. 123. (*b*)

Now, in the right-angled triangle GMG', the angle GMG' is the angle of heel θ, and we see that

$$\tan\theta = \frac{GG'}{GM}.$$

Substituting in this equation the value of GG' obtained above, we have

$$\tan\theta = \frac{w \times d}{W \times GM},$$

or

$$GM = \frac{w \times d}{W \times \tan\theta}.$$

To find the value of $\tan\theta$, long plumb-lines are suspended down hatchways. Thus, if ab is the position of a plumb-line

when the vessel is upright (Fig. 124), and bc is a transverse scale, and ac is the position of the plumb-line when the ship is inclined, we have

$$\tan\theta = \frac{bc}{ab}.$$

In practice, several determinations of GM are made by shifting ballast across the deck in various quantities, both to port and to starboard, and the mean of the results is taken.

The value of GM thus obtained enables us to fix the position of G if the position of M is already known. As a matter of fact, the position of M for any ship can be calculated from its dimensions, but we cannot here enter into the principles and calculations involved. The calculation of the position of G, however, is much more laborious and uncertain, since for this purpose we require to know the weight of every item of the hull, machinery, armour, armament, equipment, etc., and the distance of the C.G. of each item above the keel line. Although in the case of a ship of new design this calculation is actually made, it is essential that the position of the C.G. of the ship so estimated should be verified; to do this is the main object of the inclining experiment.

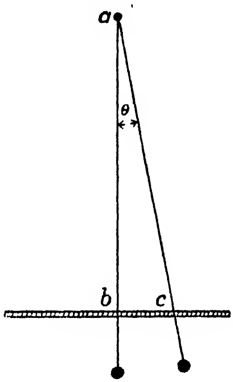


Fig. 124.

100. Experiments with a model. The following experiments have been designed to enable you to verify for yourself the principles of flotation we have discussed. A convenient wooden model for this purpose is that shewn floating in an inclined position in Fig. 125. It has a uniform external cross-section, a weight of about 36 lbs., a length of about 18″, and a breadth at the top of about 15″. The centre lines of the ends are graduated in inches from the bottom. On each side of

the centre there are slots in which 2-lb. flat iron weights can be clamped. A light plumb-line is suspended from a central mast below which a transverse scale is fixed.

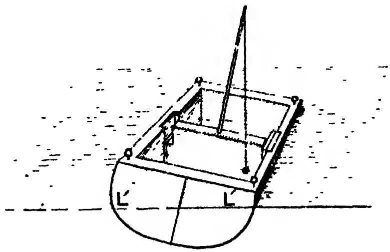


Fig. 125.

Exp. 1. *To calculate the displacement and to compare it with the weight of the model.*

Stand the model on its end on a large sheet of cardboard and run a pencil round this end, thus obtaining a section. After drawing in the centre line, cut out the section. This cardboard section, which is shewn in Fig. 126, will be required in the next three experiments.

Having clamped a 2-lb. weight at each side, float the model in a tank and see that it comes to rest in an upright position. Read off the draught at each end. Mark off on the cardboard section the mean draught and draw in the water-line *LL*. Determine, by Simpson's rule, or by the Engineers' rule, the area of the section below this line, *i.e.* the cross-section of the displacement. Measure the length of the model; calculate the volume, and hence the weight, of the water displaced.

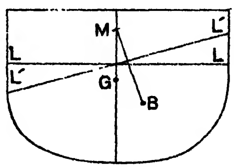


Fig. 126.

Find the weight of the model by suspending it from a spring-balance or by placing it on a weighing machine, and compare this weight with the calculated displacement.

Exp. 2. *To obtain G.*

Hang the model, by the two eyelets on one side, from a bar

supported horizontally on brackets, and at one end suspend a plumb-line from a ring on this bar. Read off the point where the plumb-line crosses the centre line of the end of the model. Repeat this by hanging the model up by the other side. Taking the mean of your measurements, mark on the cardboard section the position of G.

Exp. 3. *To obtain B', the centre of buoyancy when the model is inclined.*

Shift one of the 2-lb. weights from one side to the other, and float the model in water (Fig. 125). When it is at rest, obtain the position of the new water-line at one end by marking the points L', L' on the edge; and, by measuring the respective distances of these points from the top edge, mark these points on the cardboard section, and join them. With a sharp knife cut the cardboard along this line $L'L'$ (Fig. 126). Now find by experiment the C.G. of the lower portion. This point B' is the centre of buoyancy for the inclined position.

Exp. 4. *To obtain M, and hence GM.*

Fit together the two portions of the cardboard section. Now from B' draw $B'M$ at right angles to $L'L'$ to cut the centre line of the section in M (Fig. 126).

Measure GM, the metacentric height.

Exp. 5. *To obtain GM by inclining experiment.*

Carry out the 'inclining experiment' as described in the preceding article. Measure the transverse distance through which the C.G. of the 2-lb. weight has moved when shifted across into its new position beside the other weight.

Float the model. Measure on the scale the transverse displacement of the plumb-line and measure the distance of its point of support from this scale; then calculate the tangent of the angle of heel.

Calculate the value of GM.

Repeat this experiment after shifting the two weights to the other side.

Compare the mean of the values of GM so obtained with the value measured off the cardboard section in Exp. 4.

EXAMPLES VII.

1. If a body be suspended from any point, what condition must be fulfilled for equilibrium?

What further condition is necessary so that the equilibrium shall be (i) Stable; (ii) Unstable; (iii) Neutral?

2. Illustrate the meaning of stable, unstable, and neutral equilibrium from an egg resting on a table.

3. Draw diagrams shewing a triangular sheet of cardboard in positions of stable, unstable, and neutral equilibrium, when supported by a horizontal pin.

4. State, giving reasons, the nature of the equilibrium in the following cases:

(*a*) A pendulum.

(*b*) A ruler balanced on the edge of a knife.

(*c*) A ball in a hemispherical basin.

(*d*) A rocking-chair.

5. How is a compass-needle supported so that it is in stable equilibrium in a vertical plane?

What are the positions of stable and unstable equilibrium of the needle in a horizontal plane?

6. A body may be in different states of equilibrium at the same time according to the direction in which we suppose it to be displaced.

Give an example of a body which is at the same time in (*a*) stable and neutral equilibrium, (*b*) stable and unstable equilibrium.

7. If, on displacing a body from its position of equilibrium, the path of the C.G. is curved, the equilibrium is stable or unstable according as this path curves upwards or downwards. Illustrate by examples the truth of this statement.

What happens in the case of neutral equilibrium?

8. A body comes to rest with its C.G. as low as possible. In the light of this principle explain

(*a*) the construction of a spirit-level;

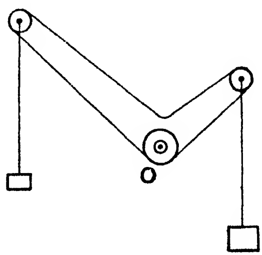
(*b*) the method of supporting a ship's compass.

9. If the line of the hinges of a door is not vertical, the door will tend either always to open or always to close. Why is this?

10. A straight rod rests with its lower end on the floor, and is supported in an inclined position by a string connecting its upper end to a staple in the ceiling, friction being sufficient to prevent the foot of the rod from slipping. If the staple and the foot of the rod are in the same vertical line, shew that for displacements round this line the equilibrium is neutral.

What are the positions of stable and unstable equilibrium for such displacements when the staple is not vertically above the foot of the rod?

11. (Fig. *a*.) Weights are suspended from the arms of a bent lever which is pivoted at *O*. If the lever is in equilibrium in the position shewn, explain why the equilibrium is unstable, even if the weight of the lever is negligible. Sketch the position in which the equilibrium would be stable.



(*a*)

12. A wooden disc or cylinder is loaded with lead near its circumference, and is placed on a plane which is slightly inclined. Shew that the cylinder can be so placed that, when released, it will begin to roll uphill, providing no slipping occurs.

Make a sketch of the cylinder on the plane shewing the position of the C.G. when it comes to rest.

13. A plumb-line consists of a $\frac{1}{2}$-lb. weight on the end of a light string 4 ft. long. If it is displaced from its vertical position through an angle of $20°$, what is the moment of the weight tending to restore it to its original position?

14. A uniform rod, 3 ft. long, is supported by a horizontal pin passing through it at a distance of 1 ft. from one end. What is the position of unstable equilibrium?

If the rod suffers a displacement of $10°$ from this position, what is the overturning moment, if the weight of the rod is 6 lbs.?

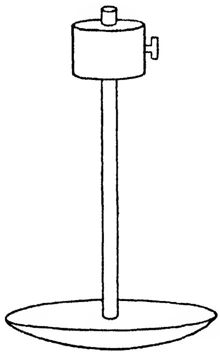
What weight must be fastened to the end of the rod to render its equilibrium neutral?

15. A uniform circular hoop hangs on a horizontal peg. If a weight equal to that of the hoop is now attached to it at one end of its horizontal diameter, sketch the position the hoop will assume (*a*) if the friction at the peg is sufficient to prevent slipping, (*b*) if there is no friction at the peg.

16. A uniform straight bar is hung up by two strings of equal length, one attached to each end of the bar, the other ends of the strings being fastened to the same fixed point. What position does the bar assume, and why?

If the bar when a weight has been attached to it comes to rest inclined at 20° to the horizontal, find (by drawing) the position of the C.G. of the loaded bar, given that the length of the bar is 4 ft., and the length of each string 3 ft.

17. A uniform straight rod, 5 ft. long, is hung up by two cords, one attached to each end of the rod, the other ends of the cords being fastened to a nail. If the lengths of the cords are 3 and 4 ft. respectively, find (by drawing) at what angle to the horizontal the rod will come to rest.



(*b*)

18. The arrangement shewn in Fig. (*b*) rests on a horizontal surface. If the base consists of a portion of a sphere, describe and explain the condition necessary for (i) Stable equilibrium, (ii) Unstable equilibrium, (iii) Neutral equilibrium.

19. A cylinder, which weighs 10 lbs. and has its C.G. 2″ from its axis, rests on its side on a horizontal surface. If the cylinder is rolled through an angle of 40°, and then released, what forces will be acting upon it, and what will be the moment of its weight tending to restore it to its original position?

20. A uniform hemisphere rests with its curved surface on a horizontal surface. Shew that, when tilted, the righting moment is proportional to the sine of the angle of tilt.

If such a hemisphere weighs 8 lbs., and its C.G. is 3″ below the centre of its flat surface, find the righting moment when the hemisphere is tilted through 30°.

21. What is meant by the *base* of a body standing on a plane surface? Sketch the base in the case of

(*a*) A motor cycle with side car.

(*b*) A standing screen consisting of three parts hinged together.

(*c*) A glass tumbler standing mouth downwards on the edge of a circular table with part of it projecting over the edge.

22. When a cyclist feels himself falling to one side, in which direction does he turn his front wheel, and why?

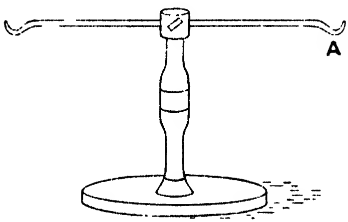
23. In balancing a sword on the end of your finger, why is it easier to maintain a balance when the sword is on its tip than on the end of its hilt?

24. When is a body said to be 'top-heavy'?

Explain the advantages resulting from loading the base of a candlestick or lamp with lead.

25. Fig. (*c*) represents a stand having a circular base of 4-in. radius. The weight of the stand is $1\frac{1}{2}$ lbs. and its C.G. lies on the central vertical axis.

What weight suspended from *A*, distant 7 ins. from the central axis, will just be sufficient to upset the stand?



(*c*)

26. A square table, weighing 100 lbs., stands on four vertical legs situated at the middle points of its sides.

What is the least force which, acting vertically downwards on one corner, will cause the table to tilt?

27. In a travelling crane (p. 216) the weight of the truck alone is 10 tons; the weight of the revolving platform and everything upon it is 8 tons and its C.G. is distant 6 ft. from the axis of rotation, which passes through the centre of the truck. On a circle of what radius does the C.G. of the whole crane lie, when the jib is slewed round, its inclination remaining constant?

28. A travelling crane (p. 216), having a total weight of 24 tons, raises a load situated midway between the rails and at a distance of 20 ft. from the front edge of the wheel base.

On what vertical line must the C.G. of the whole crane lie in order that it may raise a load of 6 tons without upsetting?

29. A uniform cylinder, of height 8″ and diameter 6″, stands on a table. If the weight of the cylinder is 4 lbs., what horizontal force applied to the top will just start it tilting, if no slipping occurs?

Through what angle can it be tilted before it topples over?

30. A uniform rectangular block of wood, height 2 ft., stands on a plank with an edge of the block, whose length is 9 ins., parallel to the edge of the plank. Through what angle can the plank be tilted before the block upsets, providing no slipping takes place?

31. A round table, with top 30″ in diameter, is supported on three feet which form the angular points of an equilateral triangle, having a side of 16″. The weight of the table is 25 lbs.

Find (*a*) the *least* weight which placed on the edge of the table will upset it; (*b*) the *greatest* weight which may be placed on the edge of the table which will just not upset it.

32. A circular table is supported by three vertical legs spaced at equal distances round the edge of its top.

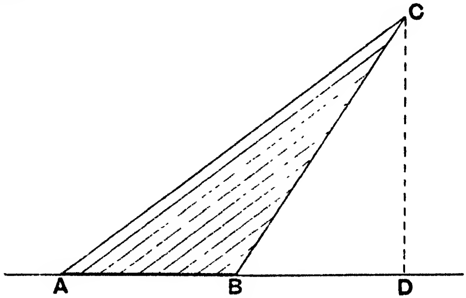
The weight of the table is 30 lbs., its height 27″, and the diameter of the top 28″.

Find the *least* horizontal force which, applied to the top, will just upset the table providing no slipping takes place.

33. A pedestal, 3 ft. high, has a circular base of radius 9 ins. It is found that a horizontal force of 36 lbs. wt. applied to the top is just sufficient to start it tilting. What is the weight of the pedestal?

When the pedestal is tilted through 35° it is found to be in unstable equilibrium. What is the height of the C.G. when the pedestal is upright?

34. In Fig. (*d*), *ABC* represents the section of a prism of uniform material,



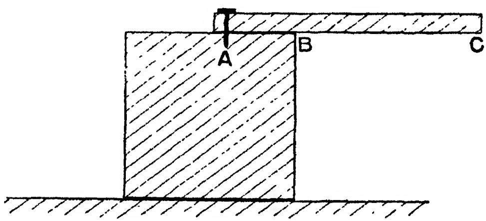
(*d*)

resting with its base AB on a horizontal surface. Shew that the prism will be on the point of upsetting when the horizontal overhang BD of the vertex is equal to the base AB.

35. A uniform circular hoop is laid flat on a circular table with so much projecting over the edge that the hoop is on the point of falling over. If the radius of the hoop is 1 ft. and the radius of the table 20 ins., what is the distance between the centre of the hoop and the centre of the table?

36. The wheels of a dogcart are 4 ft. apart and the C.G. is 5 ft. above the ground. To what height may the wheel on one side rise up a bank before the dogcart upsets?

37. A uniform cubical block, having an edge of 2 ft., rests on the floor (Fig. *e*). A bar AC, fastened to the top by a single nail at A, projects over the edge. The weight of the block is 60 lbs., the distance AB is 8 ins., and the distance BC is 18 ins. Neglect the weight of AC.



(*e*)

If a downward force is applied at C sufficient to just tilt the block, what is the force tending to extract the nail?

38. A uniform cone, having a base of diameter 6 ins., stands on a horizontal board, $2\frac{1}{2}$ ft. long. It is found that when one end of the board is raised through a vertical distance of $1\frac{1}{2}$ ft., the cone is on the point of upsetting?

At what distance from the board is the C.G. of the cone?

39. A cricket bowling screen is 15′ high and 40′ wide, and stands on wheels extending 5′ on either side of the screen. When the wind pressure on the screen is 8 lbs. wt. per sq. ft., the windward wheels are seen to be just lifting.

What is the weight of the screen?

40. A uniform symmetrical solid stone column consists of a cubical base having an edge of $2\frac{1}{2}$ ft., surmounted by a cylinder of height 8 ft. and diameter $1\frac{1}{2}$ ft.

Calculate the height of the C. G.

Through what angle can the column be tilted about one edge of its base before it topples over?

To upset the column, a rope is attached to the top and pulled in a direction inclined at 45° to the ground. What pull will start the column tilting, given that a cubic foot of stone weighs 144 lbs.?

41. A rectangular tank (without a lid) containing some water rests on the floor and is gradually tilted by raising one edge. Neglecting the weight of the tank, find the angle through which it can be tilted without falling over. Does it depend on the quantity of water put in?

42. A book is placed on an equal book, which projects over the edge of a table, the ends of the books being parallel to the table edge. What is the position of the books when the upper one projects beyond the table edge as far as possible?

43. In a pile of equal note-books, each 8 ins. long, the end of each book projects $\frac{1}{2}$ inch beyond the end of the one below it. How many books will the pile contain when it is just on the point of upsetting?

44. A uniform cylinder, the height of which is equal to its diameter, stands on a board which is inclined at 30° to the horizontal. If the weight of the cylinder is 10 lbs., find the force which must be applied to its top, parallel to the board, just to cause it to tilt (a) down the plane, (b) up the plane, assuming that sliding is prevented.

45. How many pennies can stand in a cylindrical pile on a plane inclined to the horizontal at an angle, the tangent of which is 0·4, if there is no slipping?

The thickness of a penny is $\frac{1}{18}$th of its diameter.

46. A lamp, which is symmetrical about its central vertical axis, stands with its circular base on a table. The weight of the lamp is 8 lbs., the diameter of its base is 7 ins., and the coefficient of friction for the contact surfaces of the base and the table is 0·25.

An increasing horizontal force is applied to the lamp at a height of 10 ins. above the table until motion results.

Which will the lamp do first, tilt or slide?

At what height must the force be applied so that the lamp will begin to tilt and slide at the same moment?

47. A cubical block of uniform material, having a weight of 120 lbs. and an edge of 3 ft., is pulled at uniform speed along a horizontal plane by a horizontal force applied to the middle of one of the top edges and at right angles to it. If the coefficient of friction is 0·2, what are the forces acting on the block?

Through what point does the total normal reaction of the plane on the block act?

48. On applying a steadily increasing horizontal force to the middle of one of the top edges of a cubical block of uniform material, it is found that the block begins to tilt and slide at the same moment. What is the coefficient of friction?

49. State and explain the chief considerations in the construction of a good chemical balance.

50. What is meant by the *sensitiveness* of a balance, and upon what does it depend?

51. A chemical balance, when accurately adjusted, is found to come to rest with its beam inclined at an angle of 2° to the horizontal when the loads in the two pans differ by 0·5 milligram. If the length of each arm of the balance is 4 ins., and the weight of the beam is 50 grams, how far is the C.G. of the beam from the central knife edge?

52. A flywheel weighs 6 tons and is *out of balance.* It is balanced by attaching to the wheel a weight of 80 lbs. at a distance of 25″ from the axis. How far was the C.G. of the flywheel from the axis?

53. A flywheel weighs 8 tons and its C.G. is $\frac{1}{20}''$ from the centre of the shaft. What weight at a radius of 2′ will be required to balance the flywheel?

54. A ship, when floating in sea water, displaces 70,000 cubic feet. Under the action of what two forces is the ship in equilibrium?

1 cubic foot of sea water weighs 64 lbs.

55. Explain the terms *displacement, centre of buoyancy, metacentre, metacentric height,* of a ship.

56. What are the two conditions necessary for a body to float in equilibrium?

What further condition is necessary to ensure that this equilibrium shall be stable?

57. Explain why a test-tube will not float upright in water.

Explain also why it can be made to do so by loading it with shot.

58. Shew that a uniform cylinder floating on its side is in neutral equilibrium.

For what displacements is the equilibrium of the cylinder stable, and why?

59. If a sailing-boat is found to be not 'stiff' enough, how could this be remedied?

60. A uniform wooden hemisphere is floating in water.

Where is the metacentre?

Shew that, when inclined, the 'righting' moment is the same as when the hemisphere rests on a horizontal surface.

Shew also that this is true only so long as the edge of the floating hemisphere is above the surface of the water.

61. What is meant by saying that a vessel has (*a*) a 'righting,' or (*b*) a 'capsizing,' moment?

A vessel of 1000 tons has a metacentric height of 2 ft.

What is the righting moment (or moment of stability) when this vessel heels through an angle of 10′?

62. The moment of the couple required to heel a certain battleship, of 22,000 tons displacement, through an angle of $3\frac{1}{2}°$ is found to be 4697 tons ft.

What is the metacentric height?

63. A ship of 4000 tons has its C.G. 18′ above the keel. 200 tons of additional ballast are added so that the C.G. of this ballast is 3′ above the keel. Assuming that the position of the metacentre remains unaltered, find the effect produced on the metacentric height.

64. What features in the design of a vessel are most important in influencing the metacentric height?

65. What opposing conditions have to be considered in settling the metacentric height of a warship?

66. A vessel of 5000 tons has a metacentric height of 2 ft.

Plot the stability curve of the vessel for angles of heel up to 15°, that is, plot the value of the righting moment in terms of the angle of heel.

67. Explain how the equilibrium of a submarine, when totally submerged, is stable.

Shew also that the transverse and longitudinal stabilities are the same in this case.

68. Why is the transverse stability of a submarine less when floating on the surface than when submerged?

69. A three-masted schooner, 2000 tons displacement, has a metacentric height of 2 ft. The sail area is 10,000 sq. ft. and the height of the centre of effort above the centre of lateral resistance is 50 ft.

Assuming that the sails are braced fore and aft, and that the wind produces a pressure upon them of 3 lbs. wt. per sq. ft., calculate the approximate angle of heel produced.

70. Describe the 'inclining experiment.'

Explain clearly the steps in the calculation of the metacentric height from the results of the experiment.

71. A load of 80 tons is shifted 60′ across the deck of a battleship, of 24,000 tons displacement.

What is the effect on the position of the C.G.?

72. In an inclining experiment on a certain warship of 14,800 tons displacement, it was found that the shifting of a load of 90 tons (already on board) across the deck through a distance of 50′ caused an angle of heel of 5°. Calculate the metacentric height.

73. A vessel, of 1800 tons displacement, is inclined by shifting 6 tons transversely across the deck through 20′.

The end of a plumb-line, 16′ long, moves through 6″. Determine the metacentric height.

74. In a vessel of 7200 tons, a 'boom-boat,' weighing 16 tons, is hoisted out by a derrick through a transverse distance of 40′. Calculate the transverse displacement of the C.G.

If the metacentric height of the vessel is 2′, calculate the angle of heel produced.

CHAPTER VIII

FORCE-TRIANGLE

101. The lines of action of three inclined forces meet in a point. The greater part of this chapter will be devoted to the consideration of the equilibrium of bodies under the action of three forces, the lines of action of which are inclined to one another.

The first condition which these forces must satisfy can be proved as follows.

Let the rigid body in Fig. 127 be in equilibrium under the action of three inclined forces P, Q, and R.

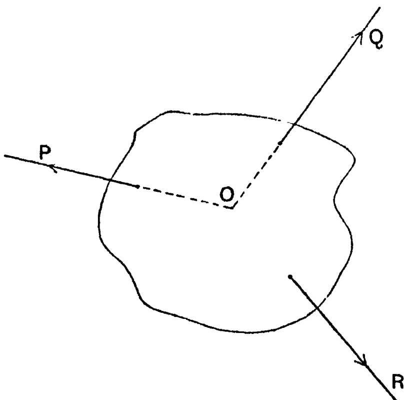


Fig. 127.

By the principle of moments, we know that, if we take moments of these forces about any point in the plane in which they act, the sum of the clockwise moments must equal the sum of the counter-clockwise moments; or, in other words, the algebraical sum of all these moments must be zero (Art. 60).

Produce the lines of action of two of the forces, P and Q, to meet in O, and consider the moments of all the forces about this point. Since the lines of action of both P and Q pass through O, the moment of each of these forces about this point is zero. This leaves us with the moment of R, and, since the sum of all the moments must be zero, this must be zero also. But the moment of R about O can only be zero if its line of action passes through this point, which accordingly it must do.

We can therefore state: *If a body is in equilibrium under the action of three forces, which are not parallel, the lines of action of the forces meet in a point.*

In our next experiment we shall find that this condition is satisfied (Art. 103).

Note that, since we are confining our attention in this book almost entirely to forces acting in one plane, we have considered the above forces as so acting. As a matter of fact, it is a further essential condition for equilibrium that the lines of action of the three forces should lie in one plane. How would you verify this by experiment?

At the same time we may add that neither of the above conditions, which apply to three forces, is essential in the case of a body in equilibrium under the action of four or more forces.

One instance of the practical significance of the condition proved above is this: if we know the lines of action of two of

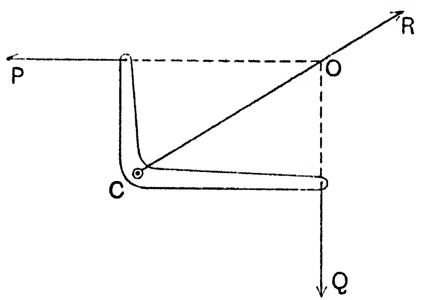


Fig. 128.

the three forces, which produce equilibrium, we can find the line of action of the third, providing we know its point of application. For example, consider the equilibrium of the bell-crank lever in Fig. 128. The lever is pivoted at C, and its arms are pulled, in the directions shewn, by the forces P and Q. If the weight of the lever is negligible, the only force acting upon it, in addition to P and Q, is the reaction of the pivot acting through C. To find the direction of this reaction, we produce the lines of action of P and Q to meet in O. Now, since the lines of action of all three forces must meet in a point, the reaction R of the pivot must act through O, and hence its line of action is given by CO.

102. Representation of a force by a straight line. In drawing the position-diagram of the forces acting on a body, it has been our custom to represent each force in *direction* by a straight line, and in *sense* by an arrow-head placed on this line. But to specify a force completely we must also give its *magnitude*. This information about a force we can also convey on paper by the same line as that which indicates its direction, by cutting off a measured length of the line to represent, on any convenient scale, the number of lbs. wt. in the force. For instance, if we take 1 inch to represent 1 lb. wt., a force of 3 lbs. wt. will be represented in magnitude by a straight line 3 inches long; on a scale of 1 cm. to 1 lb. wt., a straight line, 10·3 cms. in length, can be employed to represent a force of 10·3 lbs. wt. In Fig. 129, the line xy, 1·2 ins. long, represents on a scale of 1 inch to 10 lbs. wt., a force of 12 lbs. wt. acting in a direction making an angle of 35° with the horizontal, the arrow-head indicating that the force acts from x to y.

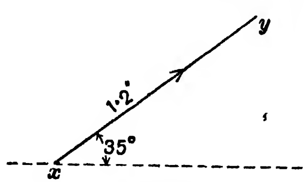


Fig. 129.

The beginner must be careful always to bear in mind that the straight line is merely a symbol for the force, designed to present in a graphic form what we know

about that force. It is advisable therefore at first to avoid such abbreviations as 'the force xy' and to use always the full expression 'the force represented by xy.'

In drawing any force-diagram, that is, a diagram in which lines are used to represent forces *both in magnitude and direction*, we shall as a rule draw the diagram quite separate from the diagram of the body on which the forces act. The object of this is to prevent any risk of confusing lines which represent forces with lines which do not.

103. Practical demonstration of the Principle of the Triangle of Forces. We will now describe an experiment with a body which is in equilibrium under the actions of three inclined forces, the object of the experiment being not only to verify that the lines of action of the forces meet in a point, but to demonstrate the law governing the relative magnitudes of the forces.

Exp. 1. Set up the apparatus shewn in Fig. 75 *a* on p. 133. Instead of a sheet of cardboard, however, it is better to use a sheet of glass having its upper surface ground or frosted, and having a number of holes drilled through it at various points near its edge. Place the sheet of glass on the steel balls which rest on the glass top of the table, and level the table carefully.

Notice that the body is quite free to move horizontally, for its weight and the reaction of the balls are vertical forces which have no direct effect in a plane at right angles to them; their indirect effect, namely, rolling friction, is also negligible. If we now maintain the body in equilibrium by applying to it horizontal forces, we may disregard the balanced vertical forces, and may consider the equilibrium as brought about by the horizontal forces only.

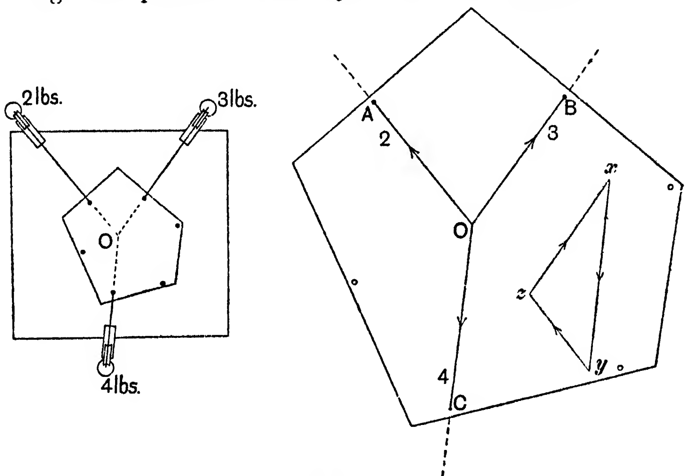
Attach *three* strings to the sheet of glass by passing hooks, fastened to their ends, through any three of the holes. Pass these strings over pulleys clamped to the edge of the table and attach to their lower ends any three weights which will keep the

body at rest. You will find that this is always possible provided that any two of the weights selected are together greater than the third.

Allow the sheet of glass to take up its position of equilibrium, being careful to adjust the plane of each pulley wheel so that the string enters its groove in this plane.

Make certain that the friction of the pulleys is negligible by displacing the body slightly in various directions and observing that it always comes to rest in the same position, when released.

Fig. 130*a* presents a bird's-eye view of the apparatus. Now,



(*a*) Fig. 130. (*b*)

placing a ruler alongside each horizontal string in turn, produce the directions of these strings on to the surface of the glass, thus obtaining the lines of action of the three pulls.

You will find in all cases that these lines of action intersect at one point, thus verifying the condition which we have already proved necessary for equilibrium (Art. 101).

Mark against each line the magnitude of the pull which acts along it, and remove the sheet of glass.

Suppose that the weights selected are 2, 3 and 4 lbs., so that the respective forces acting on the body are 2, 3 and 4 lbs. wt. The lines of action of these forces are indicated by OA, OB and OC in the position-diagram in Fig. 130*b*.

Now make the following construction on the surface of the glass, separate from the position-diagram. Draw a line parallel to OC and, taking a scale of 1 inch to 1 lb. wt., mark off along this line a length xy of 4 inches. xy will represent in magnitude and direction the force of 4 lbs. wt. Indicate the sense of this force by an arrow. Through the ends of this line xy draw lines parallel to the directions of the two remaining forces. Thus, in the figure, yz is drawn parallel to OA, and zx to OB. These lines intersect in z and a triangle xyz is obtained.

Place arrows on yz and zx to represent the senses of the forces of 2 and 3 lbs. wt., acting in these directions; note that *the three arrows follow one another round the triangle.*

Measure yz. You will find that its length is 2 inches. yz therefore represents the force of 2 lbs. wt., not only in direction, but also in magnitude.

Similarly, on measuring zx, you will find that its length is 3 inches, and that this line therefore represents fully the force of 3 lbs. wt.

The triangle xyz, the sides of which represent the three forces in magnitude, direction and sense, is called the *force-diagram* or the *triangle of forces.*

It should be noted that this triangle can also be constructed by drawing yz parallel to OB, and zx parallel to OA.

Draw the triangle in this manner, and shew that, in this case also, the sides represent in magnitude the forces to which they are respectively parallel.

This experiment should be repeated, first using the same weights and taking a different scale on which to represent the forces, and then with other sets of weights.

You will find that similar results are obtained in every case.

From these experiments we arrive at the following conclusions:

If a body is in equilibrium under the action of three inclined forces, and straight lines are drawn parallel to the lines of action of these forces, so as to form a triangle, then the sides of this triangle will represent, on some scale, the magnitudes of the forces; or in other words, the lengths of the sides of the triangle will be proportional to the forces.

Also, if the senses of the forces are indicated by arrows placed on the sides, the arrows follow one another round the triangle.

We will call this the *Principle of the Triangle of Forces.*

Seeing that any three inclined forces which keep a body in equilibrium can be represented by the sides of a triangle, it is clear that the sum of the magnitudes of any two of the forces must be greater than that of the third force which balances them, for any two sides of a triangle are together greater than the third side.

What are the directions and senses of three forces producing equilibrium, when the sum of the magnitudes of two of the forces is equal to that of the third?

Many problems dealing with the equilibrium of bodies under the action of three forces may be conveniently solved by the application of the above principle. It is clear that we only need sufficient data to enable us to draw the force-diagram for the forces acting on the body in any given case, since this diagram, when drawn, gives us the relative directions and magnitudes of all the forces.

104. Miscellaneous exercises and experiments on the Triangle of Forces. *To find the tensions in two inclined cords supporting a load.*

Exp. 2. Fasten three cords to a ring O (Fig. 131). Attach the ends of two of these cords to spring-balances which are suspended from fixed supports at A and B. On the end of the third cord hang a weight of 7 lbs.

Now use the principle of the triangle of forces to determine the tensions in the cords OA and OB.

To do this, consider the equilibrium of the ring. The weight of this ring is so small in comparison with the other forces acting upon it, that we may neglect its effect. With this proviso, the only forces acting on the ring are the pulls of the three cords; and we know that the pull of the vertical cord is 7 lbs. wt.

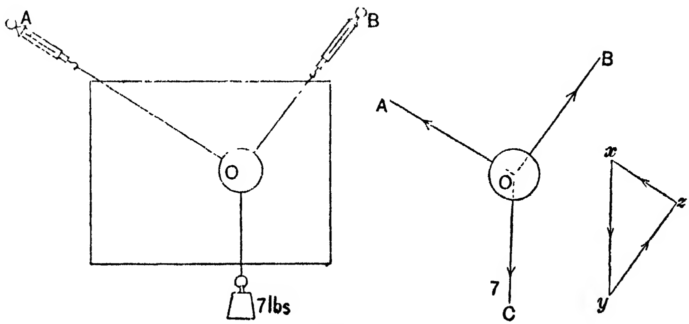


Fig. 131.

To obtain the directions of these forces, pin a sheet of paper to a board placed vertically behind the cords and mark upon it two points under each string, taking these points as far apart as possible. Remove the paper and draw straight lines through these pairs of points. If this is done accurately these straight lines will be found to meet in a point. You will thus obtain the position-diagram giving the lines of action of the forces. To draw the force-diagram, choose a convenient scale, such as 1 cm. to 1 lb. wt., and, drawing a line parallel to OC, mark off on this line a length xy of 7 cms., to represent the known vertical force of 7 lbs. wt. Indicate by an arrow on this line the sense of the force.

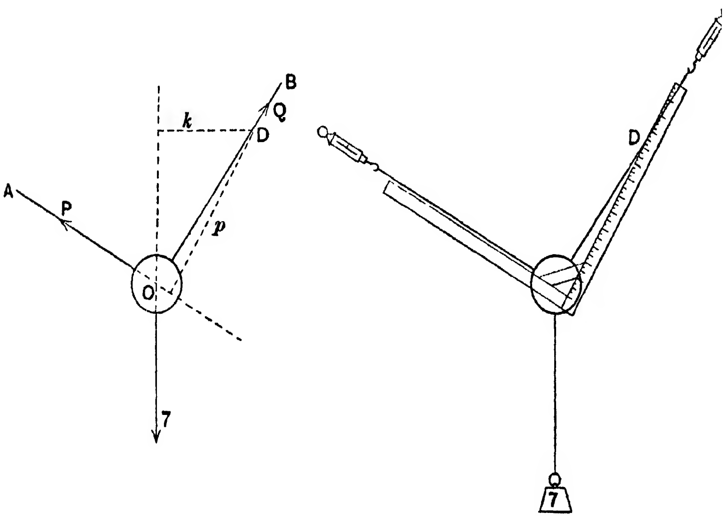
Through the ends of this line draw lines parallel to OA and OB respectively so as to form a triangle xyz.

On yz and zx place arrows so that they follow the first arrow round the triangle.

Measure yz and zx. The lengths of these sides in cms. represent in lbs. wt. the magnitudes of the pulls of the cords OB and OA, acting on the ring. The values of these pulls also measure the tensions in these cords respectively, for these pulls are equal and opposite to those which the ring exerts on the cords. Your results will be found to agree with the readings of the balances at B and A respectively.

105. Alternative solution using the Principle of Moments. To keep in mind the application of the principle of moments, and to illustrate its agreement with that of the triangle of forces, you are advised to find the tensions in the cords OA and OB by applying this principle also. Proceed as follows.

Exp. 3. Denote by P lbs. wt. the tension in OA, and by Q lbs. wt. the tension in OB. Consider the forces acting on the ring (Fig. 132a), and take moments of all these forces about some point D in OB. Since D lies in the line of action of Q, the



(*a*) Fig. 132. (*b*)

moment of this force is zero, and you have

$$P \times p = 7 \times k,$$

where p and k are the perpendiculars drawn from D to the lines of action of the forces P and 7 lbs. wt. respectively.

These perpendiculars can be drawn and measured on the position-diagram; or may be found directly from the apparatus with sufficient accuracy by marking a point D on the cord OB with a pin, and by measuring the perpendiculars from this point to the directions of the other cords with a large set square consisting of two scales fastened at right angles to one another. This will be understood from Fig. 132*b*, which shews how the set square is held to measure the perpendicular p. Measure the perpendicular k in the same manner and hence work out the value of the force P.

Similarly by taking moments about some point in the line OA, find the value of Q.

The results thus obtained will be found to agree with those obtained by applying the principle of the triangle of forces.

106. Jib Crane. The principal parts of a crane are the jib AB, the post AC, and the tie-rod BC (Fig. 133).

The jib is a beam pivoted at A to the lower part of the post.

The post is mounted on a horizontal table which can rotate,

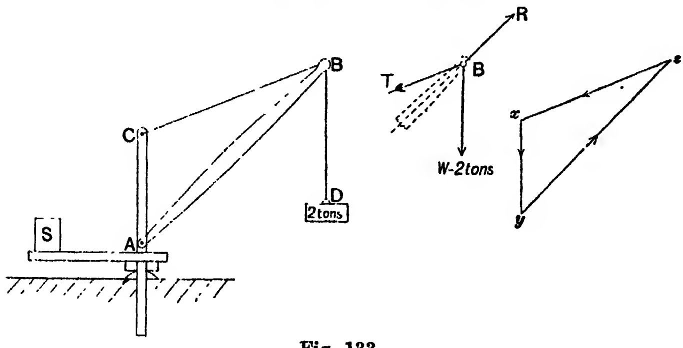


Fig. 133.

so that the crane can hoist a load out of a ship, and then, turning through an angle, lower it on to the wharf.

The tie-rod supports the jib by connecting its upper end to the top of the post. S is a heavy weight which helps to counterpoise the weights of the load and the jib.

The load is hoisted at the end of a chain which passes over a pulley at the end of the jib down to a winch. In the diagram the winch is omitted and, for simplicity, the load is considered to be suspended from the pin B.

Suppose this load to be 2 tons. Our purpose is to shew how to determine the stresses set up in the jib and the tie.

Assuming that we can neglect the weights of these members, we see that they are kept in equilibrium only by the forces exerted upon them by the pins at their ends and hence the stresses in them must be in the directions of their lengths (Art. 18, p. 26), and it is clear that the jib is in compression and the tie in tension. These stresses will be known when we know the force with which the jib pushes, and the force with which the tie pulls, on the pin B.

Our best plan therefore will be to consider the equilibrium of the pin B. The forces acting on this pin are (1) 2 tons wt. acting vertically downwards, (2) the pull T of the tie acting in the direction BC, (3) the thrust R of the jib acting in the direction AC.

The relative directions of these forces are shewn in the position-diagram, which can be drawn to scale when the angles CBA and DBA have been measured.

We now proceed to draw xyz, the triangle of forces, as before.

The lengths of yz and zx in inches give us the magnitudes in tons wt. of the thrust R of the jib and the pull T of the tie respectively. Since these forces are equal and opposite to those which the pin exerts on the jib and the tie, the values of R and T give us the stresses in these members.

Exp. 4. Use a model crane such as that shewn in Fig. 134. This model is provided with spring-balances by means of which

the stresses in the jib and tie can be read off. It is also constructed so that the inclinations of the jib and tie can be varied.

Suspend a known weight from the end of the jib, and adjust the balances so that there is a direct pull on the upper one and a direct thrust on the lower one.

State what forces are acting on the pin at the head of the jib.

Proceed, as described above, to draw the position-diagram, and by means of this, the force-diagram for these forces.

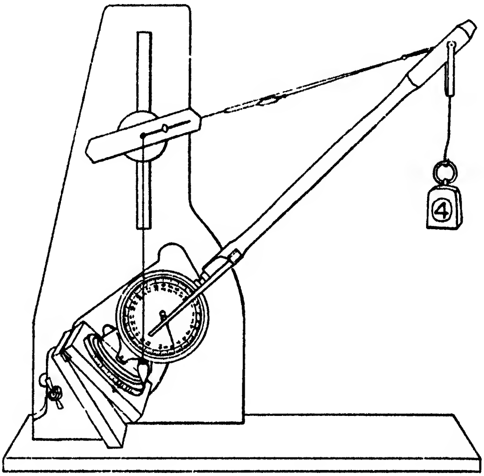


Fig. 134.

By measuring this force-diagram, determine the thrust of the jib, and the pull of the tie, on the pin. The magnitudes of these forces denote also the stresses in these members.

These results will be found to be somewhat smaller than the readings of the spring-balances owing to the effect produced by the weight of the jib itself. However, if the readings of the balances before the weight is put on be subtracted from the readings after, the differences will be found to agree closely with the results determined by the graphic method.

107. Wall Crane. In Fig. 135 is shewn a wall crane such as is used in smithies. The horizontal jib AB and the tie-rod CB are pinned together at B, while their other ends are pinned at A and C to a vertical rod which is supported by, and can turn in, vertical bearings.

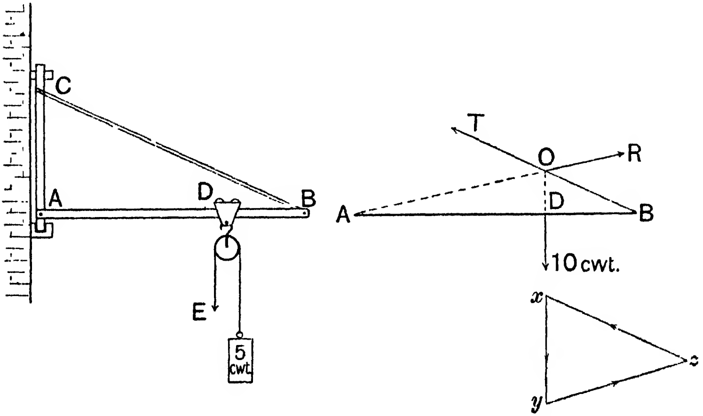


Fig. 135.

A pulley is suspended from the trolly which runs on the jib, and the diagram shews a load of 5 cwt. being hoisted by a pull exerted vertically downwards on the end E of a chain which passes over the pulley and carries the load at its other end. Neglecting the friction of the pulley, the tensions in the two sides of the chain are the same and hence produce a downward force of 10 cwt. on the jib at D.

Note that this force tends to bend the jib which is therefore not subjected to simple compression only, and the reaction of the pin A on the jib will not be in the direction of its length AB.

Our purpose is to shew how to find the direction and magnitude of this reaction, and also the tension in the tie. We shall leave out of consideration effects produced by the weights of the various parts of the crane itself.

In this case we consider the equilibrium of the jib, and proceed to draw to scale the position-diagram giving the lines of action of the three forces acting upon it. The line of action of the pull T of the tie on B intersects the line of action of the force of 10 cwt. at O. AO now gives us the line of action of the reaction R of the pin at A, since the lines of action of all three forces must intersect at the same point.

Knowing the directions of all three forces and the magnitude of one of them, we can now draw the force-triangle xyz, as in the preceding examples.

yz and zx now represent the forces R and T respectively, on the same scale as xy represents the force of 10 cwt.

108. Wheelbarrow on incline. As a further example to illustrate the application of the triangle of forces we will shew how to solve graphically the following problem.

Fig. 136 *a* is a diagram (drawn to scale) representing a wheelbarrow being pushed up an incline. We are given the total weight of the barrow, namely 150 lbs., and the position of the centre of gravity G.

The forces acting on the barrow are (1) the weight of 150 lbs. acting vertically downwards through G, (2) the normal reaction R of the plane on the wheel, and (3) the force P which must be applied at A.

(There are really two equal forces acting on the two handles at A, but we shall suppose these forces to be replaced by an equivalent single force P acting midway between them.)

It is required to find the direction and magnitude of P, and the magnitude of R.

Since the diagram is drawn to scale we can draw upon it the position-diagram of the forces acting on the barrow.

The reaction R acts at right angles to the plane and intersects the line of action of the weight at O. AO gives the line of action of P, for the lines of action of all three forces must pass through the same point.

Knowing the directions of all three forces and the magnitude of one of them we can now draw, as before, the force-triangle xyz (Fig. 136*b*). That is, we draw xy to represent the known weight of

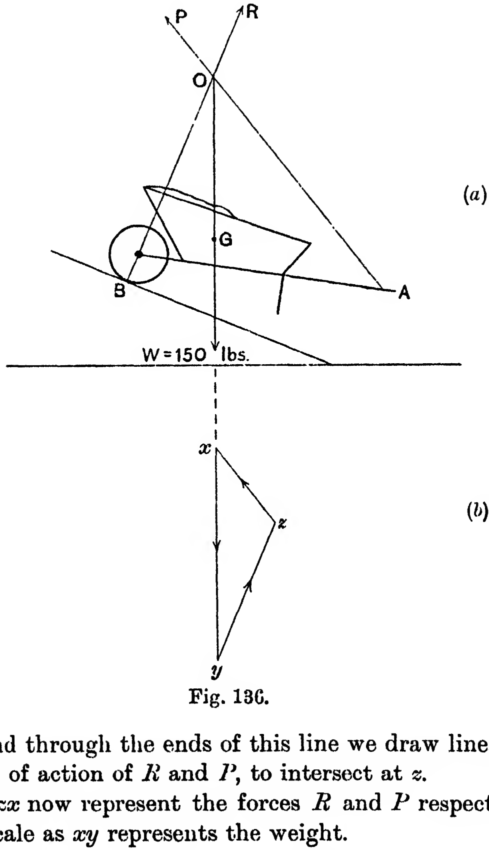


Fig. 136.

150 lbs., and through the ends of this line we draw lines parallel to the lines of action of R and P, to intersect at z.

yz and zx now represent the forces R and P respectively, on the same scale as xy represents the weight.

109. Balancing-force and Resultant of two inclined forces. Referring to any of the foregoing cases of a body in equilibrium under the action of three forces, it is obvious that we

can look upon any one of these forces as balancing the other two. Also, since the forces are in each case represented by the sides of the corresponding force-triangle, it is clear that if two sides of a triangle are drawn to represent in magnitude, direction and sense two given forces, then the third side will represent their *balancing-force.*

Exp. 5. To illustrate the graphical method of finding the balancing-force of two given forces, suspend a load of 4 lbs. from about the centre of a stiff light rod, the weight of which is comparatively small and need not be taken into account. Attach a

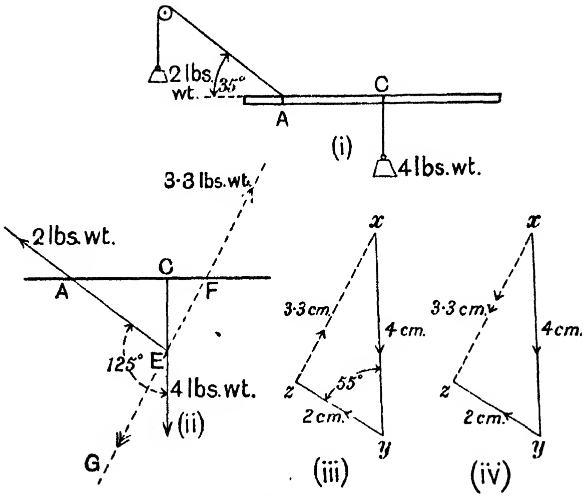


Fig. 137.

string to the rod at a point *A* near the end, pass it over a fixed pulley and load its other end with 2 lbs. Fig. 137 (i).

We will suppose that it is required to keep the rod in the horizontal position shewn in the figure, by balancing the two forces acting upon it with a third force applied to the rod by means of a string.

Where must this string be attached, and in what direction, and with what force, must it be pulled?

Draw to scale the position-diagram for the two known forces acting on the rod, and produce the lines of action of these forces to meet in E. Fig. 137 (ii).

To obtain the force-triangle xyz, Fig. 137 (iii), draw the lines xy and yz parallel to the directions of the forces of 4 lbs. wt. and 2 lbs. wt. respectively, representing them in sense and in magnitude to a scale of 1 cm. to 1 lb. wt. Join zx; this third side represents in magnitude, direction, and sense, the balancing-force required. Measuring zx we find it is 3·3 cms. long, and therefore represents a force of 3·3 lbs. wt. The line of action of the balancing-force of 3·3 lbs. wt. must pass through E (Art. 101), and is therefore the line EF, drawn parallel to zx and cutting AC in F. If a string is attached to the rod at the point represented by F in the diagram, and this string is pulled in the direction EF with a force whose magnitude is 3·3 lbs. wt., the rod will be found to be in equilibrium. To verify this, connect a spring-balance by a string to the rod at the point indicated. You will find that on pulling the other end of the balance in a certain direction the rod can be kept at rest in the position shewn in the figure. Having obtained the position, read the balance, and measure the angle which the string makes with the rod. Compare these results with those obtained graphically.

In addition, since this force of 3·3 lbs. wt. could itself be balanced by a force of 3·3 lbs. wt. of opposite sense, it is clear that the two given forces of 4 and 2 lbs. wt. acting together are equivalent to a force equal and opposite to their balancing-force; in other words, their *resultant* is a force of 3·3 lbs. wt. acting in the direction and sense indicated in Fig. 137 (ii) by the line EG.

We see then that the third side of the force-triangle represents also the resultant of the two given forces, provided that the arrow placed on this side *opposes* the arrows on the other two sides.

When using this graphic method for finding the resultant of forces we shall signify the line representing this resultant by placing a *double* arrow upon it as shewn in Fig. 137 (iv).

To sum up: *if two forces, acting on a rigid body, be represented in magnitude, direction and sense by two sides of a triangle, with arrows following, then the third side of this triangle represents their balancing-force or their resultant, according as the arrow on this line follows, or opposes, the arrows on the first two sides.*

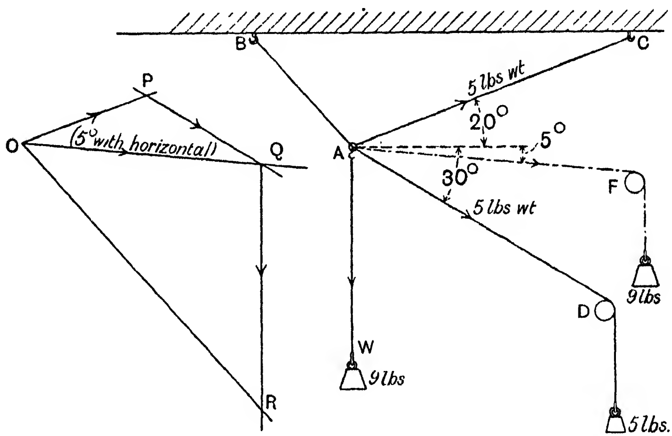
In Article 71 we shewed how to find the resultant of parallel forces. Articles 71 and 72 should be read again carefully now. The conclusions of Article 71 are given again here for convenience; they are:

When a body is in equilibrium under the action of a number of parallel forces in one plane, the resultant of these forces is their algebraic sum, and the line of action of the resultant is such that the moment of the resultant about any point in the plane is equal to the sum of the moments of the constituent forces about the same point.

110. Polygon of Forces. We have shewn how to find the resultant of two forces by the 'triangle of forces.' We may wish to find the resultant of three, four, or more forces which all meet at a point. Suppose the forces are A, B, C, D, etc., in certain known directions. First, by the triangle of forces we find the magnitude and direction of the resultant of one pair of these forces, say A and B, and we know that the line of action of the resultant of A and B passes through the common meeting point of all the forces; so we may replace A and B by their resultant and call this resultant R. Next, and again by the triangle of forces, we find the resultant of R and the force C, and call this second resultant P, then P is the resultant of A, B, and C; and so we go on, one step at a time, until we have found the resultant of all the forces. The following is an example in which we require to find the resultant of three forces.

In Fig. 138 (a), B and C are two fixed supports; at A there is a pin through a light frictionless pulley; D is a frictionless guide pulley. A weight of 9 lbs. is hung from the pin at A. A weight of 5 lbs. is hung from a cord which passes over the pulley at D,

round the pulley at A and is secured to C. The pin at A is secured by a cord to B. The pulley A comes to a position of equilibrium as shewn in the figure. AC makes an angle of 20° above the horizontal and AD an angle of 30° below the horizontal. We require to find the direction and magnitude of the pull in the cord AB. We suppose the pulley at A is very small and that all the forces meet in one point.



(b) Fig. 138. (a)

Four forces act on the pulley at A. We know the magnitude and direction of three of these four forces; they are:

(1) A force of 9 lbs. wt. vertically downwards.

(2) A force of 5 lbs. wt. along AD making an angle of 30° below the horizontal.

(3) A force of 5 lbs. wt. along AC making an angle of 20° above the horizontal. (In practice these angles are measured with a protractor placed behind the cords.)

The fourth force is the pull of the cord AB; we do not know its magnitude and we will suppose that we have not measured its direction.

The resultant of the first three forces is equal and opposite to the fourth, that is to say it acts through A, along the direction BA and is equal in magnitude to the pull of the cord AB; so when we have found the resultant of the first three forces we can verify it experimentally by measuring with a spring-balance the pull of the cord AB, and with a protractor the direction of this cord.

Our problem then is to find the resultant of the three forces along AW, AD, and AC.

Take first the forces along AC and AD and find their resultant by the triangle of forces; this triangle is shewn in Fig. 138 *b* drawn to scale. OP is drawn parallel to AC and 5 units long to represent a force of 5 lbs. wt., PQ parallel to AD and also 5 units long. OQ therefore gives us the magnitude and direction of the resultant of the two forces of 5 lbs. wt. and the line of action of this resultant is through A. OQ makes an angle of 5° below the horizontal and its length is 9 units; so the resultant of the two forces of 5 lbs. wt. is a force of 9 lbs. wt. through A at an angle of 5° below the horizontal. We may test our result up to this point as follows:

Attach a light cord to the pin at A and pass it over a frictionless pulley at F which is so arranged that the cord from A to F makes an angle of 5° below the horizontal; now hang a weight of 9 lbs. to the cord over the pulley at F and at the same time remove the cord CAD and you will find that the pulley A comes into the same position of equilibrium as before.

OQ represents in magnitude and direction the force of the cord along AF which is the resultant of the two pulls of 5 lbs. wt. along AC and AD which it has replaced.

Now draw the triangle of forces to find the resultant of the force of 9 lbs. wt. along AF and that of 9 lbs. wt. vertically downwards. QR is drawn parallel to AW and 9 units long. OR now represents in magnitude and direction the resultant of the weight of 9 lbs. and the force along AF; therefore it represents the resultant of the three original forces 5 lbs. wt. along AC, 5 lbs. wt. along AD, and 9 lbs. wt. vertically downwards.

Since OR represents the resultant of the three forces it follows that RO represents the pull in the cord AB to which the resultant of the three forces must be equal and opposite. Measuring in Fig. 138 *b* we find that RO makes with QR the same angle that BA does with AW. OR is 13·3 units long and therefore represents a force of 13·3 lbs. wt. and by inserting a spring-balance in the cord AB we find that the pull in this cord is also 13·3 lbs. wt.

The pin at A is in equilibrium under the action of four forces. In the quadrilateral $OPQR$, OP was drawn to represent in magnitude, direction and sense the force along AC, PQ represents the force along AD, and QR represents the force along AW. We have drawn three sides of a quadrilateral to represent successively in magnitude, direction and sense three out of the four forces which keep the pin at A in equilibrium. We find that the fourth side of the quadrilateral represents in magnitude, direction and sense the fourth force; or, the fourth side taken in the opposite direction represents in magnitude, direction and sense the resultant of the first three.

This method of finding the resultant of three forces may be extended to four or five or any number of forces, and accordingly our quadrilateral becomes a pentagon, hexagon or other polygon, having as many sides as there are forces keeping the body in equilibrium. If we are to find the resultant of N forces acting on a body we draw N sides of a polygon representing respectively in magnitude, direction and sense the N forces; then the line which closes the polygon represents in magnitude, direction and sense:

(*a*) the resultant of the N forces,

or (*b*) the equilibrant of the N forces

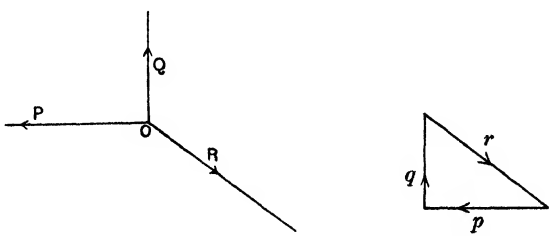
according as it is taken in a direction opposite to or the same way round the polygon as the directions of the N forces.

No new principle is involved here. We have only extended the triangle of forces by successive steps. In combining the forces two at a time by the 'triangle' it clearly makes no difference in what order we select our pairs, and so if we draw a polygon without actually completing any of the constituent triangles we may

take the forces in any order provided the sense of the line on the polygon is the same as the sense of the force on the body.

· It is very important at this point to be sure what we have done and what we have left undone. We have shewn how to find the resultant of parallel forces; we have also shewn how to find, by the 'triangle,' the resultant of two forces in one plane which are not parallel (*i.e.* two forces which meet at a point) and as an extension of this we have shewn how, by the polygon, we may find the resultant of three, four or any number of forces in one plane which meet at a point. We have not considered a number of forces which meet at a point and are not in one plane, neither have we considered a number of forces which are in one plane and not parallel but do not all meet in one point. We may be able with a little ingenuity, and with the knowledge we already have, to solve certain problems on forces not all in one plane or not concurrent but in this book we shall not consider these cases in a general way. It is just as well however to realise that the polygon rule, as we have expressed it on page 268, does not apply to non-concurrent forces.

For example, suppose a body is in equilibrium under the action of three forces P, Q and R whose directions meet at a point O,



(*a*) Fig. 139. (*b*)

Fig. 139 *a*, and these forces are represented in direction and magnitude by the sides p, q and r of the triangle in Fig. 139 *b*.

Suppose also that the forces P, Q and R remain of the same magnitude and direction, but that one of them, say R, is moved in its

line of action so that P, Q and R no longer meet at a point, but now their lines of action intersect in pairs at a, b and c, Fig. 139 c. Clearly the body shewn in Fig. 139 c is not in equilibrium, for the resultant of any pair of the three forces is a force equal and in the opposite direction to, but not in the same straight line as, the third force; and so this system of three forces reduces to a couple whose moment is equal to the product of any one of the forces and the perpendicular distance on to this force from the point of intersection of the other two. Yet the forces P, Q and R in Fig. 139 c are represented in direction and magnitude by the lines p, q and r in Fig. 139 b just as those forces are in Fig. 139 a.

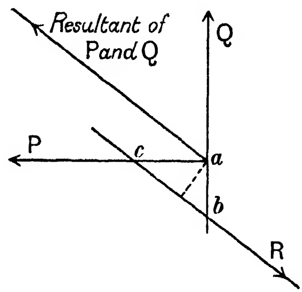


Fig. 139 (c).

If we had attempted by means of the 'polygon' to find the resultant of the three forces represented in Fig. 139 c, we should have found that the first three sides of our polygon closed and might have come to the quite wrong conclusion that these three forces had no resultant, whereas their resultant is a couple.

111. Examples illustrative of the Triangle and Polygon.

Ex. 1. In Fig. 140 a, which is not drawn to scale, a gangway AB, 30 feet long, has its centre of gravity at G, distant one foot from the middle point of AB. The gangway is hinged to a wall at A, and at high water rests on rollers which are mounted on a floating landing stage at D. At low water the top of the landing stage falls to the horizontal position D', and the gangway then rests on it by means of the rollers B. We shall neglect the dimensions of the rollers and all friction and find the direction and magnitude of the thrust of the wall at A and of the landing stage upon the gangway, at high and at low water.

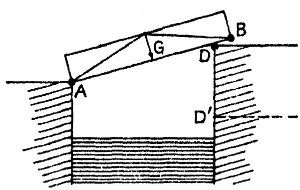


Fig. 140 (a).

D and D' are eight feet above and below the level of A respectively, and A is 20 feet distant from DD'. The weight of the gangway is 2000 lbs.

The position-diagrams of the forces are drawn to scale for both the high water and the low water conditions, in Fig. 140 *b*.

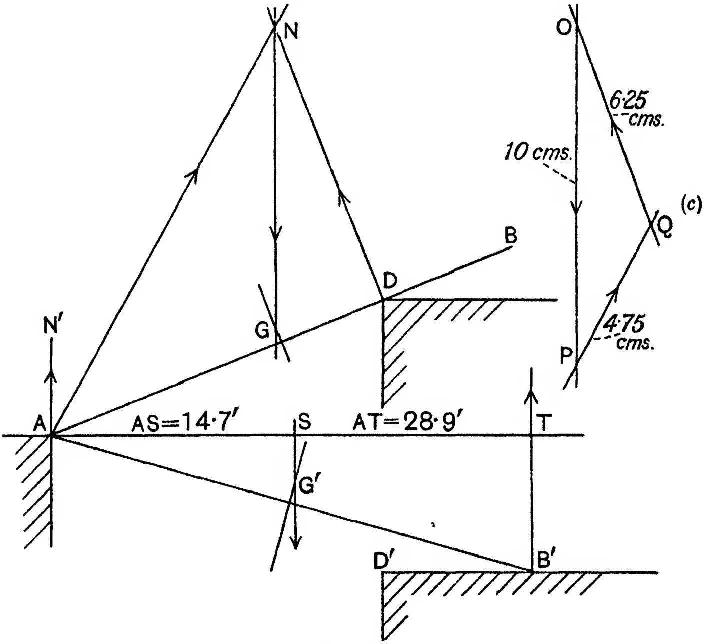
(1) *High water condition.*

The gangway is acted upon by three forces:

(*a*) The weight of 2000 lbs. vertically downwards through G.

(*b*) The thrust of the roller at D; and since the roller is frictionless the direction of this thrust must be at right angles to the line AB; so DN is drawn perpendicular to AB.

(*c*) The thrust of the hinge at A.



(*b*) Fig. 140.

The directions of the first two forces meet at N and it therefore follows that the third and only remaining force through A must also pass through N. AN is therefore the direction of the thrust at A at high water.

In order to find the magnitude of the reaction at A we draw the triangle

of forces for the three forces on the gangway under which this is in equilibrium. We know the directions of all the three forces and the magnitude of one of them, so we can draw the force-triangle OPQ, Fig. 140 *c*. OP is drawn 10 cms. long to represent on a scale of 1 cm. = 200 lbs. the weight of the gangway. PQ is drawn parallel to AN and QO is drawn parallel to DN.

By scale PQ is 4·75 cms. long and therefore represents a force of 950 lbs. wt. QO is 6·25 cms. long and therefore represents a force of 1250 lbs. wt. So the reaction at A is a force of 950 lbs. weight and makes an angle of 29° with the vertical. The reaction at D is 1250 lbs. wt. and makes an angle of 22° with the vertical.

(2) *Low water condition.*

In this condition the end of the gangway is supported on the rollers at B', and since this roller is on the gangway and frictionless, the thrust of the stage on the gangway is vertical; the weight of the gangway is vertical, therefore the only remaining force, which is the reaction of the hinge at A, must also be vertical. So in the low water condition the gangway is in equilibrium under the action of three forces, but this time the three forces are parallel.

In order to find the magnitude of the forces we take moments about A.

The reaction at A has no moment about A.

The moment of the weight of the gangway about A is 2000 lbs. wt. $\times AS = 2000 \times 14{\cdot}7$ lbs. ft. and is clockwise.

The moment of the thrust of the stage at B is $X \times AT = X \times 28{\cdot}9$ lbs. ft. and is counter-clockwise.

Hence $X = \dfrac{2940}{28{\cdot}9}$ 1017 lbs. wt.

It follows that the thrust of the hinge at A is a force of 983 lbs. wt.

Ex. 2. In Fig. 141 *a* there is shewn a pair of pulley blocks carrying a load of 600 lbs. and itself supported by two ropes along AD and AE. We will find the tensions in the supporting ropes, assuming that the pulleys are frictionless.

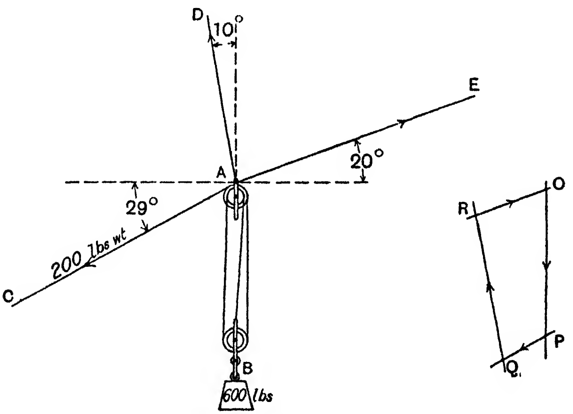
Since the pulleys are frictionless, the pull on the free end of the rope from the upper block will be 200 lbs. wt.

The three pieces of rope which lie between the upper and lower blocks exert a resultant force on the lower block which must be vertically upwards and equal to the weight of 600 lbs.; therefore they exert on the upper block a resultant force equal to the weight of 600 lbs. vertically downwards. (We are assuming that the blocks themselves are of negligible weight.)

Thus we may regard the upper block as in equilibrium under the action of four forces which meet in the point A. If therefore we represent three of these four forces by three sides of a polygon (quadrilateral), taken in order,

the fourth side of the polygon taken in the same order will represent the fourth force.

In Fig. 141 *b* the line *OP* is drawn 6 cms. long, vertically to represent in magnitude, direction and sense the resultant force of the three vertical cords upon *A*; *PQ* is drawn parallel to *AC* and 2 cms. long. The force along *AD* is represented in direction by a line in the direction *QR* parallel to *AD*. We do not yet know the magnitude of the force in *AD*, so we cannot yet determine the length of *QR*. But wherever *R* may be, we know that the fourth side of the polygon, that is to say *RO*, must be parallel to *AE*. So we draw a line through *O* parallel to *AE*, and the point where this line meets the line through *Q* parallel to *AD* is *R*; so now *QR* and *RO* represent in magnitude, direction and sense the forces exerted by the ropes along *AD* and *AE*. By scale *QR* is 6 cms. long, therefore the rope *AD* exerts a force of 600 lbs. wt.; *RO* is 3 cms. long, therefore the rope *AE* exerts a force of 300 lbs. wt.



(*a*) Fig. 141. (*b*)

112. Problems solved by drawing consecutively two or more force-diagrams. Suppose that, having drawn the force-diagram for the forces acting on a body *A*, we have thereby found the force exerted upon it by another body *B*. Since *A* reacts on *B* with an equal and opposite force, we now know one of the forces acting upon *B*, and this may possibly

enable us to draw the force-diagram for this body. This applies also to cases in which the two bodies are not in direct contact, but act upon one another through the medium of a link, such as a cord or light rod, this link being in simple tension or compression. The following examples will make this clear.

Loaded cord (Fig. 142). A cord is fastened to fixed supports

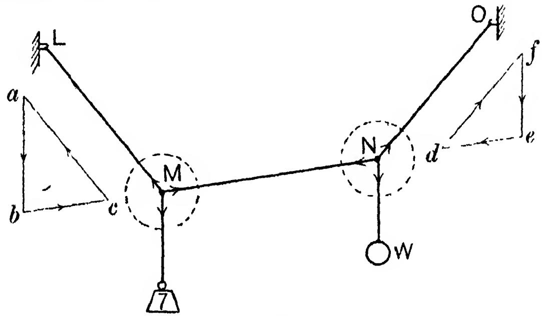


Fig. 142.

at L and O and is loaded at M with a weight of 7 lbs., and at N with an unknown weight W.

Given the directions of the parts of the cord, it is required to find the tensions in them and also the weight W.

We first consider the equilibrium of the knot at M. The position-diagram for the forces acting upon it is shewn within the circle drawn round it. By drawing ab to represent the known force of 7 lbs. wt. and then drawing through the ends of this line parallels to MN and ML we obtain the force-triangle abc. The side bc represents the pull of the part of the cord MN and hence gives us the tension in this part. Likewise ca gives us the tension in ML.

We now consider the forces acting on the knot at N. Having found the tension in MN we know the pull of this part of the cord on N and hence can draw the force-triangle def. By measuring df and fe we obtain the value of the tension in NO and the magnitude of W, respectively.

Exp. 6. Fit up the above arrangement, introducing a spring-balance into each part of the cord. Draw on paper the directions of all the parts of the cords by measuring the angles between them at M and N with a protractor. Proceed, as shewn above, to find graphically the tensions and the weight W.

Compare these results with the readings of the balances and the magnitude of W found by direct weighing.

113. Framework of jointed bars. Fig. 143 *a* shews a braced support consisting of four bars freely pinned to one

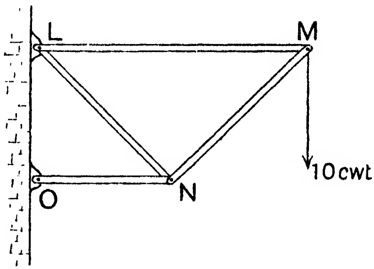


Fig. 143 (*a*).

another at their ends, the pins at L and O being carried by brackets fixed to a vertical wall.

It is required to find the stresses set up in the various bars when a load of 10 cwt. is suspended from the pin at M.

We have already explained (Art. 18, p. 26) that, leaving out of consideration the weights of the bars and any friction at the joints, each bar is in equilibrium under the action of two equal and opposite pulls or pushes, exerted upon it in the direction of its length, by the pins at its ends. That is, each bar is a simple strut or tie, and is itself exerting equal and opposite reactions on these pins.

By considering the equilibrium of successive pins we can find graphically what these forces are and hence the stresses in the various bars.

We first proceed to draw a diagram of the structure to scale

(Fig. 143 *b*). We then start with the pin *M*, for we know the magnitude of one of the forces acting upon it. By drawing *ab* to represent the force of 10 cwt. and then drawing through the ends of this line parallels to *ML* and *MN*, we obtain the force-triangle *abc*.

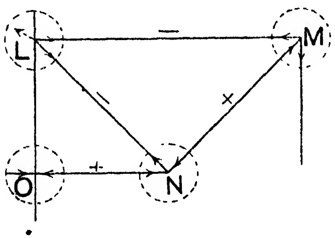


Fig. 143 (*b*).

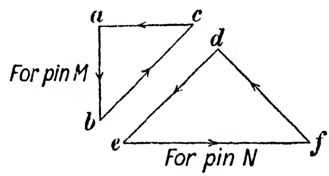


Fig. 144.

If the scale chosen is 1 cm. to 1 cwt., it will be found that *ac* measures the same as *ab*, namely 10 cms., and *bc* measures about 14·1 cms. We learn then that the bar *ML* is pulling on the pin *M* with a force of 10 cwt., and is therefore under a tension of 10 cwt.; and the bar *MN* is subjected to a compressive stress of 14·1 cwt., for this is the magnitude of the force with which it is pushing the pin. But this must also be the magnitude of the force with which this bar pushes on the pin *N* in the direction *MN*. Hence, we can now draw the force-triangle *def* for the forces acting on *N*.

If, using the same scale as before, *de* is made 14·1 cms. long, it will be found that *fd* has the same length, and that *ef* measures 20 cms. We learn then that the member *NL* is a tie under a tension of about 14·1 cwt. for this is the magnitude of the pull it exerts on *N*; and *ON* is a strut under a compressive stress of 20 cwt.

If the directions and senses of the forces represented in the force-diagrams are shewn on the diagram of the structure, it is advisable to draw the arrows near the pins and enclose them in a circle, as shewn in Fig. 143 *b*, to make it quite clear that these refer to the forces acting *on* the pins. These arrows, however, are

unnecessary if we denote by a symbol the nature of the stress in each bar as soon as it is known. Thus, as soon as we find that a bar is pushing on a pin, we put the sign + against it to signify that it is a strut. Similarly, if we find a bar is pulling on a pin, we signify that it is a tie by putting the sign -- against it.

The stresses in the various members should be tabulated thus:

Member	Strut or tie	Stress
LM	tie	10 cwt.
MN	strut	14·1 ,,
LN	tie	14·1 ,,
ON	strut	20 ,,

Additional exercises.

(1) Determine the reaction of the wall on the pin *L*, both by drawing the force-triangle for this pin with the help of the above results; and also, considering the whole structure as a single body, by drawing the force-triangle for the external forces acting upon it.

(2) Check the above result for the compressive stress in *ON* by considering the equilibrium of the whole structure *LMNO*, and taking moments, about *L*, of the external forces acting upon it.

Structures, such as the above, which are built up of bars or rods jointed together at their ends, are called 'framed structures' or 'frames'; they are much used in practice, especially for supporting roofs and bridges. The engineer, in designing such a structure to support a given load, or system of loads, first utilises the principles of Statics to determine (as we have indicated) the maximum stresses to which the various members will be subjected. Having ascertained by experiment the strengths of the materials he intends to use, he is then in a position to design the shape and size of each member so that it shall be sufficiently stiff and strong for its purpose. Moreover, by making the margin of safety the same for all members, he avoids waste of material and ensures that no part of the framework shall be unnecessarily cumbrous.

CHAPTER IX

RESOLUTION OF FORCES

114. The effects of a single force in two directions at right angles to one another. In some statical problems it is convenient to think of a force as replaced by two other forces which are at right angles to one another, and together in their effect equivalent to the single force. The convenience lies in the fact that of two forces at right angles to one another, neither has any effect in the direction of the other. It often happens that a body is constrained to move in a particular direction, though the force which produces the movement may be inclined at an angle with the direction of the movement.

For example the effective pressure of the wind on the sails of a boat may be regarded as of two parts or components, mutually at right angles; one, in the direction of the keel towards the bows —the useful part—drives the boat along; the other, at right angles to the keel—the useless part—causes the boat to drift to leeward; the effect of the latter is minimised by the large surface which the side of the boat presents to the water in this direction.

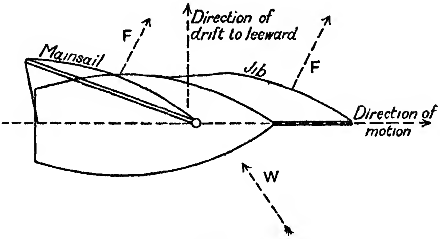


Fig. 145.

In Fig. 145 the direction of the effective force exerted by the wind is shewn by the arrows FF while the direction of the wind itself is shewn by the arrow W.

Another example is of a barge drawn by a horse along a canal. The horse pulls in a direction inclined to that in which the barge is to go. The effect of the horse's pull may be regarded as of two components, one in the direction of motion along the canal, and the other at right angles to the line of the canal; the component in the direction of motion is useful; the component at right angles to the line of the canal would pull the boat into the bank, and must be counteracted by the rudder which, setting the bows of the boat away from the towing path, induces the water of the canal to press on the side of the boat nearest the towing path, and so keep the boat away from the bank.

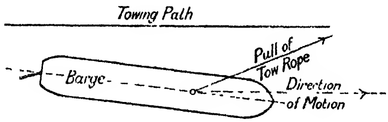


Fig. 146.

Another example is of a roller pulled up an inclined plane by a rope whose direction is horizontal and therefore inclined to the direction of motion. It is convenient to regard the force of the rope as equivalent to two forces at right angles to one another; one, the useful component, parallel to the plane, and the other perpendicular to the plane which has no direct effect in the direction of motion.

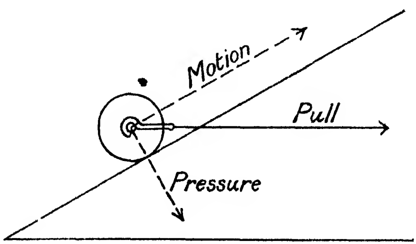


Fig. 147.

Another good example is found in the mechanism of the piston-rod, slipper, and connecting-rod of a reciprocating engine.

In this instance, in the position shewn in Fig. 148, the back thrust of the connecting-rod on the crosshead is inclined to the direction of motion of the crosshead. At any instant the thrust of the connecting-rod is balanced by two forces, that exerted by the piston-rod, and that exerted by the slide on the slipper. The piston-rod can only exert a force along its own length and there-

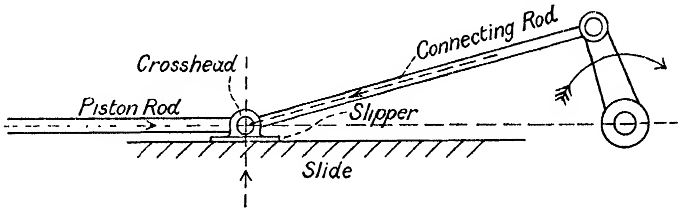


Fig. 148.

fore can only balance the 'component' in this direction of the thrust of the connecting-rod. The slide, if it is very well lubricated, can only exert a force on the slipper at right angles to the plane of its surface and at right angles to the direction of the force of the piston-rod, and is balanced by the 'component' in this direction of the thrust of the connecting-rod.

115. To measure experimentally the rectangular components of a force. We intend now to shew how we may find the value of two forces at right angles to one another, and together exactly equivalent to a single force. We shall use the name 'Components' for the two forces which are mutually at right angles and together equivalent to the single force. The components may be in any direction provided they are at right angles to one another.

A hole is bored with a gimlet in the centre of a large board, and the board is supported on a nail driven through the hole into the wall as shewn in Fig. 149. A weight W is suspended by a string from a ring, and the ring is hung on the nail in front of the board. The nail now exerts a force of W lbs. wt. vertically

upwards upon the ring. We can replace the single force of the nail on the ring by two forces exerted on the ring through strings by two spring-balances attached to nails at A and B, so arranged that the lines of these two strings are at right angles to one

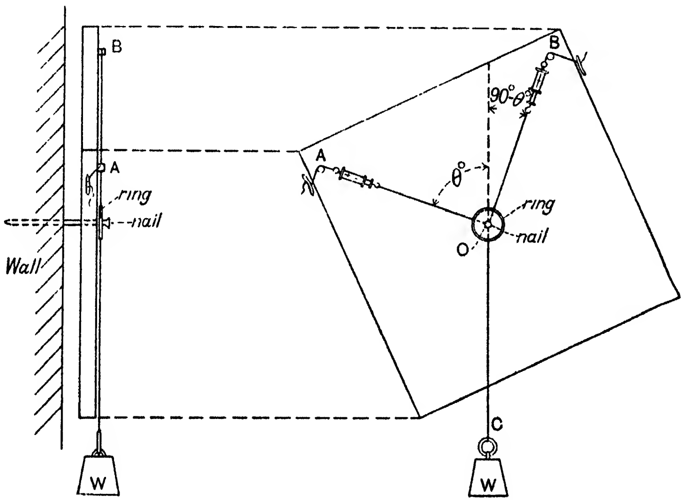


Fig. 149.

another. The readings of the spring-balances tell us the components in two directions of the vertically upward force of W lbs. wt. formerly exerted by the nail. The particular values of the two components will depend on the angle at which the board is set on the nail. If we set A vertically over O, the spring-balance at A will read W lbs. wt. and that at B will read zero; if we set OA horizontal, the spring-balance at A will read zero and that at B will read W lbs. wt. We may arrange OA to make with the vertical any angle we like between these two positions.

(Whatever angle $\theta°$ OA makes with the vertical, OB will make an angle with the vertical which is the complement of this, $90° - \theta°$.)

Before we make a series of measurements, we may notice two things:

(1) The more nearly one of the components is parallel to the original single force, the more nearly is it equal to that force.

(2) Except when one of the components is parallel to the original single force and the other consequently zero, the sum of the two components is always greater than the single force.

The original single force of which we find the components is commonly called the 'resultant' of these components.

(As the board is swung round on the nail, the forces exerted by the spring-balances change, causing the spring-balances to extend more or less according to the tension which is upon them. This extension of the spring-balances may be enough to distort the right angle between the strings; if so, the difficulty may be overcome by attaching the outer ends of the spring-balances by strings, not tied to A and B, but over A and B to small cleats or catches like those used for window-blind cords; in this way the lengths of AO and BO may be quickly adjusted so as to bring the ring central over the nail and therefore preserve the right angle.)

Let us set the board of Fig. 149 in position so that the pull of the spring-balance at A makes an angle of 30° with the vertical line; and let us hang a weight of 10 lbs. at C. The spring-balance at A registers 8·7 lbs. wt. and that at B registers 5 lbs. wt. If the ring were supported on the nail, the nail would exert a force vertically upwards on the ring equal to 10 lbs. wt. The forces of 8·7 lbs. wt. along OA and 5 lbs. wt. along OB exactly replace a vertical upward force of 10 lbs. wt., therefore they are, in this position (*i.e.* when OA makes an angle of 30° with the vertical), the components of a force of 10 lbs. wt.

116. Relation between a force and its components. In order to find the relation between a force and its components, we use the apparatus and the results of the experiment described in the previous article. First, for convenience of drawing, we pin a piece of paper on the board behind the strings.

With centre O and radius 10 centimetres, draw the arc of a circle as shewn in Fig. 150. Produce the vertical line of the string CO to cut the circle at D. OD represents in magnitude,

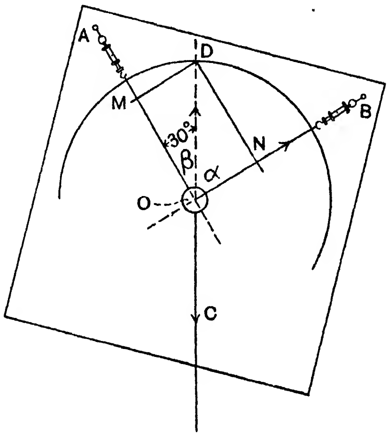


Fig. 150.

direction and sense, to a scale of 1 cm. = 1 lb. wt., the vertical force of 10 lbs. wt. with which in the first instance the nail supported the ring.

Draw DM, DN, perpendicular to the lines OA and OB.

The ring is in equilibrium under the action of three forces, (1) the weight of 10 lbs. vertically downwards, (2) the force of 8·7 lbs. wt. along OA, and (3) the force of 5 lbs. wt. along OB. Taking moments about the point D, and observing that the weight of 10 lbs. has no moment about D, we have

$$8{\cdot}7 \times MD = 5 \times ND$$

or
$$\frac{8{\cdot}7}{5{\cdot}0} = \frac{ND}{MD}.$$

Measuring ND and MD, we find they are 8·7 cms. and 5 cms. respectively. If we had drawn the circle with a different radius,

the lengths of ND and MD would have been different though still *in the same proportion.*

We make the length, in cms., of OD represent the magnitude, in lbs. wt., of the vertical upward force, and we find that in these circumstances the lengths of the sides ND and MD, or, for it is the same thing, the lengths of the sides OM and ON, represent, also in cms., the magnitudes in lbs. wt. of the component forces.

If we make similar measurements for different directions of the components, that is to say for different values of the angle MOD, we always find the lengths OM and ON represent the magnitudes of the corresponding components on the same scale as that on which OD represents the force of 10 lbs. wt.

From the geometry of the right-angled triangle, it follows that the sum of the squares of the magnitudes of the component forces is equal to the square of the magnitude of the resultant.

To treat the same question by trigonometry is even simpler. In Fig. 150 OM represents in magnitude and direction one of the components of the vertical force, and ON the other on the same scale as OD represents the vertical force. If α is the angle which the right-hand component makes with the resultant, and β the angle which the left-hand component makes with it, we have

$$ON = OD \cos \alpha \text{ and } OM = OD \cos \beta,$$

that is to say the magnitude of a component of a force in any direction is equal to this force multiplied by the cosine of the angle between the force and the component: it is in fact the 'projection' of the force upon the line of the component.

117. Resolution. A particular case of the Force-Triangle. We may regard the resolution of a force into its rectangular components, or the composition of two rectangular components into a resultant, as a particular case of the Force-Triangle which we discussed in the last chapter.

Fig. 150 of Article 116 illustrates a particular case of the Force-Triangle. In this case the angle AOB is a right angle.

In the general case described in Art. 104, Fig. 131, AOB was *any* angle.

If in this particular case of Art. 116 we draw a triangle of forces xyz (Fig. 151) in which xy, yz and zx represent the forces exerted on the ring at O by the weight W, the spring-balance at B, and the spring-balance at A, the angle yzx is a right angle. yz and zx represent in magnitude and direction the rectangular components of a vertical upward force of W lbs. wt. Thus

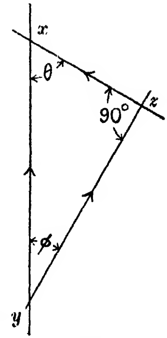


Fig. 151.

$$zx = W \cos\theta \text{ lbs. wt.}$$

$$yz = W \cos\phi \text{ lbs. wt.}$$

118. Example of resolution of a force. A body on an inclined plane. Set up a plane inclined at an angle of (say) 35° with the horizontal, and place on it a roller, whose weight we will suppose is 4 lbs. The arrangement is shewn in Fig. 152 *a*. There the roller is fitted with a stirrup pivoted loosely at the ends of a thin rod which passes axially through it. The roller is supported on the plane by a string parallel to the plane, attached to the stirrup, and passing over a pulley at the top of the plane.

Weights are attached to the end of the string so that the roller will rest wherever it is placed on the plane, and will move up and down the plane with equal ease when it is set in motion.

The forces which keep the roller in equilibrium are, (1) its weight (4 lbs.) acting vertically downwards through its centre of gravity, which is the middle point of the axis; (2) the pull P of the string which acts through the middle point of the axis

parallel to the plane; and (3) the reaction R of the plane at right angles to the plane (Fig. 152 *b*).

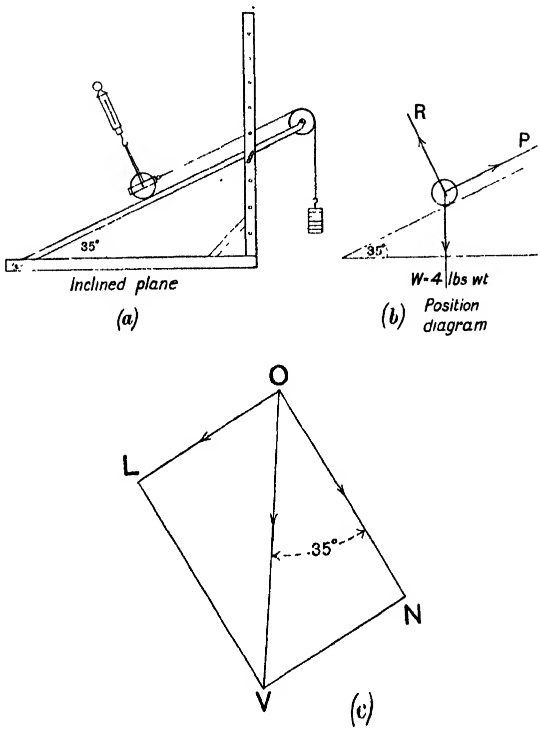


Fig. 152.

Of these three forces, two, R and P, are mutually at right angles; we will therefore calculate the components of the force of 4 lbs. wt. in directions parallel to R and P.

The construction is as follows:

Draw a line OV 4 inches long to represent the weight of the roller; draw ON and OL in the directions of the components, that is to say parallel respectively to R and P; draw VL and VN

perpendicular to OL and ON. Thus the components of the weight in directions parallel to R and P are represented in magnitude by the lengths OL and ON, Fig 152 *c*.

$$OL \text{ is } OV \cos 55° = 4 \cos 55° = 2{\cdot}29 \text{ inches,}$$

$$ON \text{ is } OV \cos 35° = 4 \cos 35° = 3{\cdot}28 \text{ inches.}$$

The components of the force of 4 lbs. wt. are therefore:

2·29 lbs. wt. parallel to the plane,

and 3·28 lbs. wt. perpendicular to the plane.

The body is in equilibrium under the action of one set of forces parallel to the plane and another set of forces perpendicular to the plane, and these sets of forces being mutually at right angles are independent of one another.

Therefore the component of the weight parallel to the plane is equal and opposite to P, and that perpendicular to the plane is equal and opposite to R; therefore

$$P = 2{\cdot}29 \text{ lbs. wt.}$$

and

$$R = 3{\cdot}28 \text{ lbs. wt.}$$

This value of P will be found to agree closely with the weight on the end of the string. The value of R may be verified experimentally with a spring-balance as drawn in Fig. 152 *a*.

119. To find the resultant of a number of concurrent forces. In Chapter VIII we have shewn how to find, by the triangle or polygon of forces, the resultant of a number of inclined forces.

It is often more convenient to choose first two directions mutually at right angles and find the components in these directions of each of the concurrent forces. In this way if we begin with 3 or 4 concurrent forces, by resolution of each of them we may have as many as 6 or 8 forces, but all of these now in two directions only. Separating the two sets, we find the resultant of each of them by algebraic addition. This leaves us with two forces at right angles to one another, and these two we

recombine into one single resultant. The following is a numerical example:

A body is under the action of three forces, 8 lbs. wt., 2 lbs. wt., and 6 lbs. wt., meeting at a point O and acting in the directions shown in Fig. 153.

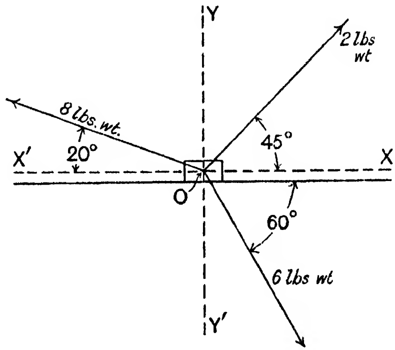


Fig. 153.

To find the resultant, let each force be replaced by its components parallel to OX and OY; the body may then be regarded as in equilibrium under the action of *six* forces, three parallel to OX and three parallel to OY.

The six forces are:

Parallel to OX

$$8 \cos 20° = 7{\cdot}9 \text{ lbs. wt. to the left,}$$
$$2 \cos 45° = 1{\cdot}42 \text{ lbs. wt. to the right,}$$
$$6 \cos 60° = 3{\cdot}0 \text{ lbs. wt. to the right.}$$

Parallel to OY

$$8 \cos 70° = 2{\cdot}74 \text{ lbs. wt. upwards,}$$
$$2 \cos 45° = 1{\cdot}42 \text{ lbs. wt. upwards,}$$
$$6 \cos 30° = 5{\cdot}2 \text{ lbs. wt. downwards.}$$

Adding separately the vertical and horizontal components, we have:

The sum of the components parallel to OX is 3·48 lbs. wt. to the left.

The sum of the components parallel to OY is 1·04 lbs. wt. downwards.

Combining these two components into one resultant, we find from Fig. 154:

The resultant R makes with the line OX' an angle whose tangent is $\frac{1·04}{3·48} = 0·3$, therefore the angle is 16° 40′.

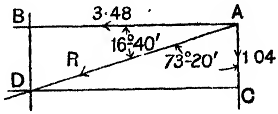


Fig. 154.

$$\therefore R = 1·04 \cos 73° \, 20' = \frac{1·04}{0·287} = 3·63 \text{ lbs. wt.}$$

120. Examples. In this paragraph we give two examples of problems solved by the method of "resolution."

Ex. 1. A uniform bar AB, 3 feet long, weighing 8 lbs., is supported in a horizontal position on a peg at C, 1 foot from B, and by a cord AD which makes an angle of 30° with the vertical; in this position the bar is on the point of slipping. We are to calculate the tension in the cord, and the coefficient of friction for the surfaces of the bar and the peg. (Fig. 155.)

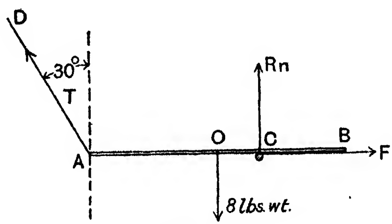


Fig. 155.

The forces which act upon the bar are:

(1) its weight, 8 lbs. vertically downwards at O;

(2) the tension, T lbs. wt., of the cord at A;

(3) the force exerted by the peg at C, R lbs. wt.

Here then we have a body in equilibrium under the action of three forces. We might deduce at once the direction of the force R at C; then solve by the triangle to find the magnitudes of the pull T and of the force R; then resolve R into its vertical and horizontal components to find the force of friction, and the vertical reaction at C, and hence find the coefficient of friction.

It is just as easy to begin by resolving all the forces into their components in two directions. Because, in order to find the coefficient of friction, we must consider the relation of the horizontal force of friction to the vertical component of the thrust of the peg at C, we choose these two directions for resolving.

We do not know the magnitude or direction of the force R, so we call its vertical and horizontal components Rn and F. Both these act through C; Rn is the normal reaction, and F is the force of friction.

Resolving T vertically and horizontally, and calling these components T_v and T_h, we have:

$$T_v = 0{\cdot}866T \text{ and } T_h = 0{\cdot}5T.$$

The weight of the bar, 8 lbs., is already in the vertical direction.

The system of forces now reduces to two,

(1) three parallel vertical forces;

(2) two opposite horizontal forces.

Using the method of Article 71 for parallel forces we have, by moments about A,

$$Rn \times 2 = 8 \times 1{\cdot}5,$$

whence $$Rn = 6 \text{ lbs. wt.};$$

also $$T_v + 6 = 8,$$

$$0{\cdot}866T = 2$$

and $$T = 2{\cdot}31 \text{ lbs. wt.}$$

From the opposite horizontal forces we have:

$$F = T_h = 0{\cdot}5T = 1{\cdot}155 \text{ lbs. wt.}$$

The coefficient of friction $= \dfrac{F}{Rn} = \dfrac{1{\cdot}155}{6} = 0{\cdot}192.$

In an example of this kind we shall be guided by circumstances in our choice of method. If we have a drawing board and instruments handy we may find it quicker to solve by drawing the triangle; if not the method of resolution is probably the quicker.

Many problems can be solved either by the triangle of forces or by resolution; sometimes one method is better, sometimes the other, and often there is little to choose between them. The following example is specially interesting, for though we cannot solve it directly by the force triangle, we can solve it easily by resolution.

Ex. 2. In a particular position of a reciprocating engine the force of the piston-rod on the crosshead is 1000 lbs. wt.; the coefficient of friction between the slide and the slipper is 0·20; the connecting-rod makes an angle of 12 degrees with the centre line of the piston-rod. We are to find the thrust of the connecting-rod on the crosshead. (We have chosen an unusually high coefficient of friction for convenience of numbers.)

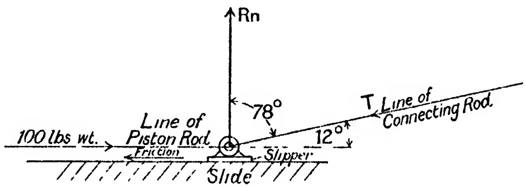


Fig. 156.

The forces acting on the crosshead are three:

(1) The force of the piston-rod, 1000 lbs. wt.

(2) The thrust of the connecting-rod, and of this we know the direction but not the magnitude.

(3) The reaction of the slide on the slipper of the crosshead, and of this we know neither the magnitude nor the direction.

Thus for the triangle of forces we know one side and the direction of another; not enough to determine the triangle and therefore not enough to solve our problem.

The reaction of the slide on the slipper of the crosshead is compound, of two components, (*a*) the normal reaction exclusive of the force of friction, and (*b*) the frictional reaction which is equal to the normal reaction multiplied by the coefficient of friction. We can prove from this that the direction of the total force of the slide on the slipper of the crosshead makes with the normal to the surface of the slide an angle whose tangent is equal to the coefficient of friction; but to prove this we must use the principle of resolution, that is to say the mutual independence of forces at right angles to one another.

Suppose T lbs. wt. is the thrust in the connecting-rod. We resolve this thrust into its components (*a*) parallel to the centre line of the piston-rod, and (*b*) normal to the surface of the slide.

The components are:

$$(a)\ T \cos 12^\circ;$$

$$(b)\ T \cos 78^\circ.$$

Taking the components in these two directions of the reaction of the slide on the slipper, these are (c) the normal reaction which we will call Rn, and (d) the force of friction parallel to the slide which is $0{\cdot}2 \times Rn$.

Of the original three forces we have replaced two by their components parallel or perpendicular to the direction of the piston-rod.

Parallel to the direction of the piston-rod there are:

1. The force of the piston-rod to the right, 1000 lbs. wt.

2. The force of friction to the left, $0{\cdot}2 \times Rn$ lbs. wt.

3. The component of the thrust of the connecting-rod to the left, $T \cos 12°$ lbs. wt.

Perpendicular to the direction of the piston-rod we have:

1. The normal reaction of the slide on the slipper, Rn lbs. wt. upwards.

2. The component of the thrust of the connecting-rod downwards, $T \cos 78°$ lbs. wt.

These two systems of forces are at right angles to one another and independent of one another, therefore

$$Rn = T \cos 78°$$

and
$$1000 = 0{\cdot}2 \times Rn + T \cos 12°.$$

Combining these two equations we have:

$$T(\cos 12° + 0{\cdot}2 \cos 78°) = 1000,$$

$$T(0{\cdot}9781 + 0{\cdot}2 \times 0{\cdot}208) = 1000,$$

$$T = \frac{1000}{1{\cdot}020} = 980 \text{ lbs. wt.}$$

The reader may very likely come across problems to be solved by resolution, in which he has some difficulty in making up his mind what pair of directions to choose for resolving. It is however, at least in real problems, usually a question of a body which is constrained to move in a certain direction though acted upon by forces inclined to this direction. Many of the more difficult examples are so because 'friction' comes into the problem and though a force at right angles to the direction in which a body must move cannot directly affect motion yet indirectly it does through its effect upon the force of friction. This consideration alone would lead us to choose for our directions of resolution, the direction of motion and the perpendicular to this.

EXAMPLES VIII AND IX.

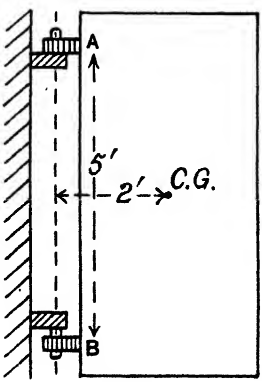
1. A weight is hung from a small ring. The ring is supported by strings attached to two spring-balances, and these strings make angles of 30° and 40° with the vertical. The spring-balance attached to the former string reads 8 lbs. wt., that attached to the latter reads 6·25 lbs. wt. Draw the position-diagram and the force-diagram and from the latter determine the weight of the body.

2. A weight of 10 lbs. is supported by two strings inclined to the vertical. One string is inclined at an angle of 55° to the vertical, the tension in the other is 10 lbs. wt. Find the tension in the former string and the angle of inclination of the latter to the vertical.

3. A piece of fine steel wire is attached to two opposite points on the walls of a room. At first the wire is slack. A weight of 10 lbs. is hung from the middle point of the wire, and then the two halves of the wire are found to be inclined to the horizontal at angles of 5°. Find the tension in the wire.

4. A body rests on a rough horizontal plane. Two horizontal forces act on the body, a force of 10 lbs. wt. towards the North, and a force of 8 lbs. wt. towards the South East. If the body remains at rest, find the force of friction between the plane and the body.

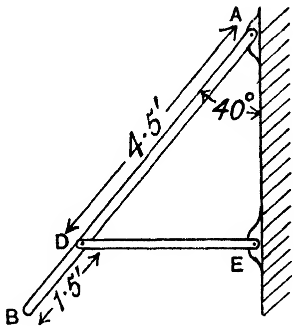
5. A door is supported by two hinges A and B (Fig. (a)), all the weight being taken by the upper hinge. The hinges are 5 feet apart. The weight of the door is 50 lbs. and its C.G. is 2 feet from the vertical line through the hinges. Find the magnitude and direction of the forces exerted by the door on each hinge. Assume that the hinges are frictionless.



(a)

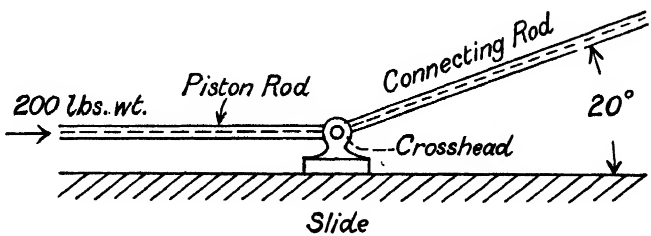
6. A uniform bar AB 6 feet long and weighing 12 lbs. is hinged to a wall at A and kept in position by a light horizontal rod DE which is hinged both at D and E (Fig. (*b*)).

What is the direction of the force exerted by the rod DE on the bar AB? Find, by drawing, the magnitude and direction of the forces on the bar at A and D.



(*b*)

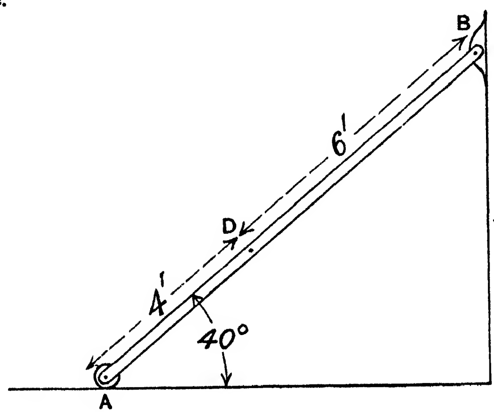
7. In a certain position the piston-rod of a steam-engine exerts on the crosshead a horizontal force of 200 lbs. wt. In this position the slide exerts on the crosshead a force which is vertically upwards. The connecting-rod is inclined to the horizontal at an angle of 20° (Fig. (*c*)). Find the magnitude of the force exerted by the slide on the crosshead and by the connecting-rod on the crosshead.



(*c*)

8. A bar AB 10 feet long weighs 8 lbs.; its C.G. is at D; it is hinged to a vertical wall at B, and rests on a roller on smooth horizontal ground at A (Fig. (*d*)).

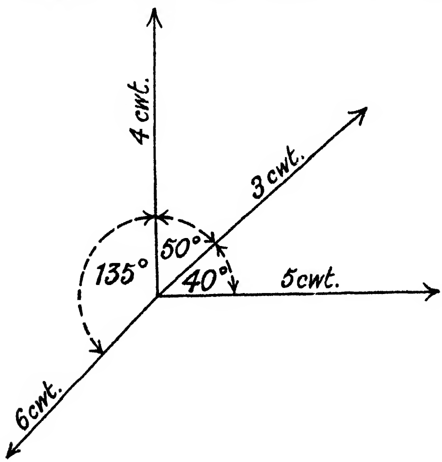
Find the magnitude of the vertical force exerted by the ground on the roller and the magnitude and direction of the force exerted on the rod at B by the hinge.



(*d*)

9. A picture weighs 20 lbs. and is supported by a cord over a hook. The angle between the parts of the cord is 70°. Find the tension in the cord.

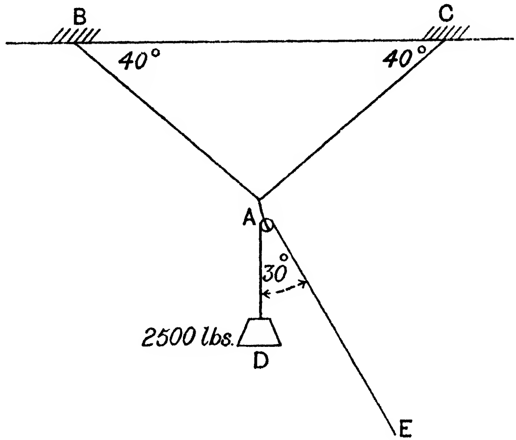
10. Find, by drawing, the resultant of the four forces shewn in Fig. (*e*).



(*e*)

11. A pulley, A, is supported by two steel cables, AB and AC, each secured to holdfasts at B and C, and inclined at 40° to the horizontal. A load of 2500 lbs. is being held as shewn, the other end of the rope E going to a windlass. The angle between the two parts of the rope is 30° (Fig. (f)).

Calculate the tensions in AB and AC.



(f)

12. A weight of 10 lbs. is suspended by two cords which make angles of 20° with the vertical and horizontal respectively. Find the tensions in the cords.

13. Two cords mutually at right angles support a body. The tensions in the cords are 21 lbs. wt. and 28 lbs. wt. respectively. Find the weight of the body which the cords support, and the angles which the cords make with the vertical line.

14. A weight of 50 lbs. is suspended by a wire from a point in the ceiling. A string attached to the weight is pulled horizontally until the wire comes to a line making an angle of 25° with the vertical. Find the tensions of the wire and the string.

15. Two strings at right angles to one another support a weight; the tension in one of the strings is equal to half the weight. Find the direction of this string.

16. The wind exerts on the sails of a boat a pressure which is equivalent to a force of 800 lbs. wt. in a direction which makes an angle of 40° with the fore and aft line of the boat. What is the component of the wind's pressure in the direction of the boat's course? What is the effect of the other component of the wind's pressure, and what is its magnitude in lbs. wt.?

17. A toboggan weighing 20 lbs. is to be pulled up a slope of 1 in 6 (1 up in 6 along the surface of the slope) by a cord parallel to the surface of the slope. Assuming that the force of friction is negligible, find the pull of the cord.

18. The toboggan of Ex. 17 is pulled up the slope by a boy with the cord over his shoulder. The cord makes an angle of 20° with the surface of the slope. Find the tension in the cord.

19. A kite whose weight is 3 lbs. floats in the air with its planes making an angle of 40° with the horizontal. The pressure of the wind on the kite is normal to the surface of the planes and equivalent to a single force of 5 lbs. wt. Find the vertical and horizontal components of the wind's pressure and the magnitude and direction of the pull of the string which secures the kite.

20. The buoyant force of the air upon an observation balloon is 800 lbs. wt. The weight of the balloon with its load is 500 lbs. The balloon is anchored at a height of 600 ft. by a line which is 1000 ft. long and almost straight. Find the pull of the line and the horizontal pressure of the wind on the balloon.

21. A block weighing 60 lbs. can be just moved on a horizontal plane by a horizontal force of 15 lbs. wt. What force acting at 30° to the plane will be required to produce motion?

22. A roller is to be pulled up an inclined plane by means of a handle which is kept parallel to the plane. If the weight of the roller is 224 lbs., and the inclination of the plane to the horizontal is 15°, determine the force, acting parallel to the plane, that must be exerted on the handle. What force does the roller exert on the plane? The friction between the roller and the plane may be neglected.

23. A block of wood weighing 6 lbs. rests on a plane which is inclined to the horizontal at an angle of 25°. What horizontal force must be applied to the block in order to cause it to move directly up the plane (*a*) neglecting friction, (*b*) having given that the coefficient of friction is 0·25?

24. A window is pushed up by a pole inclined at 30° to the vertical. If the force exerted along the pole is 12 lbs., what is the force urging the window up, and what is the force urging it against the sash?

25. A body whose weight is 16 lbs. rests on a plane which is inclined to the horizontal at an angle of 25°. What is the force of friction between the plane and the body? What is the normal component of the force exerted by the plane on the body? What is the total force exerted by the plane on the body?

26. A horse draws a sleigh, which weighs 1500 lbs., up a slope of 10°. The traces are inclined at 15° to the surface of the slope (25° to the horizontal); the pull of the traces is 360 lbs. wt.

Find the component of the pull in the traces,

(*a*) parallel to the slope;

(*b*) perpendicular to the slope;

and calculate

(*c*) the magnitude of the force of friction;

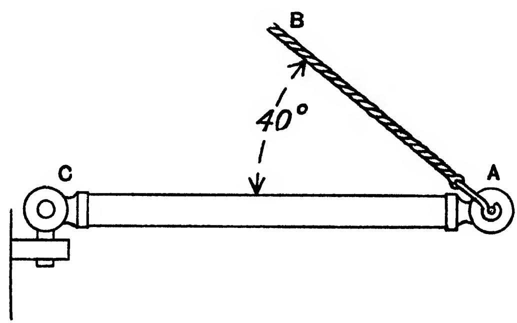
(*d*) the coefficient of friction between the surfaces.

27. Explain briefly how an aeroplane is sustained in the air.

28. Which is the easier, to pull or to push a roller across a soft lawn? Give reasons.

29. Is it easier to push a roller with a long handle or with a short one? Give reasons.

30. A uniform boom *AC* (Fig. (*g*)) weighing 5 cwt. is hinged at *C* and supported at *A* by a rope *AB*. Calculate the tension of the rope.

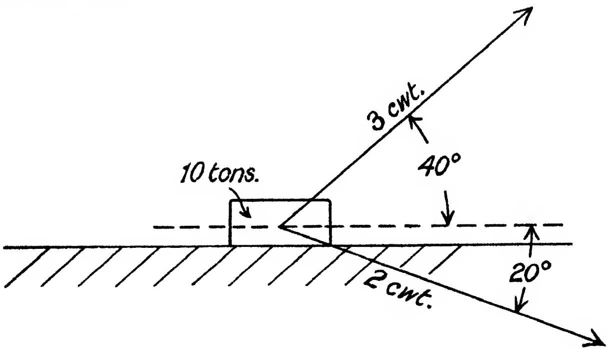


(*g*)

31. A block of metal weighing 10 tons rests on a horizontal plane. Forces of 3 cwt. and 2 cwt. are exerted on the block by ropes whose directions make angles of 40° and 20° with the plane (and with the horizontal axis of

the block) and are just sufficient to keep the block moving with uniform velocity (Fig. (*h*)).

Calculate the force of friction between the plane and the block, and the coefficient of friction.

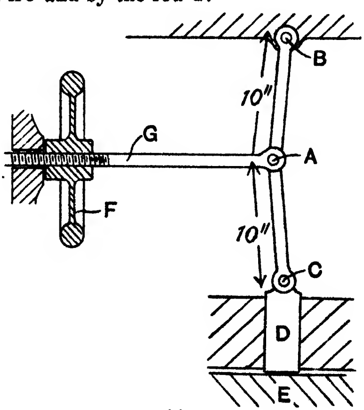


(*h*)

32. Fig. (*i*) illustrates a mechanism for producing great pressure. *B* is a fixed point. Rotation of the wheel *F* causes the screwed rod *G* to move out to the right, and thereby causes the plunger *D* to exert a pressure on *E*.

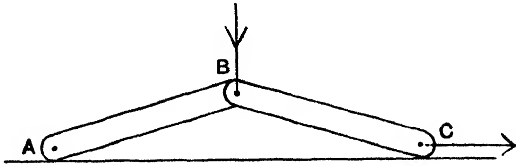
AB and *AC* are each 10 inches long. The centre of *A* is half an inch from the line joining *BC*.

If *D* exerts a force of 30 tons on *E*, calculate the forces exerted on the pin at *A* by the rod *AC* and by the rod *G*.



(*i*)

33. Fig. (*j*) shews a form of toggle joint commonly used in hand printing presses.

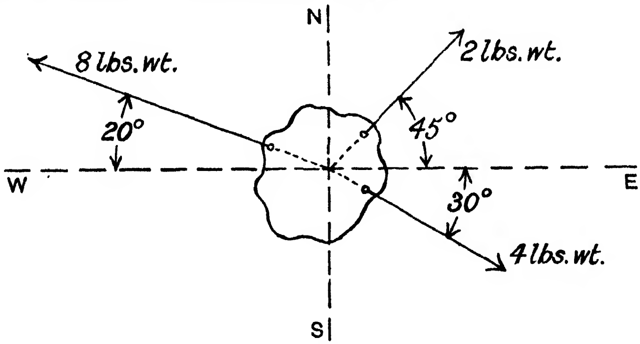


(*j*)

A is fixed. A vertical force applied at *B* causes a thrust at *C* which can only move in a horizontal direction. The link *AB* is the same length as *BC*.

What horizontal force at *C* will be caused by a vertical force of 20 lbs. wt. at *B* (friction in the joints being inappreciable) when the angle *BAC* is 15°? Shew that the force at *C* will become larger and larger as the angle *BAC* becomes smaller and smaller.

34. Forces of 8 lbs. wt., 2 lbs. wt., and 4 lbs. wt., whose directions meet at a point, act upon a body as shewn in Fig. (*k*).



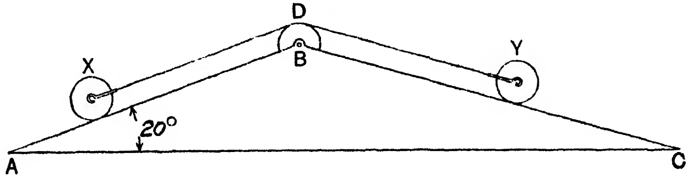
(*k*)

Resolve each of the three forces into its components along the N.S. and E.W. lines and so find the resultant of the three forces both in magnitude and direction.

35. A roller *X* weighing 10 lbs. is placed on a smooth plane *AB* inclined at 20° to the horizontal (Fig. (*l*)).

If the roller *X* is connected by a cord *XDY* passing over a pulley at *D* to a second roller *Y* whose weight is 14 lbs. and which rests on a smooth plane

BC, determine at what angle the plane BC must be inclined for the two rollers to remain at rest. XD is parallel to AB and DY is parallel to BC.



(l)

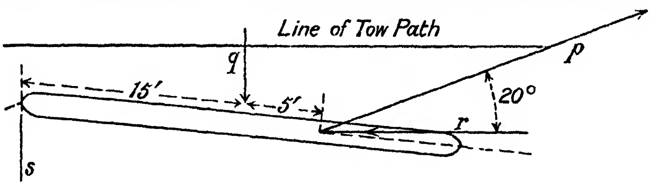
36. A ladder, AB, 30 feet long rests with one end B on rough horizontal ground, and the other end A against a smooth vertical wall. The ladder weighs 80 lbs., its C.G. is at a point C 10 feet from B, and it slopes at an angle of 30° with the vertical wall.

Find, by moments, the force exerted by the vertical wall on the ladder at A, and find, by resolution, the force exerted by the ground on the ladder at B.

37. A man drags behind him a rope 60 feet long. Part of the rope, 42 feet long, is on the ground; the rest hangs in a curve from the man's shoulder to the ground.

If, at the man's shoulder, the rope is inclined at 60° to the horizontal, calculate the value of the coefficient of friction.

38. Fig. (m) represents a barge being towed along a straight canal.



(m)

The forces acting upon the barge are as follows:

p, the tension of the tow-rope.

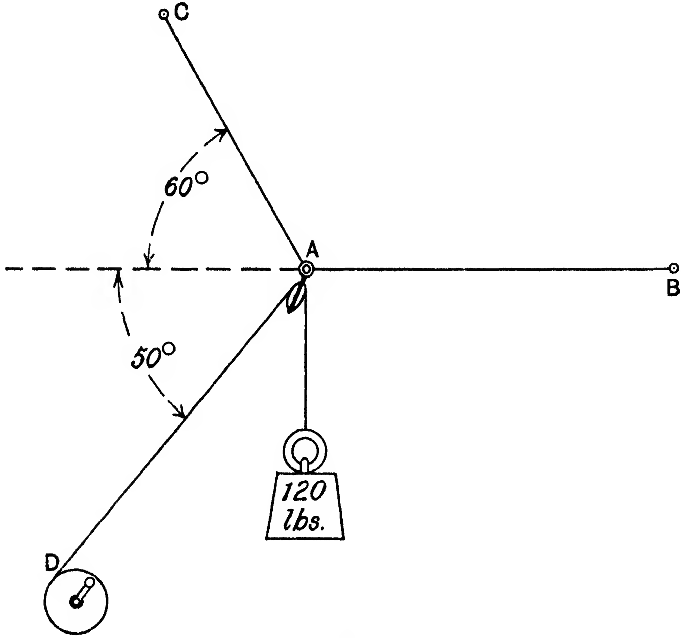
r, the component parallel to the bank of the resistance of the water acting on the barge and its rudder,

q, the component perpendicular to the bank of the resistance of the water acting on the barge,

s, the component perpendicular to the bank of the resistance of the water acting on the rudder.

Suppose p to be 100 lbs. wt., calculate the values of r and s.

39. A pulley block A is supported by two ropes AB and AC (Fig. (*n*)). A rope from a winch at D passes over the pulley and carries a weight of 120 lbs. The pulley at A is supposed to be frictionless.



(*n*)

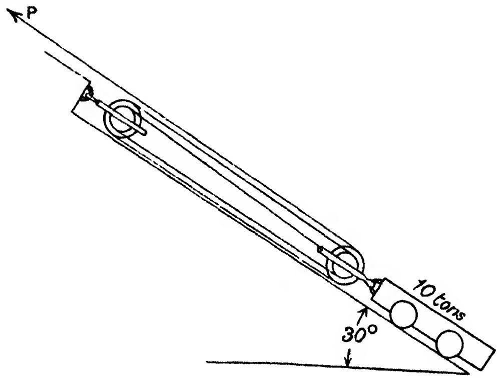
Find the tensions in the supporting ropes AB and AC.

MISCELLANEOUS EXAMPLES

1. Loads are lifted by a crane, the rope passing over a drum 2 ft. diameter. On the drum shaft there is a spur gear with 115 teeth gearing into a pinion of 23 teeth on another shaft. On this shaft is also keyed a gear of 100 teeth meshing with a pinion of 25 teeth on the shaft which carries the handle. The handle is 18 ins. long. An effort of 100 lbs. on the handle raises a weight of one ton uniformly. Find the efficiency of the crane, the velocity-ratio, and the energy wasted in raising a ton through one foot.

2. Fig. (*a*) represents an arrangement for hauling a heavy truck up a slope.

Calculate the pull at P necessary to move the truck, neglecting friction.



(*a*)

3. In a certain machine the velocity-ratio was found to be 40, and the following results were observed:

Weight lifted (W lbs.)......	150	300	450	600
Applied Force (P lbs.)	10·6	17·0	23·5	30

Plot a curve shewing the relation between P and W, and thence find an equation giving P in terms of W. Find the applied force required to lift a load of 370 lbs. and thus calculate (1) the mechanical advantage, (2) the efficiency at this load. Find also the energy lost in overcoming friction while this load is raised 2 feet.

4. What do you mean by the 'Transverse Metacentre' of a ship? Explain why the position of this metacentre, with regard to the Centre of Gravity, is a measure of the stability of the ship for small angles of heel. Why is this not the case for large angles of heel?

What heel will be caused by moving a load of 50 tons athwartships a distance of 10 feet, in the case of a ship displacing 10,000 tons and whose transverse metacentric height is 4 feet?

5. In the tenth book of Vitruvius it is stated that a device for conveying large blocks of stone to their destination was to enclose them in wooden cases

of cylindrical form, so that they could be rolled along the ground by men, or if furnished with pivots at each end, could be drawn by horses.

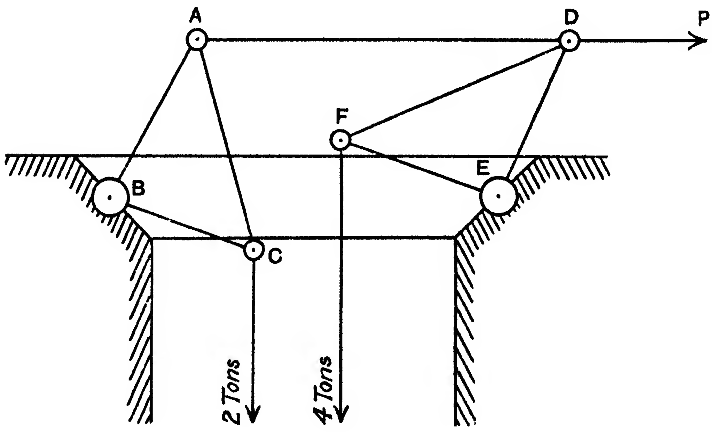
Now, suppose two similar blocks of stone, each 30 feet long, 5 feet wide and 5 feet deep (as used in the Great Pyramid), were enclosed in solid cylindrical wooden cases, one having its sides and the other its ends in the faces of the case: find the horizontal pulls (in tons) which must be applied at the centres of the cases when they meet an obstacle 1 inch high.

The diameters of the cases are 34 and 10 feet. The stone and wood weigh 140 and 50 lbs. per cubic foot respectively.

6. A uniform gate weighs 180 lbs., and its centre of gravity, G, is $3\frac{1}{2}$ feet from the line joining the hinges, which are 3 feet apart. The weight of the gate is entirely supported by the lower hinge. Find, graphically or otherwise, the forces exerted on the gate at the hinges in magnitude and direction.

7. If a rigid body is kept in equilibrium by three forces, what is the relation between the moments of the three forces about any point in their plane?

Fig. (*b*) illustrates the mechanism operating the vertical pump rods of a deep well. For the given load and for the position shewn, determine, by taking moments about B and E successively, the thrust in the coupling-rod AD and the value of the force P necessary to maintain equilibrium. The positions of the points may be obtained by pricking through on to your paper; friction and the weights of the mechanism may be disregarded.



(*b*)

8. For various loads W lifted by a crane the corresponding efforts E applied are as below. If the velocity-ratio is 220, find the law of the machine and draw a load-efficiency curve.

W............tons	1	2	2·5	5	7	10
E............ lbs.	43	58	63	95	120	157

9. A steam launch has a water-plane of 280 sq. ft.

The launch is being hoisted out of the water on a single rope.

Calculate the tension of the rope when the launch has been raised 10 inches from its normal position, assuming the area of the water-plane to remain unchanged.

10. A horizontal shaft carries a flywheel and is turning in bearings which are an easy fit and unworn. Shew that contact occurs along a horizontal line such that the plane through this line and the axis of the shaft is inclined at an angle $\tan^{-1}\mu$ to the vertical, μ being the coefficient of friction between the shaft and bearings. The only load on the shaft is its weight and the weight of the flywheel.

11. The vertical ram of an hydraulic lift is 30 square inches in section. The friction of the guides and leather packing for the ram amounts to a vertical force of $100 + 6p$ lbs., where p is the water-pressure in lbs. per square inch. The total weight of the ram and lift is 1900 lbs., and of this, 1000 lbs. is balanced by weights hung over frictionless pulley wheels.

If a weight of 11,000 lbs. is placed in the lift, calculate the least water-pressure which will cause it to ascend, and determine the efficiency of this mechanism, regarded as a means of raising the 11,000 lbs. load.

12. A sailing cutter has a dead weight of 1 ton. Taking the pressure of the wind on the cutter as equivalent to a horizontal force of 200 lbs. weight making an angle of 40° with the fore and aft line, and acting 14 feet above the centre of buoyancy when heeled, find the angle of heel produced. The cutter's transverse metacentric height is 3 feet and you may assume that the direction of the resultant thrust of the water on the cutter passes through the centre of buoyancy.

13. A piece of metal is being ground down on the rim of a carborundum wheel of diameter 8 inches.

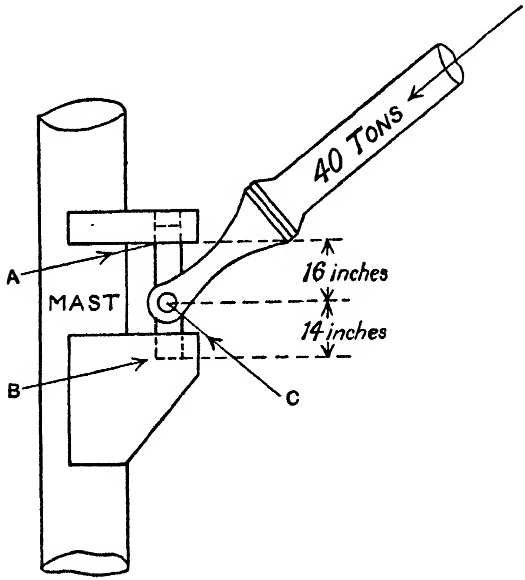
Determine the heat generated per minute if the metal is pressed up against the wheel with a radial force of 5 lbs. weight, the speed of the wheel being 2500 revs. per minute and the tangential force on the wheel being assumed to be half the radial force.

[1 B.Th.U. is equivalent to 778 foot lbs.]

14. A light bar ABC is pivoted at its centre B and carries a 50 lb. weight at A, being kept at an angle of 30° with the horizontal by a horizontal force P at C; find the value of this force. What is the reaction at the pivot B in magnitude and direction?

15. Comment on the statement made by the Greek mathematician Archimedes—"Give me a fulcrum and I will move the earth."

16. Fig. (c) represents the coupling by means of which a derrick is attached to a mast. A pin C passes through the jaws of the derrick and through a vertical spindle AB, which fits into brackets secured to the mast.



(c)

The derrick is inclined at 40° to the horizontal and a thrust of 40 tons acts along its axis. Find the reactions of the brackets on the spindle, assuming that they act at the points A and B, and that the direction of the reaction at A is horizontal. The diameter of the spindle is 6 inches.

17. The Roman steelyard consists of a lever supported at C, the weight W to be measured being suspended at D. The position of the constant weight w on the graduated bar AC gives the required weight. The steelyard

itself weighs 5 lbs., its centre of gravity being at G. $CG=4$ ins., $CD=8$ ins. and $CA=30$ ins. w weighs 8 lbs.

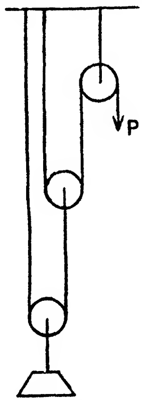
Find (*a*) The greatest weight that the steelyard can weigh.

(*b*) The point along CA that should be graduated zero, *i.e.*, the position on the bar CA that w will occupy when there is no weight at D.

18. A ship weighs 10,000 tons. A weight of 5 tons (originally on board) is shifted 200 feet aft along the fore and aft line, and then 20 feet vertically downwards. Find the change in the position of the centre of gravity of the ship.

Explain each step in your working.

19. Derive a relationship between the 'mechanical advantage,' the velocity-ratio' and the 'efficiency' of a machine.



(*d*)

Find the velocity-ratio of the system of pulleys shewn in Fig. (*d*).

If a load of $\frac{1}{2}$ ton was just moved by an effort of 3 cwt., what would be the efficiency of the machine in this particular case?

20. From mechanical principles explain

(1) why a horse zig-zags when dragging a heavy load up a hill;

(2) what is the force that propels a rowing boat.

In a sculling boat the length of the sculls is 6 ft. from blade to button and 3 ft. from button to hand-grip.

If an oarsman exerts a force of 16 lbs. weight with each hand when the oar is at right angles to the direction of the boat, determine

(*a*) the thrust on the rowlocks;

(*b*) the total propulsive force on the boat.

21. A ship of 1000 tons heels through 10°. Find the righting moment if the distance between the centre of gravity and transverse metacentre is 4 feet 6 inches, indicating by a sketch how the forces act.

22. A vertical circular boiler, 5 feet in diameter, has its centre of gravity in the axis of figure and stands unsecured on a horizontal bed-plate. In order to remove the boiler it is necessary to turn it so that its axis is horizontal, and for this purpose a rope is to be secured to the boiler and pulled horizontally. If the coefficient of friction between the boiler and bed-plate is ·4, find the least height at which the rope should be secured in order that the boiler may just turn over without sliding.

23. A ship has a displacement tonnage of 12,000 tons, and her water-plane area is 18,000 square feet.

Calculate how much the ship sinks in passing from sea-water, whose density is 64 lbs. per cubic foot, to fresh water, whose density is 62·4 lbs. per cubic foot.

24. Explain what you mean by—

(*a*) the velocity-ratio of a machine;

(*b*) the mechanical advantage of a machine.

Describe some form of differential pulley block with which you are acquainted, and give directions for calculating from the dimensions its velocity-ratio and its efficiency at some assumed load.

25. A load of 1 ton is carried by a tackle from the top of a derrick 20 feet long. The rope from the tackle is led through a snatch-block which is made fast to the foot of the derrick. The top of the derrick overhangs the foot by 6 feet. The back-guy is secured to a holdfast 40 feet horizontally behind the foot of the derrick. A pull of 8 cwt. on the rope of the tackle raises the load of 1 ton.

Find the stresses in the back-guy and in the derrick, neglecting the weight of the derrick.

26. Part of the trunk of a tree is 16 feet long and rests on two supports. The support near the thin end of the trunk is 3 feet from that end; the other support is 5 feet from the thick end.

A force of 200 lbs. wt. is required to lift the thin end of the trunk. A force of 300 lbs. wt. is required to lift the thick end.

Find the weight of the trunk.

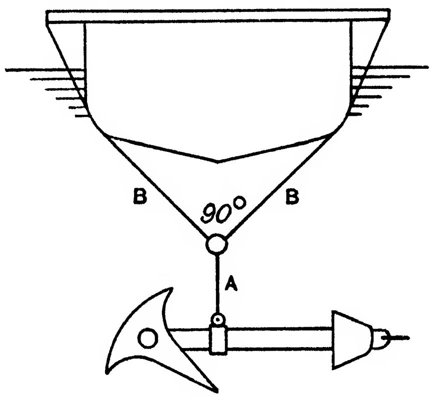
27. Explain, with a diagram, what you mean by the 'Metacentre' and what by the 'Metacentric Height' of a ship.

Shew that of two vessels otherwise alike, that with the greater metacentric height will be the more stable.

AB and BC are two spars held together by a tie-rod AC. The structure is supported at A and C. B is vertically above AC and carries a load of 200 lbs.

Find the supporting forces at A and C, and the tension in the tie-rod AC.

28. In order to lay out a sheet anchor weighing 105 cwt., it is suspended under a boat by means of a wire sling in the manner shewn below. The anchor is of wrought iron, weighing 480 lbs. per cubic foot (Fig. (*e*)). Find in cwts. the tensions in the parts A, B of the sling.



(*e*)

If the water-plane area of the boat is 360 square feet, how many inches will the boat rise after the anchor has been released?

29. For the purpose of raising loaded and lowering empty trucks, two parallel lines of rails are laid on a plane inclined at 10° to the horizontal. At the top of the plane is a pulley, 5 feet in diameter, round which passes a rope which, running parallel to the plane, has its ends fastened to the loaded and empty trucks. The shaft of the pulley is geared up with a steam-engine.

Suppose that five loaded trucks of total weight 8 tons are being raised and five empty ones of total weight 1 ton lowered at a steady speed of 3 miles per hour, and that the frictional resistance is 10 lbs. wt. per ton.

Find the tensions in the parts of the ropes that come from the trucks, and hence the turning moment that must be applied to the shaft to keep the motion steady.

If the efficiency of the gearing is 80 °/₀, find the B.H.P. of the engine.

30. The two upper wheels of a 'Weston' differential purchase have 13 and 12 teeth respectively. Calculate the velocity-ratio of lifting. If the efficiency of the purchase is 35 %, find what effort is necessary to raise half a ton.

31. The crank of a reciprocating engine is 1 foot long and the connecting-rod is 5 feet long; the connecting-rod makes an angle of 10 degrees with the centre line of the engine and the crank an angle of 60 deg. 13 min. with the same line, the piston being on the out-stroke and before mid-stroke.

Calculate the turning moment on the shaft if the piston-rod is subject to a compressive stress of 6 tons weight.

ANSWERS TO EXAMPLES

CHAPTER I.

1. $2\frac{2}{3}$ in. **2.** $14\frac{2}{3}$ lbs. wt.
4. (*a*) 3·1 in.; (*b*) 3·8 in.; (*c*) 8·35 in. **5.** 6 in.; 7·2 in.
6. Two equal and opposite forces of 6 lbs. wt., one exerted by the load, the other by the fixed support.
7. 12 stone. **8.** 1 cwt. **10.** 5 cwt., 5 cwt.
11. Balance reads 5 lbs. wt. **12.** 2 lbs. wt.; 2 lbs. wt.
13. 105 lbs. wt.; 105 lbs. wt. **16.** 8 lbs. wt.; 15 lbs. wt.; 11 lbs. wt.
17. (i) 1·5 lbs. wt. in such a direction as to oppose any motion; (ii) 2·8 lbs. wt.; (iii) left 10·3 lbs. wt., right 4·7 lbs. wt.; (iv) 2·8 lbs. wt. in each case.
18. $\frac{1}{2}$ lb. wt. **22.** 11 lbs. wt.
25. 8 lbs. wt.; 8 lbs. wt.; two forces, each of 8 lbs. wt., acting inwards.
26. Two forces, acting inwards, each of 80 lbs. wt.
27. 10 lbs. wt.; 6 lbs. wt.
28. 60 lbs. wt. exerted by the pole, and 60 lbs. wt. by the stake. 60 lbs. wt. on the pole, and 60 lbs. wt. on the stake.
29. 300 lbs. wt.; 300 lbs. wt.; 312 lbs. wt.

CHAPTER II.

1. 400 ft. lbs. **2.** 600 ft. tons. **3.** 4480 ft. lbs., or 2 ft. tons.
4. $221\frac{2}{3}$ ft. lbs. **5.** $\frac{3}{4}$ ton wt. **6.** $3\frac{4}{7}$ ft. **7.** $3\frac{1}{5}$ ft.
8. 5 stone. **9.** 660 ft. lbs. **10.** 475,200 ft. lbs., or 212 ft. tons.
11. 5200 ft. lbs.; 1000 ft. lbs. **12.** 1500 ft. tons.
13. 33,264 ft. lbs.; or 28·8 ft. tons. **14.** $8\frac{1}{4}$ lbs. wt.
15. 13,200 ft. lbs. **16.** 270 ft. lbs. **17.** 2800 ft. lbs.; or $1\frac{1}{4}$ ft. tons.
18. 30,800 ft. lbs. **19.** 7333 ft. lbs. **20.** 6114 ft. lbs.
21. 2016 ft. lbs.; 16·8 lbs. wt. **22.** 900 ft. lbs.; 14,230 ft. lbs.
23. 14·4 lbs. wt. **24.** 28 ft. lbs.; 28 lbs. wt.
25. 5 lbs. wt.; 2·5 lbs. wt. **26.** The slope is 3 in 10.
27. 1509 ft. lbs.; $17\frac{1}{7}$ lbs. wt. **28.** 1120 ft. lbs. **29.** 52 lbs. wt.
30. 2200 ft. lbs.; $23\frac{1}{3}$ lbs. wt. **31.** 388·8 ft. lbs.; 39·5 lbs. wt.
32. 720 ft. lbs.; 45 lbs. wt. **33.** 320 lbs. wt. **34.** $8\frac{1}{3}$; ·417.
35. ·606; ·720. **36.** 20; $8\frac{1}{3}$; ·417. **37.** ·347; ·388; ·471. **38.** 6.
39. 5760 lbs. wt. **40.** 21 lbs. wt.; 11·9. **41.** 11·2; ·417.

42. 1500 ft. lbs.; 940 ft. lbs. **43.** 1·58; ·791; 2 lbs. wt.
44. 22 lbs. wt.; 1232 ft. lbs. **45.** 3; 5; 6; 3.
46. 90 lbs.; 150 lbs.; 180 lbs.; 90 lbs.
47. 62·5 lbs. wt.; 104·2 lbs. wt.; 62·5 lbs. wt.; 52·1 lbs. wt. **50.** 720.
51. $\frac{1}{6}$. **52.** 70; $\frac{1}{5}$. **53.** 50. **56.** 7 lbs. wt.; 11 lbs. wt.
57. 6·6; 6·05. **58.** 142·2. **59.** $5W = 33P$.
60. 10,180 lbs. wt.; or $4\frac{1}{2}$ tons wt. (approx.).

CHAPTER III.

5. 15; 7 in.; 528. **6.** 11·1 in. **7.** 31·8 lbs. wt.; 6947 ft. lbs.
9. 2682 lbs. wt. **10.** 16,090 lbs. wt.; or 7·2 tons wt. (approx.).
11. 128 lbs. wt. **12.** 16·8 lbs. wt.; 792 ft. lbs.
13. Friction effect = 1·08 lbs. wt. **14.** 24. **15.** vel. ratio = 270.
16. 25·2; 576 to 121; ·934. **17.** 18·9.
18. $P = 5 + \cdot 054W$ lbs. wt.; 96 lbs. wt. (approx.).
19. $P = \cdot 5 + \cdot 18W$ lbs. wt. **20.** $P = \cdot 5 + \cdot 008W$ lbs. wt.; 18·4 lbs. wt.
21. $P = 5 \cdot 2 + \cdot 06W$ lbs. wt. **22.** Friction effect $= 3 \cdot 3 + \cdot 17W$ lbs. wt.
23. No. The efficiency tends to a maximum value of about ·25.
24. 22·4 lbs. wt. **26.** 10° (approx.).

CHAPTER IV.

1. 1 lb. wt.; 1 lb. wt.; 8 lbs. wt.; 2·8 lbs. wt. **3.** 45·5 lbs. wt.; ·337.
4. 3 lbs. wt.; 1 lb. wt.; 4 lbs. wt.; $\frac{4}{3}$ lbs. wt.; 12 lbs. wt. **5.** No.
6. 4·5 lbs. wt. **7.** ·269. **8.** 224 lbs. wt.; 4480 ft. lbs.
9. 12 lbs. wt. **11.** 11·2 lbs. wt.; 112 ft. lbs., and 280 ft. lbs.
12. ·6 lbs. wt. **13.** 132 lbs. wt. **14.** 7·89 lbs. wt.
15. 44·8 lbs. wt. **17.** 9·8 lbs. wt. **18.** 25 lbs. wt. **19.** ·5.
20. ·96 ton; 288 ft. tons. **21.** 44·8 lbs. wt.
22. $3\frac{3}{4}$ cwt., or 672 lbs. wt.; ·1. **23.** 15·2 cwt. **24.** 1299 lbs. wt.
25. 5488 ft. lbs. **26.** 280 lbs. **27.** 11·1 lbs. wt. **28.** 5 tons wt.
29. 1064 tons. **30.** 6·86 lbs. wt.; 1·25 lbs. wt.
31. 6 lbs. wt.; 4 lbs. wt. **32.** 4·4 lbs. wt.; 2·4 lbs. wt.; 3·6 lbs. wt.
33. 99,000 ft. lbs. **34.** 64 lbs. wt.; 57,600 ft. lbs.
35. 2·1 lbs. wt.; 5·1 lbs. wt.
36. Between the hands and the outside books; $16\frac{2}{3}$ lbs. wt.
37. 43·2 lbs. wt.; 25,920 ft. lbs. **38.** 2095 ft. lbs. **39.** 739·2 ft. lbs.
40. 18·85 B.T.U.'s. **41.** 622 B.T.U.'s. **42.** $\mu = \cdot 192$.
43. 7·5 lbs. wt. **45.** 1182 ft. lbs.
46. 23·57 ft. lbs. **47.** 16 lbs. wt.; $53\frac{1}{3}$ lbs. wt.
48. $\mu = \cdot 178$.

CHAPTER V.

1. 20 lbs. wt. **2.** 10 lbs. wt. **3.** 9·6 lbs. wt.
4. 90 lbs. in.; 68·94 lbs. in. **5.** $13\frac{1}{2}$ in.
6. 12·6 stone; 29·6 stone. **7.** 120 lbs. wt. **8.** 19·69 lbs. wt.
9. $4\frac{2}{7}$ ft. **10.** 1080 lbs. in. **11.** 13·5 lbs. ft. **12.** 36·9 lbs. wt.
13. 20 lbs. wt. **15.** 192 lbs. **17.** 30·5 lbs. wt.
18. 5 ft. from the centre; 19 lbs. wt. **19.** 6 in.; 10 lbs. wt.; 2 lbs. wt.
20. $2\frac{3}{4}$ lbs. wt.; $4\frac{1}{4}$ lbs. wt. **21.** 18·4 lbs. wt. **22.** 64 lbs. wt.
23. 143 stone; 156 stone. **24.** 9·77 lbs. wt.
26. 18·2 lbs. wt.; 13·8 lbs. wt. **27.** $4\frac{3}{11}$ in. from one end.
30. 270 lbs. wt.; $38\frac{2}{11}$ lbs. wt. per sq. in. **31.** 61·4 lbs. wt.
33. 82·58 lbs. wt. **35.** 35 lbs. wt.; 38·9 lbs. wt.
36. 2547 lbs. in.; 2065 lbs. in. **37.** 8·81 lbs. wt.
38. 34 8 lbs. wt. **39.** 13 lbs. wt.; 22·52 lbs. wt. **40.** 22·9 lbs. wt.
41. $3\frac{3}{8}$ in. **42.** 80 lbs. wt. **43.** The ratio of the radii is 7 : 2.
44. 140 lbs. in.; 70 lbs. wt. **45.** 84 lbs. wt. **46.** 43·5 lbs. wt.
47. 1495 lbs. wt. **48.** 12 lbs. wt. **49.** 63 lbs. wt.
50. 430 lbs. wt. **51.** 20 oz. wt.; 42 oz. wt.; 58 oz. wt.
53. 18 lbs.; 27 lbs. **54.** 90 lbs. wt.; no; 112·5 lbs. wt. and 67·5 lbs. wt.
55. Left, 11·6 cwt.; right, 8·4 cwt. **56.** 4 ft.
57. 19 cwt.; 5·9 ft. from the left-hand end; right 8 cwt., left 11 cwt.
58. 4·17 tons wt.; 4·83 tons wt. **59.** $213\frac{1}{3}$ lbs. wt.; $123\frac{1}{3}$ lbs. wt.
60. 22 lbs. wt.; 4·14 feet. **61.** 158·1 lbs. wt.; 31·9 lbs. wt.
62. 80 lbs. wt.; 100 lbs. wt. **63.** 4·964 tons wt.; 4·536 tons wt.
64. 32 lbs. wt.; 124 lbs. wt. **65.** 9 lbs. wt.; 7 lbs. wt.; 2 ft.
66. 3 ft. from one end; 200 lbs. wt.; 100 lbs. wt.
67. 1047 lbs. wt.; 392·7 lbs. wt. **68.** $22\frac{6}{7}$ lbs. wt.
69. 28·92 lbs. wt.; 14·1 lbs. wt. **70.** 84·98 lbs. wt. **71.** $\frac{5}{7}$ lbs. wt.
72. 50 cwt. **73.** 1·414 cwt. in each rope. **76.** 30·9 lbs. wt.
77. 3·59 cwt. **79.** 12 lbs. wt.; 7·5 in. **81.** $213\frac{1}{3}$ lbs. wt.
82. $26\frac{2}{3}$ lbs. wt. **83.** 277·1 lbs. wt.; 368·3 lbs. wt.
85. 24·5 lbs. wt. **86.** 192 lbs. wt. **87.** 32 lbs. wt.

CHAPTER VI.

7. 5 in.; 3 in. **9.** $2\frac{2}{3}$ in. **13.** 11 in. **15.** 3·34 in.
18. 4 in. from the 3 lb. weight. **19.** 9 in. **20.** 95 cms.
21. 1440 lbs. wt.; 800 lbs. wt. **22.** 1·2 ft. **23.** 6·53 ft.
24. $\frac{5}{9}$ of the length from the 20 lb. weight. **25.** 5·68 in. **26.** 4·92 in.
27. 1·06 in. **28.** ·71 in. **29.** 14·95 in. **30.** 2·32 in.
31. 3·74 ft. **32.** 112 lbs. wt.; $11\frac{3}{4}$ ft. **33.** 14 in. **34.** $4\frac{8}{9}$ in.

35. 8 in. from B. **36.** $5\frac{1}{4}$ in. from A. **37.** 7·64 in. from A.
38. $11\frac{1}{7}$ in. **39.** $5\frac{5}{8}$ in. **40.** $4\frac{2}{3}$ in. **41.** 1·69 ft.; 3 ft. 1 in.
42. $6\frac{1}{3}$ in.; $10\frac{1}{3}$ in. **43.** $11\frac{1}{4}$ in. **44.** 8 in.; 4 in.
45. $11\frac{2}{3}$ in.; 10 in. **46.** 3 in. from AB; 10 in. from BC.
47. $1\frac{6}{7}$ in. from AB; $3\frac{6}{7}$ in. from BC. **48.** 2·5 in. from the corner.
49. $7\frac{1}{2}$ in. from AB; $6\frac{3}{4}$ in. from BC. **50.** $2\frac{1}{3}$ ft.; 7 ft.
51. 4 in.; 2 in. **52.** 2 in.; 3 in. **53.** $7\frac{1}{3}$ in. from A. **54.** 21·2 ft.
55. $7\frac{7}{8}$ in. **56.** 4·33 in. from the base. **57.** 7·92 in.
58. 6·2 ft. from the mast; 9·6 ft. from the deck. **59.** 2·2 ft.
60. ·35 ft. **61.** Vertical = ·31 ft., horizontal = 1·15 ft.
62. 10 in. from A. **63.** 36 in. from the centre. **64.** 5·12 in.
65. 15·5 ft. above the keel. **66.** 2·4 ft.
67. 5·6 in. from AB; 5·8 in. from AC. **68.** ·19 in.
69. 1·2 in. from the centre. **70.** $1\frac{1}{3}$ ft. **71.** ·35 ft. aft.
72. ·6 ft. **73.** $\frac{1}{15}$ ft.

MISCELLANEOUS EXAMPLES AT END OF CHAPTER VI.

75. 6·76 in. **76.** 1 lb.; $\frac{1}{2}$ lb.; $1\frac{1}{2}$ lbs.; 707′. **77.** 135·7 lbs. wt.
78. $\frac{2}{7}$ ft. lower; 18·6 ft. **79.** $1\frac{3}{4}$ ft. **81.** 5·74 ft. **82.** 8·13 cms.
85. 1 ton. **86.** Vertical = ·52 ft., horizontal = ·39 ft.
87. $656\frac{2}{3}$ lbs. wt.; $493\frac{1}{3}$ lbs. wt. **88.** $3\frac{1}{7}$ in.

CHAPTER VII.

13. ·68 lbs. ft. **14.** ·52 lb. ft.; 3 lbs. wt. **17.** 16° 16′.
19. 12·86 lbs. wt. **20.** 12 lbs. in. **25.** 2 lbs. wt.
26. 100 lbs. wt. **27.** $2\frac{2}{3}$ ft. **28.** 5 ft. **29.** $1\frac{1}{2}$ lbs. wt.; 36° 52′.
30. 20° 33′. **31.** (*a*) 11·12 lbs. wt.; (*b*) 39·2 lbs. wt. **32.** 7·8 lbs. wt.
33. 288 lbs. wt.; $6\frac{3}{7}$ in. **35.** 16·2 in. **36.** 1·48 ft.
37. 90 lbs. wt. **38.** 1 in. from the base. **39.** 7200 lbs. wt.
40. 3·74 ft.; 18° 28′; 721·9 ft. **41.** 45°; no.
42. One quarter of the lower book, and three quarters of the upper project over the edge of the table.
43. 16 books. **44.** 1·83 lbs. wt.; 6·83 lbs. wt. **45.** 45.
46. Slide; 14 in. **47.** ·9 ft. from the edge of the block. **48.** ·5.
51. ·011 cm. **52.** ·149 in. **53.** 448 lbs. wt.
54. Its own weight, 2000 tons wt.; a force equal to the weight of the displaced water, acting upwards.
61. 5·9 tons ft. **62.** 5·5 ft.
68. The increase in the metacentric height is 8·52 in. **69.** 19° 34′.
71. ·2 ft. **72.** 3·47 ft. **73.** 1·07 ft. **74.** ·9 in.; 2° 32′.

CHAPTERS VIII AND IX.

1. 11·5 lbs. wt. **2.** 11·5 lbs. wt.; 70°. **3.** 57·4 lbs. wt.
4. 7·1 lbs. wt.; S. 52·5° W.
5. 53·9 lbs. wt. at 21° 50′ with vertical; 20 lbs. wt. horizontal.
6. 13·75 lbs. wt. at 29° 15′ with vertical; 6·7 lbs. wt.
7. 72·8 lbs. wt.; 213 lbs. wt. **8.** 4·8 lbs. wt.; 3·2 lbs. wt. vertical.
9. 12·2 lbs. wt. **10.** 3·5 lbs. wt. at 29° with the force of 5 cwt.
11. AB 4440 lbs. wt.; AC 2810 lbs. wt. **12.** 9·4 lbs. wt.; 3·42 lbs. wt.
13. 35·1 lbs. wt.; 53°; 37°. **14.** 55·2 lbs. wt.; 23·3 lbs. wt.
15. 60° with vertical. **16.** 613 lbs. wt.; 514 lbs. wt.
17. 3·7 lbs. wt. **18.** 3·9 lbs. wt.
19. 3·83 lbs. wt.; 3·21 lbs. wt.; 3·32 lbs. wt. at 14·5° with the horizontal.
20. 500 lbs. wt.; 400 lbs. wt.
21. A pull of 15·14 lbs. wt., or a push of 20·24 lbs. wt.
22. 58 lbs. wt.; 216 lbs. wt. **23.** (*a*) 2·8 lbs. wt.; (*b*) 4·86 lbs. wt.
24. 10·4 lbs. wt.; 6 lbs. wt.
25. 6·76 lbs. wt.; 14·5 lbs. wt.; 16 lbs. wt. vertical.
26. 348 lbs. wt.; 93 lbs. wt.; 88 lbs. wt.; 0·063. **30.** 3·9 cwt.
31. 4·18 cwt.; 0·021. **32.** 30·03 tons wt.; 3·03 tons wt.
33. 37·31 lbs. wt. **34.** 34·2 lbs. wt.; N. 51° W. **35.** 14° 9′.
36. 15·4 lbs. wt.; 81·5 lbs. wt. at 11° with vertical. **37.** 0·248.
38. $r \equiv 94$ lbs. wt.; $s \equiv 11{\cdot}4$ lbs. wt. **39.** 199 lbs. wt.; 245 lbs. wt.

ANSWERS TO MISCELLANEOUS EXAMPLES.

1. 74·7 %; 30 : 1; 760 ft. lbs. **2.** 1 ton wt.
3. $P = 4 + 0{\cdot}043W$; 20 lbs. wt.; 18·5 : 1; 46·2 %; 863 ft. lbs.
4. 43′. **5.** 13·4 tons wt.; 15·3 tons wt.
6. 210 lbs. wt. horizontal; 278 lbs. wt.; 40½° with horizontal.
7. 3·8 tons wt.; 4·6 tons wt. **8.** $E = 32 + 12{\cdot}6W$.
9. 6·7 tons wt. **11.** 500 lbs. per sq. in.; 73·4 %. **12.** 15°.
13. 16·9 B.T.U. **14.** 86·7 lbs. wt.; 100 lbs. wt. at 30° with horizontal.
16. 12·1 tons wt. horizontal; 31·8 tons wt. at 36½° with vertical.
17. 27½ lbs.; 2½ in. left of C. **18.** 0·1 foot aft; 0·01 foot down.
19. 4 : 1; 83·3 %. **20.** (*a*) 24 lbs. wt. each; (*b*) 16 lbs. wt.
21. 793 tons ft. **22.** 6·25 ft. **23.** 9·64 ins.
25. 7·83 cwt. tension, 32·2 cwt. compression. **26.** 763 lbs. wt.
28. 91 cwt.; 64·3 cwt.; 5·3 in. **29.** 4530 lbs. ft.; 18·1 H.P.
30. 26 : 1; 123 lbs. wt. **31.** 5·74 tons ft.

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